Kinematics Tips & Tricks

https://qilinxue.github.io/physics-problems/Kinematics/Standalone.pdf

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Contents

Chapter 1

Kinematics

(To view the full document with answers, go to https://qilinxue.github.io/physics-problems/Kinematics/Standalone.pdf)

Kinematics is the study of the motion of objects.

In this chapter, we will be using the constants x, v, a to represent position, velocity, and acceleration, respectively. Unless otherwise stated, subscripts "0 *and* f" will be given to represent the original and final value of the variable. (e.g. v_0 is the starting velocity and v_f is the final velocity)

It is important to know that the problems we will be dealing with have *uniform accelerated motion*, meaning the acceleration does not change.

1.1 Equations

1.1.1 Uniform Accelerated Motion

The four elementary kinematic equations are:

$$\Delta x = (\frac{v_o + v_f}{2})t \tag{1.1}$$

$$v_f = v_0 + at \tag{1.2}$$

$$\Delta x = v_0 t + \frac{1}{2}at^2 \tag{1.3}$$

$$v_f^2 = v_0^2 + 2a\Delta x \tag{1.4}$$

You should know these equations by heart. To strengthen your understanding of them, attempt to do an unit analysis. If you're feeling extra brave, attempt to derive these as well.

1.1.2 Calculus

Acceleration is the time derivative of velocity and velocity is the time derivative of position, as shown by the following equations:

$$\Delta x = \int dx = \int_{t_1}^{t_2} v(t)dt \tag{1.5}$$

$$\Delta v = \int dv = \int_{t_1}^{t_2} a(t)dt \tag{1.6}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} \tag{1.7}$$

Vector algebra also tells us:

$$\begin{aligned} \vec{v}_{p\ relative\ to\ b} &= \vec{v}_{p\ relative\ to\ a} + \vec{v}_{a\ relative\ to\ b} \\ \vec{a}_{p\ relative\ to\ b} &= \vec{a}_{p\ relative\ to\ a} + \vec{a}_{a\ relative\ to\ b} \end{aligned} \tag{1.8}$$

$$\Delta r = \int dv$$

1.2 Tricks

1.2.1 General Steps

The most difficult part about kinematics is collecting the information, and interpreting the question correctly. Here are the general steps to solve any kinematic problem:

1. List down all the variables you are given, and their respective values.
2. Determine and list the unknown variable you want to solve for.
3. Track down the appropriate equation that involves only the variables you listed above.
4. Substitute in the values, and solve for the unknown.
5. Double check your work. Do an unit analysis and check if it makes sense.

1.2.2 Free-Fall Problems

One of the most common types of kinematic problems involve objects falling only under the force of gravity (air resistance ignored), such that their acceleration is g. The following tricks are useful in solving this type of problem:

- At the peak of any trajectory, the object's y-component of velocity is zero. This is often used to calculate the peak height (by setting the velocity equal to zero).

- Because of conservation of energy, and as reflected mathematically in the symmetry of parabolas, an object's *speed* as it passes a certain height is exactly the same on the way up as it is on the way down.
- Also because of the symmetry of parabolas, the magnitude of the time difference between when the object is at the peak of its trajectory (t_{peak}) and when it is at a particularly y-position below its peak is the same whether the object is ascending or descending at the latter position.
- You should memorize $g = 9.8\frac{m}{s^2}$ however for the purposes of this guide, you can use $g = 10\frac{m}{s^2}$. You can set g to be negative or positive, as long as you keep your positive direction constant throughout each question!

1.3 Problems

Regular

Exercise 1.3.1 Do a dimensional analysis on the four elementary kinematic equations presented in 1.1 (View Solution)

Exercise 1.3.2 An object slides off a roof 10 meters above the ground with an initial horizontal speed of 5 meters per second. What is the time between the object's leaving the roof and hitting the ground? (View Solution)

Exercise 1.3.3 Derive $v_f^2 = v_0^2 + 2a\Delta x$ (View Solution)

Exercise 1.3.4 Justin Bieber jumps off a 20m high cliff with an initial upwards velocity of 1 m/s. He knows that an impact force of anything higher than 20m/s will be sufficient to kill him. However, he didn't learn Kinematics and didn't bother to do the math. Will Bieber die? (View Solution)

Exercise 1.3.5 To celebrate his birthday, Albert jumps off a plane at a height of 3000m. However, right as he exited, he realized he forgot his parachute.

(a) Will he have enough time to sing happy birthday to himself before he falls to his death? (Singing Happy Birthday take 30 seconds)

(b) How fast will he impact the ground?

(View Solution)

Exercise 1.3.6 An object initially at rest at position x=0 starts moving with constant acceleration. After 1s, the object is located at x=2. What is the object's velocity at t=2s? (View Solution)

Exercise 1.3.7 A car can accelerate from rest to a final speed v_1 over a distance d. To what speed can the car accelerate within a distance of 2d (starting from rest)? Assume the same value of acceleration in both cases. (View Solution)

Exercise 1.3.8

In order to calculate the maximum range of a rocket, you fire the rocket straight up and record the time it takes for it to return to the ground, $t_{trajectory}$. Based on this single piece of data, how long would it take for the rocket to return to the ground when fired at a 45 degree angle?

(View Solution)

Exercise 1.3.9 James Bond is racing Forrest Gump. After the race begins, it takes Forrest 3s to remove his metal leg braces before he starts running. If Gump accelerates with a constant $3\frac{m}{s^2}$ once he begins to run and Bond accelerates with a constant $1.5\frac{m}{s^2}$, what will be the difference in their speeds when Forrest passes Bond? (View Solution)

Exercise 1.3.10 A kiwi is dropped into a well, and the splash is heard 20 s later. What is the depth of the well? (Take the speed of sound to be 340 m/s) (View Solution)

Calculus

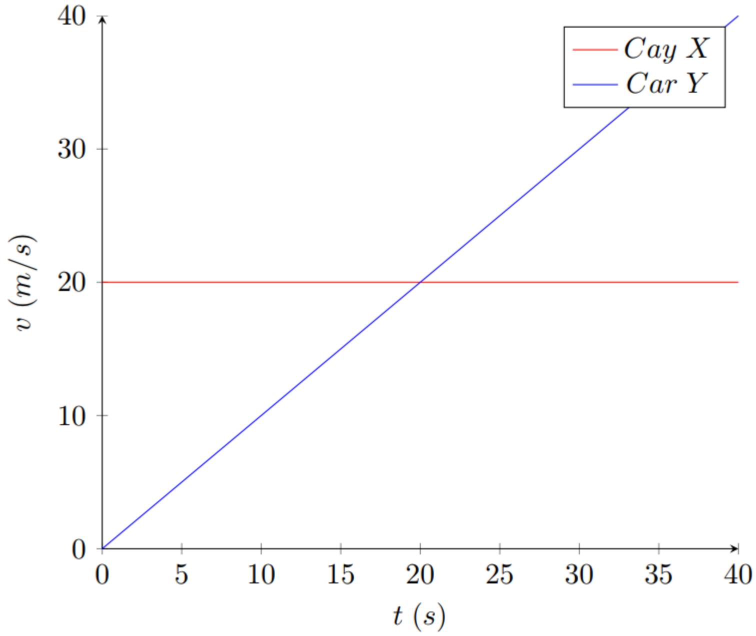
Exercise 1.3.11 Derive (1.3) $\Delta x = v_0 t + \frac{1}{2}at^2$ by solving the differential $\frac{dx}{dt} = v_0 + at$ (View Solution)

Exercise 1.3.12 We know $y(t)$ is parabolic for projectile motion. Prove $y(x)$ is also parabolic. (View Solution)

Exercise 1.3.13 The Concorde supersonic aircraft is moving in still air with a constant velocity of $200\frac{km}{h}\hat{\imath} + 20\frac{km}{h}\hat{\jmath}$, where $\hat{\imath}$ points east and $\hat{\jmath}$ points north. Suddenly at $t = 0$ the wind gusts with a velocity of $20\frac{km}{h^2}t\hat{\imath} + 30\frac{km}{h^3}t^2\hat{\jmath}$ Assuming the pilot makes no attempt to compensate for the wind, what will the plane's displacement be in 1 h with respect to the ground? (View Solution)

Exercises

Only answers and hints will be provided to these exercises, with no full solution. Good luck!



