PHY372: All-Photonics Quantum Repeaters

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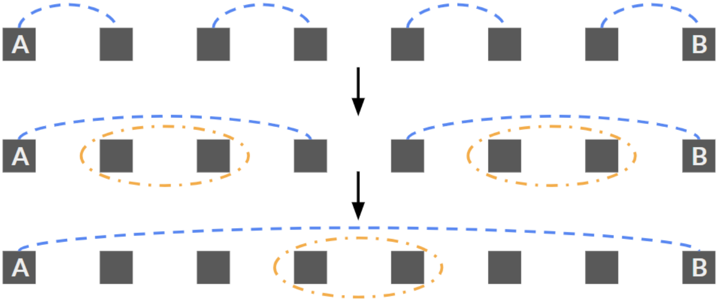
Motivation: Overview

- Current cryptography relies on unproven hardness of certain problems
- Quantum cryptography uses the laws of physics for unconditional security.
- **Challenge:** Channel loss and noise increase with distance.

Classical vs Quantum Repeaters

- Classical repeaters amplify signals to overcome loss in transmission medium.
- Quantum signals cannot be amplified due to the **no cloning theorem.**
- Instead, create long-distance entanglement linking distant nodes through 'entanglement swapping'.

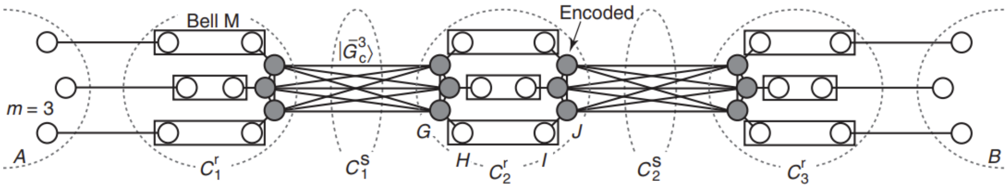


Challenges with Quantum Repeaters

- Quantum Memory
- Ability to convert between staionary and flying qubits
- $\implies$ Could be as hard as quantum computers!

All-Photonics Quantum Repeater

- No need for quantum memory.
- Multiple channels for creating entanglement.

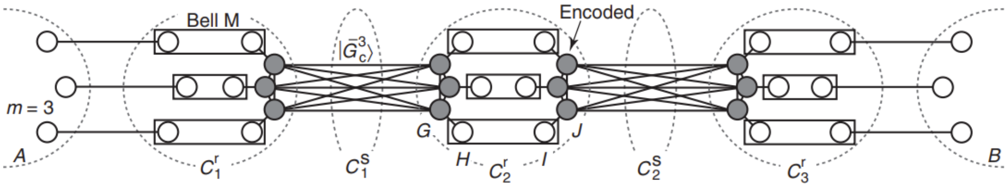


Overview of Photonic Quantum Repeaters

- Two stages:

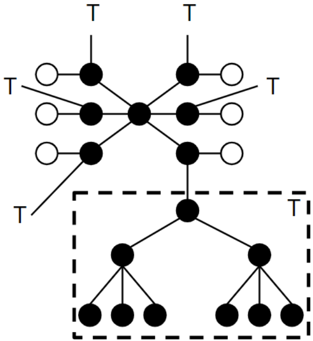
  Prepare Photonic Clusters $\longrightarrow$ Measurement + Error Correction

- **Preparation:** Compute P_{cn}, the probability to prepare the clusters
- **Measurement:** Compute the key rate R by looking at success probability of measurements.



Building a Cluster State: Repeater Nodes

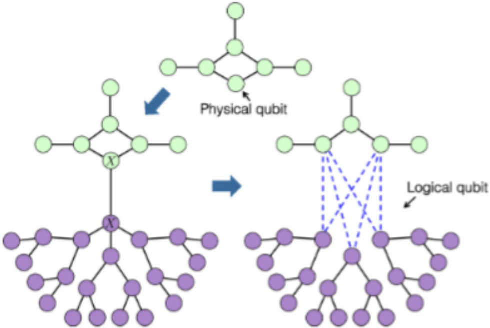
- Attach error protection trees with branching vector $\vec{b} = [b_0, b_1]$.
- White qubits are *outer qubits* that are transmitted to receiver nodes.