At time t=0, car X traveling with speed v_0 passes car Y, which is just starting to move. Both cars then travel on two parallel lanes of the same straight road. The graphs of speed v versus time t for both cars are shown above. Use this information for the next two questions.

Exercise 1.3.14 (AP) Which of the following is true at time t=20 seconds?

(a) Car Y is behind car X

(b) Car Y is passing car X

(c) Car Y is in front of Car X

(d) Both Cars have the same acceleration

(e) Car X is accelerating faster than car Y

(View Solution)

Exercise 1.3.15 (AP) From time $t = 0$ to time $t = 40$ *seconds*, the areas under both curves are equal. Therefore, which of the following is true at time $t = 40$ *seconds*?

(a) Car Y is behind car X

(b) Car Y is passing car X

(c) Car Y is in front of Car X

(d) Both Cars have the same acceleration

(e) Car X is accelerating faster than car Y

(View Solution)

Exercise 1.3.16 (AP) A body moving in the positive x direction passes the origin at time $t = 0$. Between $t = 0$ and $t = 1$ *sec*, the body has a constant speed of $24\frac{m}{s}$. At $t = 1$ *sec* the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The positive x of the body at t=11 seconds is:

(a) +99 m (b) +36 m (c) -36 m (d) -75 m (e) -99 m

(View Solution)

Exercise 1.3.17 (AP) An object released from rest at time t=0 slides down a frictionless incline a distance of 1 meter during the first second. The distance traveled by the object during the time itnerval from t=1 second to t=2 seconds is:

(a) 1 m (b) 2 m (c) 3 m (d) 4 m (e) 5 m

(View Solution)

Exercise 1.3.18 (AP) Two people are in a boat that is capable of a maximum speed of 5 kilometers per hour in still water, and wish to cross a river 1 kilometer wide to a point directly across from their starting point. If the speed of the water in the river is 5 kilometers per hour, how much time is required for the crossing?

(a) 0.05 hr (b) 0.1 hr (c) 1 hr (d) 10 hr (e) impossible

(View Solution)

Exercise 1.3.19 (AP) A projectile is fired from the surface of the Earth with a speed of 200 meters per second at an angle of 30 deg above the horizontal. If the ground is level, what is the maximum height reached by the projectile?

(a) 5 m (b) 10 m (c) 500 m (d) 1000 m (e) 2000 m

(View Solution)

Exercise 1.3.20 (AP) A rock is dropped from the top of a 45 meter tower, and at the same time a ball is thrown from the top of the tower in a horizontal direction. Air resistance is negligible. The ball and the rock hit the ground a distance of 30 meters apart. The horizontal velocity of the ball thrown was most nearly:

(a) 5 m/s (b) 11 m/s (c) 14.1 m/s (d) 20 m/s (e) 28.3 m/s

(View Solution)

Exercise 1.3.21 (AP) In the absense of air friction, an object dropped near the surface of the Earth experiences a constant acceleration of about $9.8\frac{m}{s^2}$. This means that the

(a) speed of the object increases 9.8 m/s during each second

(b) speed of the object as it falls is 9.8 m/s

(c) object falls 9.8 meters during each second

(d) object falls 9.8 meters during the first second only

(View Solution)

Exercise 1.3.22 (AP) A 500 kilogram sports car accelerates uniformly from rest, reaching a speed of 30 meters per second in 6 seconds. During the 6 seconds, the car has traveled a distance of:

(a) 15 m (b) 30 m (c) 60 m (d) 90 m (e) 180 m

(View Solution)

Exercise 1.3.23 (AP) An object is shot vertically upward into the air with a positive velocity. Which of the following correctly describes the velocity and acceleration of the object at its maximum elevation?

	Velocity	Acceleration
A	Positive	Positive
B	Zero	Zero
C	Negative	Negative
D	Zero	Negative
E	Positive	Negative

(View Solution)

Exercise 1.3.24 (AP) A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45 deg from the vertical, what maximum height can now be reached by the projectile?

(a) $\frac{h}{4}$ (b) $\frac{h}{2\sqrt{2}}$ (c) $\frac{h}{2}$ (d) $\frac{h}{\sqrt{2}}$ (e) h

(View Solution)

Exercise 1.3.25 (AP) The velocity of a projectile at launch has a horizontal component v_h and a vertical component v_v. Air resistance is negligible. When the projectile is at the highest point of its trajectory, which of the following show the vertical and horizontal components of its velocity and the vertical component of its acceleration?

	Vertical Velocity	Horizontal Velocity	Vertical Acceleration
A	v_v	v_h	0
B	v_v	0	0
C	0	v_h	0
D	0	0	g
E	0	v_h	g

(View Solution)

Exercise 1.3.26 (AP) A target T lies flat on the ground 3 m from the side of a building that is 10 m tall. A student rolls a ball off the horizontal roof of the building in the direction of the target. Air Resistance is negligible. The horizontal speed with which the ball must leave the roof if it is to strike the target is most nearly:

(a) $\frac{3}{10}\frac{m}{s}$ (b) $\sqrt{2}\frac{m}{s}$ (c) $\frac{3}{\sqrt{2}}\frac{m}{s}$ (d) $3\frac{m}{s}$ (e) $10\sqrt{\frac{10}{3}}\frac{m}{s}$

(View Solution)

Exercise 1.3.27 (AP) An object is dropped from rest from the top of a 400 m cliff on Earth. If air resistance is negligible, what is the distance the object travels during the first 6 s of its fall?

(a) 30 m (b) 60 m (c) 120 m (d) 180 m (e) 360 m

(View Solution)

Exercise 1.3.28 (AP) A student is testing the kinematic equations for uniformly accelerated motion by measuring the time it takes for light-weight plastic balls to fall to the floor from a height of 3 m in the lab. The student predicts the time to fall using g as 9.80 m/s2 but finds the measured time to be 35% greater. Which of the following is the most likely cause of the large percent error?

(a) The acceleration due to gravity is 70% greater than $9.8\frac{m}{s^2}$

(b) The acceleration due to gravity is 70% less than $9.8\frac{m}{s^2}$

(c) Air resistance increases the downward acceleration

(d) The acceleration of the plastic balls is not uniform

(e) The plastic balls are not truly spherical

(View Solution)

Exercise 1.3.29 (AP) An object is thrown with velocity v from the edge of a cliff above level ground. Neglect air resistance. In order for the object to travel a maximum horizontal distance from the cliff before hitting the ground, the throw should be at an angle θ with respect to the horizontal of

(a) Greater than 60 deg above the horizontal

(b) greater than 45 deg but less than 60 deg above the horizontal

(c) greater than zero but less than 45 deg above the horizontal

(d) zero

(e) greater than zero but less than 45 deg below the horizontal

(View Solution)

Starting from rest, a vehicle accelerates on a straight level road at the rate of $4\frac{m}{s^2}$ for 5 seconds. Use this information for the next two questions:

Exercise 1.3.30 (AP) What is the speed of the vehicle at the end of this time interval?

(a) 1.3 m/s (b) 10 m/s (c) 20 m/s (d) 80 m/s (e) 100 m/s

(View Solution)

Exercise 1.3.31 (AP) What is the total distance the vehicle travels during this time interval?

(a) 10 m (b) 20 m (c) 25 m (d) 40 m (e) 50 m

(View Solution)

Exercise 1.3.32 (AP) If air resistance is negligible, the speed of a 2 kg sphere that falls from rest through a vertical displacement of 0.2 m is most nearly

(a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s (e) 5 m/s

(View Solution)

Exercise 1.3.33 (AP) A projectile is launched from level ground with an initial speed v_0 at an angle θ with the horizontal. If air resistance is negligible, how long will the projectile remain in the air?