In above example, $\vec{b} = [2, 3]$ and $m = 3$.

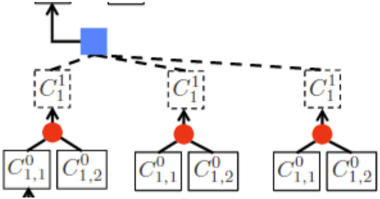
Building a Cluster State: Repeater Nodes

- $4m + 1$ basis measurements are done to turn tree into a cluster
- Allows for maximal error correction



Building a Cluster State: Fusion Operations

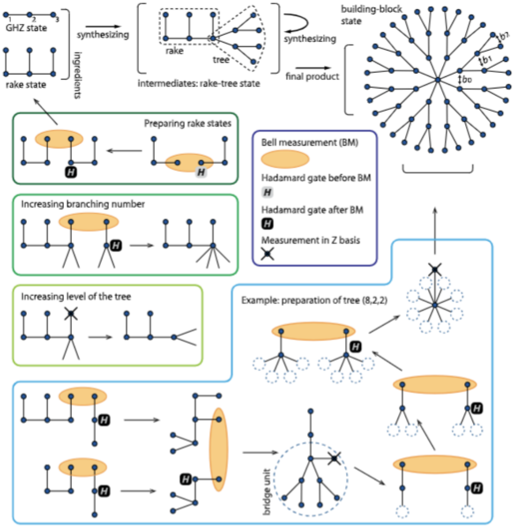
- Start from GHZ states.
- Fusion: Combine two states but remove two photons in the process.
- Combining two states of x photons will result in a state with $2x - 2$ photons.



- Given the probabilistic nature of creating GHZ states and the fusion operations, run these operations on multiple copies at each stage.

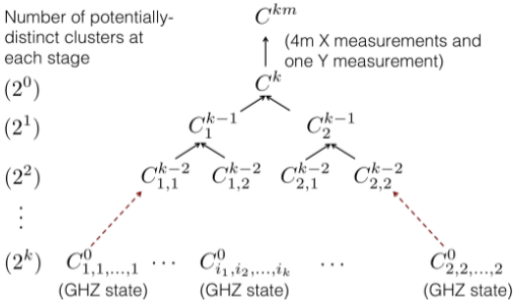
Building a Cluster State: Example

TODO: make my own better figure of more relevant example



Naive Approach

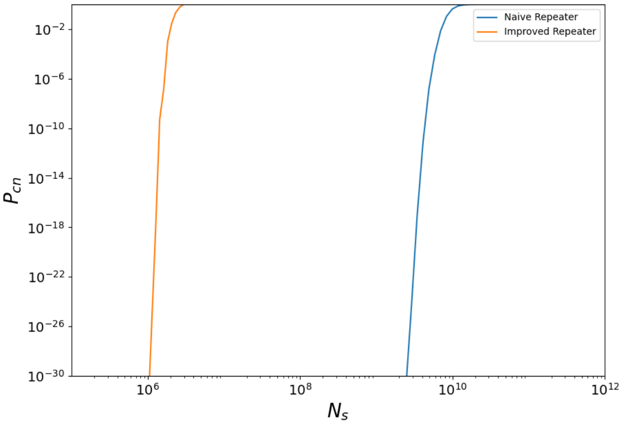
- Build up the tree, through time steps.
- Each time step, combine states of similar size to create larger intermediate states.
  - Each fusion step is attempted several times.
- Start with GHZ states.



- **Problem:** If multiple attempts at a given fusion step succeed, then the other successful ones are wasted.

Improved Approach

- **Solution:** Pool all the successful ones in a bank that any fusion step could pull from.
- Perform measurements early.



Rate Calculations

- Key rate, in terms of bits per mode, is computed following formula below.

$$R = \frac{P_{cn} P_{meas}}{N_{parallel}}$$

- For improved scheme,

$$N_{\text{parallel}} = 2m \tag{1}$$

- For naive scheme,

$$N_{\text{parallel}} = 2m + 2m(b_0 b_1 + b_0) \tag{2}$$

Determining Loss Rates

- For outer qubits, loss rate is given by:

$$\epsilon_{trav} = 1 - \eta^{1/(2n)} P_{chip}^{k+2} \eta_{GHZ} \eta_c$$

- For inner qubits, loss rate is given by:

$$\epsilon_{stat} = 1 - \eta^{1/n} P_{fib} P_{chip}^{k+2} \eta_{GHZ} \eta_c$$

Determining Probabilities

- Probability of an indriect Z masurement at ith level, ξ_i given by

$$\xi_i = 1 - [1 - (1 - \epsilon_{stat})(1 - \epsilon_{stat} + \epsilon_{stat}\xi_{i+2})^{b_{i+1}}]^{b_i}, \text{ for } i \leq \ell, \xi_{\ell+1} = b_{\ell+1} = 0.$$

- P_X, P_Z is calculated as:

$$P_X = 1 - [1 - (1 - \epsilon_{stat})^{b_1+1}]^{b_0}$$
$$P_Z = (1 - \epsilon_{stat}^{b_1+1})^{b_0}$$

- Establish success rate of Bell state measurements:

$$P_B = \left[\frac{1}{2}(\eta_s\eta_d)^2 + \frac{1}{4}(\eta_s\eta_d)^4\right] \cdot \left(P_{chip}^{k+2}\eta_{GHZ}\eta_c\right)^2 \cdot \eta^{1/n}$$

- Probability of at least one successful detection at ends is

$$P_{end} = 1 - (1 - \eta^{1/(2n)}P_{chip}^{k+2}\eta_{GHZ}\eta_c)$$

Naive Approach Modifications

- In naive approach, the number of parallel channels and success probability of Bell measurements change.

$$N_{\text{parallel}} : 2m \to 2m + 2m(b_0 b_1 + b_0)$$

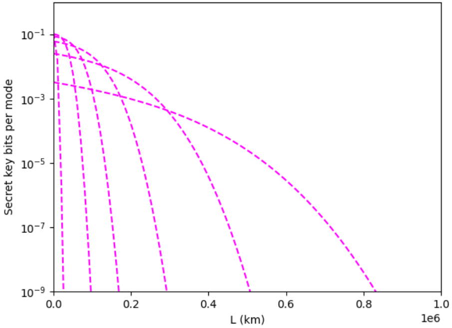
$$P_{\text{BSM}} : \frac{3}{4} \to \frac{1}{2}$$

$$\epsilon : \epsilon_{\text{stat}} \to \epsilon_{\text{trav}}$$

k	m_{naive}	$\vec{b}_{\text{naive}}$	R_{naive}	m_{improved}	$\vec{b}_{\text{improved}}$	R_{improved}
7	5	$[3, 2]$	1.91×10^{-8}	4	$[4, 2]$	3.87×10^{-6}
8	8	$[4, 2]$	6.13×10^{-6}	5	$[5, 3]$	2.98×10^{-3}
9	11	$[5, 3]$	5.77×10^{-4}	7	$[6, 4]$	2.71×10^{-2}
10	13	$[7, 4]$	7.42×10^{-4}	8	$[10, 5]$	4.93×10^{-2}

Optimizing the number of repeater nodes

- Optimal value of n is different for each value of L

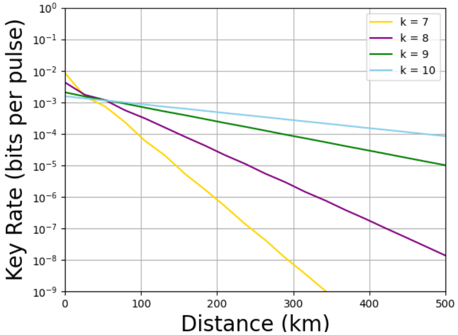


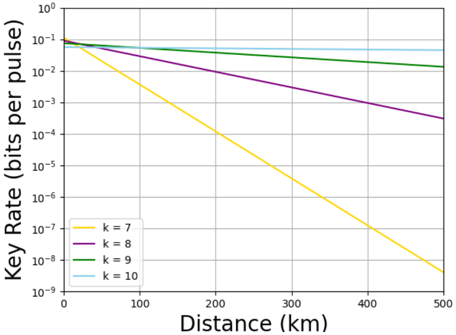
- Satisfies the relationship

$$\frac{L}{n} = 1.5 \text{ km}.$$

Comparison

- Rate vs Distance for Naive (left) and Improved (right):





The End