(a) $\frac{2v_0}{g}$ (b) $\frac{2v_0 cos(\theta)}{g}$ (c) $\frac{v_0 cos(\theta)}{g}$ (d) $\frac{v_0 sin(\theta)}{g}$ (e) $\frac{2v_0 sin(\theta)}{g}$

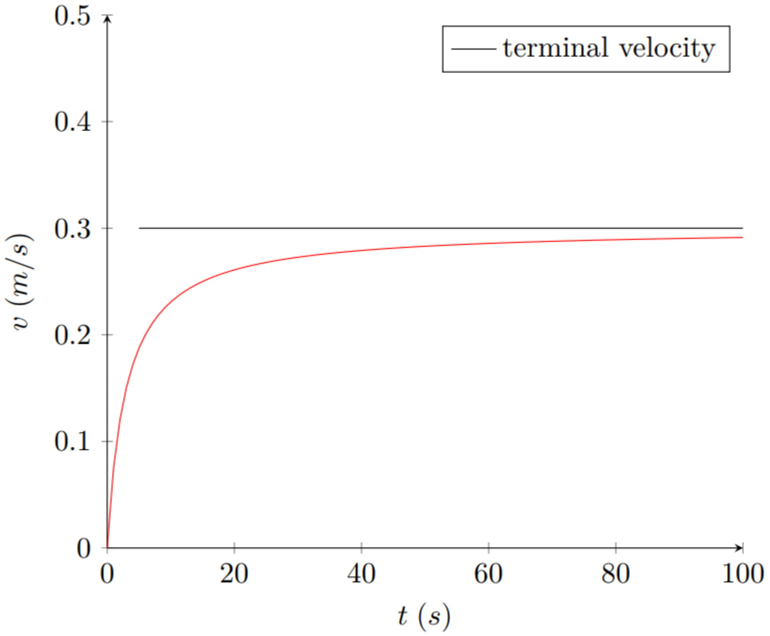
(View Solution)

Exercise 1.3.34 (AP) An object of unknown mass is initially at rest and dropped from a height h. It reaches the ground with a velocity v_1 The same object is then raised again to the same height h but this time is thrown downward with velocity v_1 It now reaches the ground with a new velocity v_2. How is v_2 related to v_1?

(a) $v_2 = \frac{v_1}{2}$ (b) $v_2 = v_1$ (c) $v_2 = \frac{v_1}{\sqrt{2}}$ (d) $v_2 = 2v_1$ (e) $v_2 = 4v_1$

(View Solution)

Exercise 1.3.35 The velocity function for an object falling from rest under the influence of gravity and air resistance is shown below. Which of the following statements about x(t) are consistent with this velocity function?



	Y-Intercept	Starting Acceleration	Asymptote
A	Zero	Positive	Slant (Oblique)
B	Positive	Positive	Horizontal
C	Zero	Negative	Slant (Oblique)
D	Negative	Negative	Horizontal
E	Zero	Positive	No

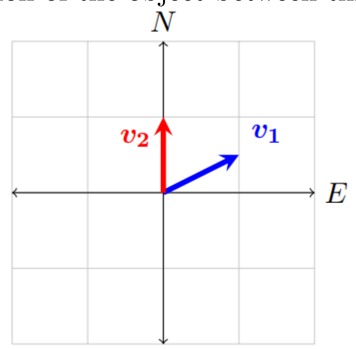
(View Solution)

Exercise 1.3.36 A particle moves along the x-axis with a nonconstant acceleration described by $a = 12t$, where a is in meters per second squared and t is in seconds. If the particle starts from rest so that its speed v and position x are zero when t=0, where is it located when $t = 2\ sec$?

1. 5 m
2. 10 m
3. 500 m
4. 1000 m
5. 2000 m

(View Solution)

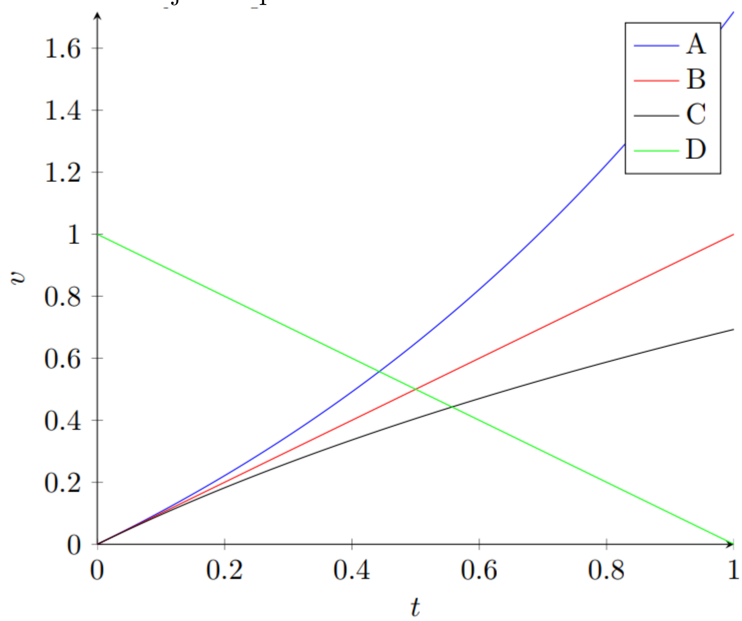
Exercise 1.3.37 Vectors v_1 and v_2 shown below have equal magnitudes. The vectors represent the velocities of an object at times t_1, and t_2, respectively. The average acceleration of the object between time t_1 and t_2 was directed:



(a) North (b) West (c) North of East (d) North of West

(View Solution)

Exercise 1.3.38 An object is thrown vertically upward in a region where g is constant and air resistance is negligible. Its speed is recorded from the moment it leaves the thrower's hand until it reaches its maximum height. Which of the following graphs best represent the object's speed v versus time t [1]



(View Solution)

[1]If you can't distinguish between the colors, A, B, C, D are labelled in order of the end behaviour of each line

An object moving in a straight line has a velocity v in meters per second that varies with time t in seconds according to the following function: $v = 4 + 0.5t^2$. Use this information to answer the next two questions:

Exercise 1.3.39 The instantaneous acceleration of the object at t = 2 seconds is:

1. $2\frac{m}{s^2}$
2. $4\frac{m}{s^2}$
3. $5\frac{m}{s^2}$
4. $6\frac{m}{s^2}$
5. $8\frac{m}{s^2}$

(View Solution)

Exercise 1.3.40 The displacement of the object between t=0 and t=6 seconds is:

1. 22 m
2. 28 m
3. 40 m
4. 42 m
5. 60 m

(View Solution)

Exercise 1.3.41 The position of an object is given by the equation $x = 3t^2 + 1.5t + 4.5$ where x is in meters and t is in seconds. What is the instantaneous acceleration of the object at $t = 3\ sec$?

1. $3\frac{m}{s^2}$
2. $6\frac{m}{s^2}$
3. $9\frac{m}{s^2}$
4. $19.5\frac{m}{s^2}$
5. $36\frac{m}{s^2}$

(View Solution)

Exercise 1.3.42 At a particular instant, a stationary observer on the ground sees a package falling with a speed of v_1 at an angle to the vertical. To a pilot flying horizontaly at constant speed relative to the ground, the package appears to be falling vertically with a speed v_2 at that instant. What is the speed of the pilot relative to the ground?

1. $v_1 + v_2$
2. $v_1 - v_2$
3. $v_2 - v_1$
4. $\sqrt{v_1^2 - v_2^2}$
5. $\sqrt{v_1^2 + v_2^2}$

(View Solution)

1.4 Solutions

Solution 1.3.1 (View Question) Position has units of m. Velocity has units of m/s. Acceleration has units of m/s^2. Time has units of s.

To perform a dimensional analysis, one method is to pretend all variables have an unit of 1, evaluate the expression, and look at the units.

$$\Delta x = (\frac{v_o + v_f}{2})t \rightarrow (\frac{1\frac{m}{s} + 1\frac{m}{s}}{2})(1\ s) = (\frac{2\frac{m}{s}}{2})(1\ s) = (1\frac{m}{s})(1\ s) = 1\ m$$

$$v_f = v_0 + at \rightarrow (1\frac{m}{s}) + (1\frac{m}{s^2})(1\ s) = (1\frac{m}{s}) + (1\frac{m}{s}) = 2\frac{m}{s}$$

$$\Delta x = v_0 t + \frac{1}{2}at \rightarrow (1\frac{m}{s})(1\ s) + \frac{1}{2}(1\frac{m}{s^2})(1\ s^2) = (1\ m) + (\frac{1}{2}\ m) = \frac{3}{2}\ m$$

$$v_f^2 = v_0^2 + 2a\Delta x \rightarrow (1\frac{m^2}{s^2}) + 2(1\frac{m}{s^2})(1\ m) = (1\frac{m^2}{s^2}) + (2\frac{m^2}{s^2}) = 3\frac{m^2}{s^2}$$

You can see for yourself the units at the very right are the same as the units on the very left.

Solution 1.3.2 (View Question)

1. We need to solve for the time t it takes for the object to hit the ground.

2. We are given the following information

   - $\Delta y = 10\ m$ (Δy is the change in position, or height)
   - $v_x = 5\frac{m}{s}$ (the starting x velocity)
   - $v_y = 0\frac{m}{s}$ (the starting y velocity)
   - $a = g = 10\frac{m}{s^2}$ (we are defining the downwards direction as positive)

   Note that because motion in the x direction *does not* affect the motion in the y direction, we can safely ignore v_x.

3. Note that (1.3) $\Delta y = v_0 t + \frac{1}{2}at$ uses all the variables listed above.

4. After substituting, we get:

   $$\Delta y = v_0 t + \frac{1}{2}at \rightarrow 20\ m = 0\frac{m}{s}t + \frac{1}{2}(10\frac{m}{s^2})t^2 \rightarrow 20\ m = 5\frac{m}{s^2}t^2$$

   Solving for t gives $t = 2\ s$

Solution 1.3.3 (View Question) Starting with the equation:

$$v_f = v_0 + at$$

$$t = \frac{\Delta v}{a}$$

Substituting into (1.3):

$$\Delta x = v_0 t + \frac{1}{2}at^2 \to \Delta x = v_0\frac{\Delta v}{a} + \frac{1}{2}a\frac{\Delta v}{a}^2 = \frac{\Delta v}{a}(v_0 + \frac{1}{2}\Delta v)$$

After simplifying:

$$\Delta x = \frac{\Delta v}{a}(v_0 + \frac{v_f}{2} - \frac{v_0}{2}) = \Delta x = \frac{v_f - v_0}{a}(\frac{v_f + v_0}{2}) = \frac{v_f^2 - v_0^2}{2a}$$

Solution 1.3.4 (View Question)

1. We're trying to find the final velocity v_f
2. We are given:
   - $\Delta y = 20\ m$
   - $v_0 = 1\frac{m}{s}$
   - $a = g = 10ms^2$ (we're setting the downwards direction as positive)
   - $v_max = 20\frac{m}{s}$
3. We see that equation (1.4) $v_f^2 = v_0^2 + 2a\Delta y$ uses all the variables listed above
4. After substituting, we get:

$$v_f^2 = (1\frac{m}{s})^2 + 2(10\frac{m}{s^2})(20\ m) \to v_f^2 = 1\frac{m^2}{s^2} + 400\frac{m^2}{s^2} \to v_f^2 = 401\frac{m^2}{s^2} \tag{1.9}$$

5. We see $v_f = \sqrt{401}\frac{m}{s} \approx 20.02\frac{m}{s}$ Since $|v_f|$ is higher than the maximum *speed* Bieber can withstand, he dies.

Solution 1.3.5 (View Question) To solve for (a):

1. We're trying to find how much more time he has left. We can figure this out by solving for the total time t_{total}, and subtract it by 9 minutes.
2. We are given:
   - $\Delta y = 3000\ m$
   - $v_0 = 0\frac{m}{s^2}$

- $a = g = 10\frac{m}{s^2}$ (setting the downwards direction as positive)
- $t_{song} = 30\ s$

3. We see that equation (1.3) $\Delta x = v_0 t + \frac{1}{2}at$ uses all the variables listed above.

4. Substituting, we get:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \rightarrow 3000\ m = (0\frac{m}{s}t) + \frac{1}{2}(10\frac{m}{s^2})t^2 \rightarrow 3000\ m = 5\frac{m}{s^2}t^2$$

5. Solving for t gives $t = 24.5\ sec$ Therefore, Albert will not be able to sing his last birthday song.

To solve for (b):

1. Now we're trying to find his impact speed.

2. We are given:

   - $\Delta y = 3000\ m$
   - $v_0 = 0\frac{m}{s^2}$
   - $a = g = 10\frac{m}{s^2}$ (setting the downwards direction as positive)
   - $t_{total} = 600\ s$

3. (1.1) (1.2) and (1.3) all use the variables above. For simplicity reasons, we will use (1.2) $v_f = v_0 + at$.

4. Substituting, we get:

$$v_f = v_0 + at \rightarrow v_f = 0\frac{m}{s} + (10\frac{m}{s})(600\ s) \rightarrow$$

5. Solving for v_f gives 6000 sec. Ouch![2]

Solution 1.3.6 (View Question)

1. We want to figure out the object's final velocity v_f, but to do that we need to figure out the object's acceleration a. Let us ignore the last sentence for now.

2. We are given:

   - $v_0 = 0\frac{m}{s}$
   - $\Delta x_{t=1\ s} = 2\ m$
   - t=1 s

[2]This is impossible in reality because air resistance will slow Albert down to a terminal velocity. This would be correct if the Earth had no air.

3. To solve for acceleration, we can use (1.3) $\Delta x = v_0 t + \frac{1}{2}at^2$

4. After substituting:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \rightarrow 2\ m = \frac{1}{2}a(1\ s^2)$$

5. Solving for a gives $a = 4\frac{m}{s^2}$. Substitute it into (1.2) $v_f = v_0 + at \rightarrow v_f = 4\frac{m}{s^2}2s$ to get $v_f = 8\frac{m}{s}$

Solution 1.3.7 (View Question) We see that the problem is divided into two parts. First, we need to figure out the acceleration of the car. Then use that to figure out its final velocity. However, there is a more elegant solution

1. Our ultimate end goal is to figure out the final velocity v_1 of the car after accelerating over a distance of $2d$.

2. We are given the final velocity, the starting velocity, and the distance. We also know the acceleration is constant.

3. The only equation that gives a relationship between these four variables is (1.4) $v_f^2 = v_0^2 + 2a\Delta x$

4. We see that since $v_0 = 0\frac{m}{s}$, the equation becomes

$$v_1^2 = 2ad \rightarrow v_1 = \sqrt{2ad}$$

5. It can be seen that if d is doubled, the final velocity increases by a factor of $\sqrt{2}$, therefore the final velocity for the second trial is $\sqrt{2}v_1$. If you still can't see why, let v_2 be the final velocity of the second trial:

$$v_2 = \sqrt{2a(2d)} \rightarrow v_2 = \sqrt{2}\sqrt{2ad} \rightarrow v_2 = \sqrt{2}v_1$$

Solution 1.3.8 (View Question)

1. We need to figure out the time t_f it takes for the rocket to return to the ground when fired at a 45 degree angle. However, before we do that, we need to solve for velocity v_0 it started with.

2. We are only explicitly given one piece of information, but we can extrapolate more data:

   - $t_{trajectory}$
   - $g = a = -10\frac{m}{s^2}$ Note we set a to be positive in previous questions because we defined the downwards direction as positive. In this question, since it's moving both down and up, we define it to be negative

- $v_f = -v_0$

3. To solve for v_0 we can use (1.2) $v_f = v_0 + at$
4. After substituting:

$$v_f = v_0 + at \rightarrow -v_0 = v_0 - 10\frac{m}{s^2}t_{trajectory} \rightarrow 2v_0 = 10\frac{m}{s^2}t_{trajectory} \rightarrow v_0 = 5\frac{m}{s^2}t_{trajectory}$$

5. Now we have an equation for the starting velocity v_0. To solve for the time it takes if the rocket was fired at a 45 degree angle, we need to seperate v_0 into its y component:

$$v_{0,y} = sin(45)v_0 = \frac{\sqrt{2}}{2}v_0$$

We can then use (1.2) again and follow the same steps to solve for t_f:

$$v_f = v_{0,y} + at \rightarrow 2v_{0,y} = 10\frac{m}{s^2}t_f \rightarrow 2(\frac{\sqrt{2}}{2}v_0) = 10\frac{m}{s^2}t_f$$

After substituting $v_0 = 5\frac{m}{s^2}t_{trajectory}$:

$$\sqrt{2}5\frac{m}{s^2}t_{trajectory} = 10\frac{m}{s^2}t_f$$

$$t_f = \frac{\sqrt{2}}{2}t_{trajectory}$$

Hmm... this looks familiar to the conversion between v_0 and $v_{0,y}$. Can you find a general formula for any angle θ?

Solution 1.3.9 (View Question) This is a classic problem, and it requires a fair bit of ingenuity to solve. As a general approach, define two equations for position, one for Bond and another for Gump, and solve for the time when they are equal (the time when Forrest catches up with Bond).

Let t=0 be when the race begins. Then Bond's position is modelled by (1.3):

$$\Delta x_{bond} = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}(1.5\frac{m}{s^2})t^2$$

Let t_g be when Gump begins to run. Then Gump's position is also modelled by (1.3):

$$\Delta x_{bond} = v_0 t_g + \frac{1}{2}at_g^2 = \frac{1}{2}(3(\frac{m}{s^2})t_g^2$$

To define their position with the same time parameter, notice $t = t_g + 3\ s$ because Gumps tarts running 3 s after Bond. Substituting this into Δx_{bond} gives:

$$\Delta x_{bond} = \frac{1}{2}(3(\frac{m}{s^2})t_g^2 = \frac{1}{2}(1.5(\frac{m}{s^2})(t-3\ s)^2$$

Solving for $x_{bond} = x_{gump}$ gives:

$$x_{bond} = x_{gump} \rightarrow \frac{1}{2}(1.5(\frac{m}{s^2})t^2 = \frac{1}{2}(3\frac{m}{s^2})(t-3\ s)^2$$

$$(1.5\frac{m}{s^2})t^2 = (3\frac{m}{s^2})(t-3\ s)^2$$

At this point, if we want to progress further, the units are going to get all muddled up, so we're going to ignore them for now. (If you ignore them, always do a dimensional analysis at the end!)

$$1.5t^2 = 3(t-3)^2 \rightarrow \sqrt{1.5}t = \sqrt{3}(t-3) \rightarrow \sqrt{1.5}t = \sqrt{3}t - 3\sqrt{3}$$

Solving this linear equation gives the answers t=10.24 s. Therefore, Gump caught up to Bond after 10.24 seconds.

Bond's velocity at this time can be figured out using (1.2) $v_f = v_0 + at$

$$v_f = v_0 + at = 0\frac{m}{s} + 1.5\frac{m}{s^2}(10.24\ s) = 15.36\frac{m}{s}$$

Similarly, Gump's velocity is:

$$v_f = v_0 + a(t-3) = 0\frac{m}{s} + 3\frac{m}{s^2}(7.24\ s) = 21.72\frac{m}{s}$$

Therefore, Gump is running faster than Bond by $21.7\frac{m}{s} - 15.4\frac{m}{s} = 6.3\frac{m}{s}$

Solution 1.3.10 (View Question) This another classic problem. We need to figure out how long it took for the kiwi to fall, and how long it took for the sound to arrive back. The crux is to realize both these time intervals depend solely on the distance. Thus, if we are able to represent both these time intervals in terms of the depth of the well, we can set the sum to 20 s and solve!

1. We need to solve for two variables, the time it takes for the kiwi to fall t_{fall} in terms of the height y_{well} and the time it takes for the sound to go back up t_{up} in terms of y_{well} and set the sum $t_{fall} + tup$ to 20 s!

2. We are given:

   - $v_0 = 0\frac{m}{s}$
   - $t_total = 20\ s$
   - $a = g = 10\frac{m}{s}$

3. First we need to find t_{fall} in terms of y which we can use (1.3) $\Delta y = v_0 t + \frac{1}{2}at^2$. The second variable t_{up} can be found using (1.1) $\Delta y = v_{sound}t$

4. Substituting to find t_{fall}:

$$\Delta y = v_0 t + \frac{1}{2}at^2 \rightarrow y_{well} = (0\frac{m}{s})t + \frac{1}{2}(10\frac{m}{s^2}t_{fall}^2 \rightarrow t_{fall} = \sqrt{\frac{y_{well}}{5\frac{m}{s^2}}}$$

Substituting to find t_{up}:

$$\Delta y = v_{sound}t \rightarrow y_{well} == 340\frac{m}{s} = t_{up} \rightarrow t_{up} = \frac{y_{well}}{340\frac{m}{s}}$$

Adding these together gives:

$$t_{fall} + t_{up} = 20\ s \rightarrow \sqrt{\frac{\Delta y_{well}}{5\frac{m}{s^2}}} + \frac{y_{well}}{340\frac{m}{s}} = 20\ s$$

$$(20\ s - \frac{y_{well}}{340\frac{m}{s}})^2 = \sqrt{\frac{y_{well}}{5\frac{m}{s^2}}}^2$$

$$400\ s^2 + \frac{y_{well}^2}{115600\frac{m^2}{s^2}} - \frac{2y_{well}}{17} = \frac{y_{well}}{5\frac{m}{s}}$$

Solving this quadratic equation yields $y_{well} = 1306m$ and $y_{well} = 35414m$. We have introduced a new solution by squaring both sides, but we see that if the well is 35414m deep, it will take sound over 100 s just to pass through. Plugging $y_{well} = 1306m$ into the expression for t_{fall} yields $t_{fall} = 16.16\ s$.

Solution 1.3.11 (View Question) We see that:

$$\frac{dx}{dt} = v_0 + at \rightarrow \int_{x_0}^{x_1} dx = \int_{t_0}^{t_1} v_0 + atdt \rightarrow x\Big|_{x_0}^{x_f} = (v_0 t + \frac{1}{2}at^2)\Big|_0^t$$

$$x_f - x_0 = v_0 t + \frac{1}{2}at^2$$

Solution 1.3.12 (View Question) For any question that has to do with the coordinate plane, remember to choose the origin. In this example, we choose our coordinate system such that the object's initial position is at the origin: $x_0 = y_0 = 0$. Thus:

$$x(t) = v_x t$$

$$y(x) = v_{y,0}t - \frac{1}{2}gt^2$$

We can covert the parametric equation $y(t)$ to $y(x)$ by making the substitution:

$$t = \frac{x}{v_x}$$

Substituting:

$$y(x) = v_{y,0}(\frac{x}{v_x}) - \frac{1}{2}g(\frac{x}{v_x})^2$$

$$y(x) = (\frac{v_{y,0}}{v_x})x + (\frac{-g}{2v_x^2})x^2$$

This is the equation pf a parabola that passes through the origin (as it should, based on the initial conditions). As long as $v_x \neq 0$, y(x) will be parabolic (If $v_x = 0$ then the path will simply be a one dimensional line up and down, which you can consider a limiting case of a parabola.

To double check, perform a dimensional analysis to see all terms have units of metres.

Solution 1.3.13 (View Question)

1. We are trying to find the plane's velocity, $v_{plane\ relative\ to\ ground}$ when we are given two sepereate pieces of information about the velocity of the aircraft in still air, and the strength and direction of the wind as a function of time. Hmm... this rings a bell. What about vector calculus?

2. We are given 2 pieces of information:

   - $v_{plane\ relative\ to\ air} = 200\frac{km}{h}\hat{\imath} + 20\frac{km}{h}\hat{\jmath}$
   - $v_{air\ relative\ to\ ground} = 20\frac{km}{h^2}t\hat{\imath} + 30\frac{km}{h^3}t^2\hat{\jmath}$

3. We can use (1.8)

4. Substituting:

$$\vec{v}_{p\ relative\ to\ b} = \vec{v}_{p\ relative\ to\ a} + \vec{v}_{a\ relative\ to\ b}$$

$$v_{plane\ relative\ to\ ground} = v_{plane\ relative\ to\ air} + v_{air\ relative\ to\ ground}$$

$$v_{plane\ relative\ to\ ground} = 200\frac{km}{h}\hat{\imath} + 20\frac{km}{h}\hat{\jmath} + 20\frac{km}{h^2}t\hat{\imath} + 30\frac{km}{h^3}t^2\hat{\jmath}$$

$$v_{plane\ relative\ to\ ground} = (200\frac{km}{h} + 20t\frac{km}{h^2})\hat{\imath} + (20\frac{km}{h} + 30t^2\frac{km}{h^3})\hat{\jmath}$$

$$\Delta r = \int dv = \int_{t=0h}^{t=1h} dv[(200\frac{km}{h} + 20t\frac{km}{h^2})\hat{\imath} + (20\frac{km}{h} + 30t^2\frac{km}{h^3})\hat{\jmath}]$$

$$\Delta r = [200t\frac{km}{h} + 10t^2\frac{km}{h^2})\hat{\imath} + (20t\frac{km}{h} + 10t^3\frac{km}{h^3})\hat{\jmath}\Big|_{t=0h}^{t=1h}$$

5. After solving, we get:

$$\Delta r = (210km)\hat{\imath} + (10km)\hat{\jmath}$$

Solution 1.3.14 (View Question) A (hint: the area underneath v(t) is position)

Solution 1.3.15 (View Question) B (hint: equal areas mean currently, they're in the same position)

Solution 1.3.16 (View Question) C (Hint: use (1.3))

Solution 1.3.17 (View Question) C (Hint: The acceleration due to gravity isn't g!)

Solution 1.3.18 (View Question) E (Hint: This is actually a stupid question. Use common sense!)

Solution 1.3.19 (View Question) C (Hint: Velocity at the top of the trajectory is zero!)

Solution 1.3.20 (View Question) B (Hint: First, you'll need to figure out the time it took, then solve for the speed)

Solution 1.3.21 (View Question) A (Hint: Go over what acceleration is again)

Solution 1.3.22 (View Question) D (Hint: First solve for acceleration, then solve for distance)

Solution 1.3.23 (View Question) E (Hint: It's going up, but slowing down!)

Solution 1.3.24 (View Question) D (Hint: It's very similar to 1.3.8)

Solution 1.3.25 (View Question) E (Hint: It's still moving forward, but it's not moving up anymore. Something's pulling it down!)

Solution 1.3.26 (View Question) C (Hint: Solve for time, then solve for velocity)

Solution 1.3.27 (View Question) D (Hint: For problems like this, double check by making sure the time it takes to fall 300 metres is over 6 seconds)

Solution 1.3.28 (View Question) D (Hint: Use a process of elimination!)

Solution 1.3.29 (View Question) C (Hint: If two trajectories ever cross paths, the one with the higher x velocity will land furthest)

Solution 1.3.30 (View Question) C (Hint: Use 1.2)

Solution 1.3.31 (View Question) E (Hint: Use 1.3)

Solution 1.3.32 (View Question) E (Hint: Use 1.3)

Solution 1.3.33 (View Question) E (Hint: The original and final speed is the same, due to the symmetry of the parabola)

Solution 1.3.34 (View Question) E (Hint: Use 1.4 to solve solve for v_1 then use 1.4 again to solve for v_2 in terms of v_1. Then substitute.)

Solution 1.3.35 (View Question) A (Hint: Sketch it out!)

Solution 1.3.36 (View Question) B (Hint: take the integral two times!)

Solution 1.3.37 (View Question) D (Hint: We're not looking at the location of v. We're looking at how v changes.

Solution 1.3.39 (View Question) A (Acceleration is the first derivative of velocity)

Solution 1.3.40 (View Question) E (Take the definite integral with 0 and 6 as bounds!)

Solution 1.3.41 (View Question) B (Hint: Acceleration is the second derivative of position)

Solution 1.3.42 (View Question) D (Hint: Pythagorean Theorem!)

1.5 Solutions

Solution 1.3.1 (View Question) Position has units of m. Velocity has units of m/s. Acceleration has units of m/s^2. Time has units of s.

To perform a dimensional analysis, one method is to pretend all variables have an unit of 1, evaluate the expression, and look at the units.

$$\Delta x = (\frac{v_o + v_f}{2})t \rightarrow (\frac{1\frac{m}{s} + 1\frac{m}{s}}{2})(1\ s) = (\frac{2\frac{m}{s}}{2})(1\ s) = (1\frac{m}{s})(1\ s) = 1\ m$$

$$v_f = v_0 + at \rightarrow (1\frac{m}{s}) + (1\frac{m}{s^2})(1\ s) = (1\frac{m}{s}) + (1\frac{m}{s}) = 2\frac{m}{s}$$

$$\Delta x = v_0 t + \frac{1}{2}at \rightarrow (1\frac{m}{s})(1\ s) + \frac{1}{2}(1\frac{m}{s^2})(1\ s^2) = (1\ m) + (\frac{1}{2}\ m) = \frac{3}{2}\ m$$

$$v_f^2 = v_0^2 + 2a\Delta x \rightarrow (1\frac{m^2}{s^2}) + 2(1\frac{m}{s^2})(1\ m) = (1\frac{m^2}{s^2}) + (2\frac{m^2}{s^2}) = 3\frac{m^2}{s^2}$$

You can see for yourself the units at the very right are the same as the units on the very left.

Solution 1.3.2 (View Question)

1. We need to solve for the time t it takes for the object to hit the ground.

2. We are given the following information

   - $\Delta y = 10\ m$ (Δy is the change in position, or height)
   - $v_x = 5\frac{m}{s}$ (the starting x velocity)
   - $v_y = 0\frac{m}{s}$ (the starting y velocity)
   - $a = g = 10\frac{m}{s^2}$ (we are defining the downwards direction as positive)

   Note that because motion in the x direction *does not* affect the motion in the y direction, we can safely ignore v_x.

3. Note that (1.3) $\Delta y = v_0 t + \frac{1}{2}at$ uses all the variables listed above.

4. After substituting, we get:

   $$\Delta y = v_0 t + \frac{1}{2}at \rightarrow 20\ m = 0\frac{m}{s}t + \frac{1}{2}(10\frac{m}{s^2})t^2 \rightarrow 20\ m = 5\frac{m}{s^2}t^2$$

   Solving for t gives $t = 2\ s$

Solution 1.3.3 (View Question) Starting with the equation:

$$v_f = v_0 + at$$

$$t = \frac{\Delta v}{a}$$

Substituting into (1.3):

$$\Delta x = v_0 t + \frac{1}{2}at^2 \rightarrow \Delta x = v_0\frac{\Delta v}{a} + \frac{1}{2}a\frac{\Delta v}{a}^2 = \frac{\Delta v}{a}(v_0 + \frac{1}{2}\Delta v)$$

After simplifying:

$$\Delta x = \frac{\Delta v}{a}(v_0 + \frac{v_f}{2} - \frac{v_0}{2}) = \Delta x = \frac{v_f - v_0}{a}(\frac{v_f + v_0}{2}) = \frac{v_f^2 - v_0^2}{2a}$$

Solution 1.3.4 (View Question)

1. We're trying to find the final velocity v_f
2. We are given:
   - $\Delta y = 20\ m$
   - $v_0 = 1\frac{m}{s}$
   - $a = g = 10ms^2$ (we're setting the downwards direction as positive)
   - $v_max = 20\frac{m}{s}$
3. We see that equation (1.4) $v_f^2 = v_0^2 + 2a\Delta y$ uses all the variables listed above
4. After substituting, we get:

$$v_f^2 = (1\frac{m}{s})^2 + 2(10\frac{m}{s^2})(20\ m) \rightarrow v_f^2 = 1\frac{m^2}{s^2} + 400\frac{m^2}{s^2} \rightarrow v_f^2 = 401\frac{m^2}{s^2} \qquad (1.10)$$

5. We see $v_f = \sqrt{401}\frac{m}{s} \approx 20.02\frac{m}{s}$ Since $|v_f|$ is higher than the maximum *speed* Bieber can withstand, he dies.

Solution 1.3.5 (View Question) To solve for (a):

1. We're trying to find how much more time he has left. We can figure this out by solving for the total time t_{total}, and subtract it by 9 minutes.
2. We are given:
   - $\Delta y = 3000\ m$
   - $v_0 = 0\frac{m}{s^2}$

- $a = g = 10\frac{m}{s^2}$ (setting the downwards direction as positive)
- $t_{song} = 30\ s$

3. We see that equation (1.3) $\Delta x = v_0 t + \frac{1}{2}at$ uses all the variables listed above.

4. Substituting, we get:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \rightarrow 3000\ m = (0\frac{m}{s}t) + \frac{1}{2}(10\frac{m}{s^2})t^2 \rightarrow 3000\ m = 5\frac{m}{s^2}t^2$$

5. Solving for t gives $t = 24.5\ sec$ Therefore, Albert will not be able to sing his last birthday song.

To solve for (b):

1. Now we're trying to find his impact speed.

2. We are given:

   - $\Delta y = 3000\ m$
   - $v_0 = 0\frac{m}{s^2}$
   - $a = g = 10\frac{m}{s^2}$ (setting the downwards direction as positive)
   - $t_{total} = 600\ s$

3. (1.1) (1.2) and (1.3) all use the variables above. For simplicity reasons, we will use (1.2) $v_f = v_0 + at$.

4. Substituting, we get:

$$v_f = v_0 + at \rightarrow v_f = 0\frac{m}{s} + (10\frac{m}{s})(600\ s) \rightarrow$$

5. Solving for v_f gives 6000 sec. Ouch![3]

Solution 1.3.6 (View Question)

1. We want to figure out the object's final velocity v_f, but to do that we need to figure out the object's acceleration a. Let us ignore the last sentence for now.

2. We are given:

   - $v_0 = 0\frac{m}{s}$
   - $\Delta x_{t=1\ s} = 2\ m$
   - t=1 s

[3] This is impossible in reality because air resistance will slow Albert down to a terminal velocity. This would be correct if the Earth had no air.

3. To solve for acceleration, we can use (1.3) $\Delta x = v_0 t + \frac{1}{2}at^2$

4. After substituting:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \rightarrow 2\ m = \frac{1}{2}a(1\ s^2)$$

5. Solving for a gives $a = 4\frac{m}{s^2}$. Substitute it into (1.2) $v_f = v_0 + at \rightarrow v_f = 4\frac{m}{s^2}2s$ to get $v_f = 8\frac{m}{s}$

Solution 1.3.7 (View Question) We see that the problem is divided into two parts. First, we need to figure out the acceleration of the car. Then use that to figure out its final velocity. However, there is a more elegant solution

1. Our ultimate end goal is to figure out the final velocity v_1 of the car after accelerating over a distance of $2d$.

2. We are given the final velocity, the starting velocity, and the distance. We also know the acceleration is constant.

3. The only equation that gives a relationship between these four variables is (1.4) $v_f^2 = v_0^2 + 2a\Delta x$

4. We see that since $v_0 = 0\frac{m}{s}$, the equation becomes

$$v_1^2 = 2ad \rightarrow v_1 = \sqrt{2ad}$$

5. It can be seen that if d is doubled, the final velocity increases by a factor of $\sqrt{2}$, therefore the final velocity for the second trial is $\sqrt{2}v_1$. If you still can't see why, let v_2 be the final velocity of the second trial:

$$v_2 = \sqrt{2a(2d)} \rightarrow v_2 = \sqrt{2}\sqrt{2ad} \rightarrow v_2 = \sqrt{2}v_1$$

Solution 1.3.8 (View Question)

1. We need to figure out the time t_f it takes for the rocket to return to the ground when fired at a 45 degree angle. However, before we do that, we need to solve for velocity v_0 it started with.

2. We are only explicitly given one piece of information, but we can extrapolate more data:

   - $t_{trajectory}$
   - $g = a = -10\frac{m}{s^2}$ Note we set a to be positive in previous questions because we defined the downwards direction as positive. In this question, since it's moving both down and up, we define it to be negative

- $v_f = -v_0$

3. To solve for v_0 we can use (1.2) $v_f = v_0 + at$
4. After substituting:

$$v_f = v_0 + at \to -v_0 = v_0 - 10\frac{m}{s^2}t_{trajectory} \to 2v_0 = 10\frac{m}{s^2}t_{trajectory} \to v_0 = 5\frac{m}{s^2}t_{trajectory}$$

5. Now we have an equation for the starting velocity v_0. To solve for the time it takes if the rocket was fired at a 45 degree angle, we need to seperate v_0 into its y component:

$$v_{0,y} = sin(45)v_0 = \frac{\sqrt{2}}{2}v_0$$

We can then use (1.2) again and follow the same steps to solve for t_f:

$$v_f = v_{0,y} + at \to 2v_{0,y} = 10\frac{m}{s^2}t_f \to 2(\frac{\sqrt{2}}{2}v_0) = 10\frac{m}{s^2}t_f$$

After substituting $v_0 = 5\frac{m}{s^2}t_{trajectory}$:

$$\sqrt{2}5\frac{m}{s^2}t_{trajectory} = 10\frac{m}{s^2}t_f$$

$$t_f = \frac{\sqrt{2}}{2}t_{trajectory}$$

Hmm... this looks familiar to the conversion between v_0 and $v_{0,y}$. Can you find a general formula for any angle θ?

Solution 1.3.9 (View Question) This is a classic problem, and it requires a fair bit of ingenuity to solve. As a general approach, define two equations for position, one for Bond and another for Gump, and solve for the time when they are equal (the time when Forrest catches up with Bond).

Let t=0 be when the race begins. Then Bond's position is modelled by (1.3):

$$\Delta x_{bond} = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}(1.5\frac{m}{s^2})t^2$$

Let t_g be when Gump begins to run. Then Gump's position is also modelled by (1.3):

$$\Delta x_{bond} = v_0 t_g + \frac{1}{2}at_g^2 = \frac{1}{2}(3(\frac{m}{s^2})t_g^2$$

To define their position with the same time parameter, notice $t = t_g + 3\ s$ because Gumps tarts running 3 s after Bond. Substituting this into Δx_{bond} gives:

$$\Delta x_{bond} = \frac{1}{2}(3(\frac{m}{s^2})t_g^2 = \frac{1}{2}(1.5(\frac{m}{s^2})(t-3\ s)^2$$

Solving for $x_{bond} = x_{gump}$ gives:

$$x_{bond} = x_{gump} \rightarrow \frac{1}{2}(1.5(\frac{m}{s^2})t^2 = \frac{1}{2}(3\frac{m}{s^2})(t-3\ s)^2$$

$$(1.5\frac{m}{s^2})t^2 = (3\frac{m}{s^2})(t-3\ s)^2$$

At this point, if we want to progress further, the units are going to get all muddled up, so we're going to ignore them for now. (If you ignore them, always do a dimensional analysis at the end!)

$$1.5t^2 = 3(t-3)^2 \rightarrow \sqrt{1.5}t = \sqrt{3}(t-3) \rightarrow \sqrt{1.5}t = \sqrt{3}t - 3\sqrt{3}$$

Solving this linear equation gives the answers t=10.24 s. Therefore, Gump caught up to Bond after 10.24 seconds.

Bond's velocity at this time can be figured out using (1.2) $v_f = v_0 + at$

$$v_f = v_0 + at = 0\frac{m}{s} + 1.5\frac{m}{s^2}(10.24\ s) = 15.36\frac{m}{s}$$

Similarly, Gump's velocity is:

$$v_f = v_0 + a(t-3) = 0\frac{m}{s} + 3\frac{m}{s^2}(7.24\ s) = 21.72\frac{m}{s}$$

Therefore, Gump is running faster than Bond by $21.7\frac{m}{s} - 15.4\frac{m}{s} = 6.3\frac{m}{s}$

Solution 1.3.10 (View Question) This another classic problem. We need to figure out how long it took for the kiwi to fall, and how long it took for the sound to arrive back. The crux is to realize both these time intervals depend solely on the distance. Thus, if we are able to represent both these time intervals in terms of the depth of the well, we can set the sum to 20 s and solve!

1. We need to solve for two variables, the time it takes for the kiwi to fall t_{fall} in terms of the height y_{well} and the time it takes for the sound to go back up t_{up} in terms of y_{well} and set the sum $t_{fall} + tup$ to 20 s!

2. We are given:

   - $v_0 = 0\frac{m}{s}$
   - $t_total = 20\ s$
   - $a = g = 10\frac{m}{s}$

3. First we need to find t_{fall} in terms of y which we can use (1.3) $\Delta y = v_0 t + \frac{1}{2}at^2$. The second variable t_{up} can be found using (1.1) $\Delta y = v_{sound}t$

4. Substituting to find t_{fall}:

$$\Delta y = v_0 t + \frac{1}{2}at^2 \rightarrow y_{well} = (0\frac{m}{s})t + \frac{1}{2}(10\frac{m}{s^2}t_{fall}^2 \rightarrow t_{fall} = \sqrt{\frac{y_{well}}{5\frac{m}{s^2}}}$$

Substituting to find t_{up}:

$$\Delta y = v_{sound}t \rightarrow y_{well} == 340\frac{m}{s} = t_{up} \rightarrow t_{up} = \frac{y_{well}}{340\frac{m}{s}}$$

Adding these together gives:

$$t_{fall} + t_{up} = 20\ s \rightarrow \sqrt{\frac{\Delta y_{well}}{5\frac{m}{s^2}}} + \frac{y_{well}}{340\frac{m}{s}} = 20\ s$$

$$(20\ s - \frac{y_{well}}{340\frac{m}{s}})^2 = \sqrt{\frac{y_{well}}{5\frac{m}{s^2}}}^2$$

$$400\ s^2 + \frac{y_{well}^2}{115600\frac{m^2}{s^2}} - \frac{2y_{well}}{17} = \frac{y_{well}}{5\frac{m}{s}}$$

Solving this quadratic equation yields $y_{well} = 1306m$ and $y_{well} = 35414m$. We have introduced a new solution by squaring both sides, but we see that if the well is 35414m deep, it will take sound over 100 s just to pass through. Plugging $y_{well} = 1306m$ into the expression for t_{fall} yields $t_{fall} = 16.16\ s$.

Solution 1.3.11 (View Question) We see that:

$$\frac{dx}{dt} = v_0 + at \rightarrow \int_{x_0}^{x_1} dx = \int_{t_0}^{t_1} v_0 + atdt \rightarrow x\Big|_{x_0}^{x_f} = (v_0 t + \frac{1}{2}at^2)\Big|_0^t$$

$$x_f - x_0 = v_0 t + \frac{1}{2}at^2$$

Solution 1.3.12 (View Question) For any question that has to do with the coordinate plane, remember to choose the origin. In this example, we choose our coordinate system such that the object's initial position is at the origin: $x_0 = y_0 = 0$. Thus:

$$x(t) = v_x t$$

$$y(x) = v_{y,0}t - \frac{1}{2}gt^2$$

We can covert the parametric equation $y(t)$ to $y(x)$ by making the substitution:

$$t = \frac{x}{v_x}$$

Substituting:

$$y(x) = v_{y,0}(\frac{x}{v_x}) - \frac{1}{2}g(\frac{x}{v_x})^2$$

$$y(x) = (\frac{v_{y,0}}{v_x})x + (\frac{-g}{2v_x^2})x^2$$

This is the equation pf a parabola that passes through the origin (as it should, based on the initial conditions). As long as $v_x \neq 0$, y(x) will be parabolic (If $v_x = 0$ then the path will simply be a one dimensional line up and down, which you can consider a limiting case of a parabola.

To double check, perform a dimensional analysis to see all terms have units of metres.

Solution 1.3.13 (View Question)

1. We are trying to find the plane's velocity, $v_{plane\ relative\ to\ ground}$ when we are given two sepereate pieces of information about the velocity of the aircraft in still air, and the strength and direction of the wind as a function of time. Hmm... this rings a bell. What about vector calculus?

2. We are given 2 pieces of information:
   - $v_{plane\ relative\ to\ air} = 200\frac{km}{h}\hat{\imath} + 20\frac{km}{h}\hat{\jmath}$
   - $v_{air\ relative\ to\ ground} = 20\frac{km}{h^2}t\hat{\imath} + 30\frac{km}{h^3}t^2\hat{\jmath}$

3. We can use (1.8)

4. Substituting:

$$\vec{v}_{p\ relative\ to\ b} = \vec{v}_{p\ relative\ to\ a} + \vec{v}_{a\ relative\ to\ b}$$

$$v_{plane\ relative\ to\ ground} = v_{plane\ relative\ to\ air} + v_{air\ relative\ to\ ground}$$

$$v_{plane\ relative\ to\ ground} = 200\frac{km}{h}\hat{\imath} + 20\frac{km}{h}\hat{\jmath} + 20\frac{km}{h^2}t\hat{\imath} + 30\frac{km}{h^3}t^2\hat{\jmath}$$

$$v_{plane\ relative\ to\ ground} = (200\frac{km}{h} + 20t\frac{km}{h^2})\hat{\imath} + (20\frac{km}{h} + 30t^2\frac{km}{h^3})\hat{\jmath}$$

$$\Delta r = \int dv = \int_{t=0h}^{t=1h} dv[(200\frac{km}{h} + 20t\frac{km}{h^2})\hat{\imath} + (20\frac{km}{h} + 30t^2\frac{km}{h^3})\hat{\jmath}]$$

$$\Delta r = [200t\frac{km}{h} + 10t^2\frac{km}{h^2})\hat{\imath} + (20t\frac{km}{h} + 10t^3\frac{km}{h^3})\hat{\jmath}\Big|_{t=0h}^{t=1h}$$

5. After solving, we get:

$$\Delta r = (210km)\hat{\imath} + (10km)\hat{\jmath}$$

Solution 1.3.14 (View Question) A (hint: the area underneath v(t) is position)

Solution 1.3.15 (View Question) B (hint: equal areas mean currently, they're in the same position)

Solution 1.3.16 (View Question) C (Hint: use (1.3))

Solution 1.3.17 (View Question) C (Hint: The acceleration due to gravity isn't g!)

Solution 1.3.18 (View Question) E (Hint: This is actually a stupid question. Use common sense!)

Solution 1.3.19 (View Question) C (Hint: Velocity at the top of the trajectory is zero!)

Solution 1.3.20 (View Question) B (Hint: First, you'll need to figure out the time it took, then solve for the speed)

Solution 1.3.21 (View Question) A (Hint: Go over what acceleration is again)

Solution 1.3.22 (View Question) D (Hint: First solve for acceleration, then solve for distance)

Solution 1.3.23 (View Question) E (Hint: It's going up, but slowing down!)

Solution 1.3.24 (View Question) D (Hint: It's very similar to 1.3.8)

Solution 1.3.25 (View Question) E (Hint: It's still moving forward, but it's not moving up anymore. Something's pulling it down!)

Solution 1.3.26 (View Question) C (Hint: Solve for time, then solve for velocity)

Solution 1.3.27 (View Question) D (Hint: For problems like this, double check by making sure the time it takes to fall 300 metres is over 6 seconds)

Solution 1.3.28 (View Question) D (Hint: Use a process of elimination!)

Solution 1.3.29 (View Question) C (Hint: If two trajectories ever cross paths, the one with the higher x velocity will land furthest)

Solution 1.3.30 (View Question) C (Hint: Use 1.2)

Solution 1.3.31 (View Question) E (Hint: Use 1.3)

Solution 1.3.32 (View Question) E (Hint: Use 1.3)

Solution 1.3.33 (View Question) E (Hint: The original and final speed is the same, due to the symmetry of the parabola)

Solution 1.3.34 (View Question) E (Hint: Use 1.4 to solve solve for v_1 then use 1.4 again to solve for v_2 in terms of v_1. Then substitute.)

Solution 1.3.35 (View Question) A (Hint: Sketch it out!)

Solution 1.3.36 (View Question) B (Hint: take the integral two times!)

Solution 1.3.37 (View Question) D (Hint: We're not looking at the location of v. We're looking at how v changes.

Solution 1.3.39 (View Question) A (Acceleration is the first derivative of velocity)

Solution 1.3.40 (View Question) E (Take the definite integral with 0 and 6 as bounds!)

Solution 1.3.41 (View Question) B (Hint: Acceleration is the second derivative of position)

Solution 1.3.42 (View Question) D (Hint: Pythagorean Theorem!)