Fintech545 Homework 1

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Question 1. Hypothesis testing on Skewness & Kurtosis Functions

As a reference, my functions are based on the sklearn library from python. Here are some introduction about preparations that I take to conduct the Hypothesis Testing.

- Generate 100 samples with 100 data points from Standard Normal Distribution N(0,1);
- Calculate skewness and Kurtosis for each samples, and calculate mean and variance of skewness and kurtosis, respectively.
- Formulate Hypothesis of Skewness and Kurtosis, and get the T-statistics to calculate p-value.

Hypothesis Testing on Skewness Functions

$$H_0 : \text{The Skewness Function is unbiased}$$
$$H_1 : \text{The Skewness Function is biased}$$

Since we already know that the skewness of standard normal distribution is 0, the Hypothesis can be transferred into a new form:

$$H_0 : \text{Skewness} = 0$$
$$H_1 : \text{Skewness} \neq 0$$

According to the simulation results,

$$t_0 = 0.94$$
$$p_0 = 2P(t(99) > 0.94) = 0.35 > 0.1$$

So, we failed to reject the H_0 at 0.1 significance level.

Hypothesis Testing on Kurtosis Functions

With the knowledge that the Kurtosis of standard normal distribution is 3, we can formulate the Hypothesis:

$$H_0 : \text{Kurtosis} = 3$$
$$H_1 : \text{Kurtosis} \neq 3$$

According to the simulation results,

$$t_0 = -2.49$$
$$p_0 = 2P(t(99) > 2.49) = 0.014 < 0.1$$

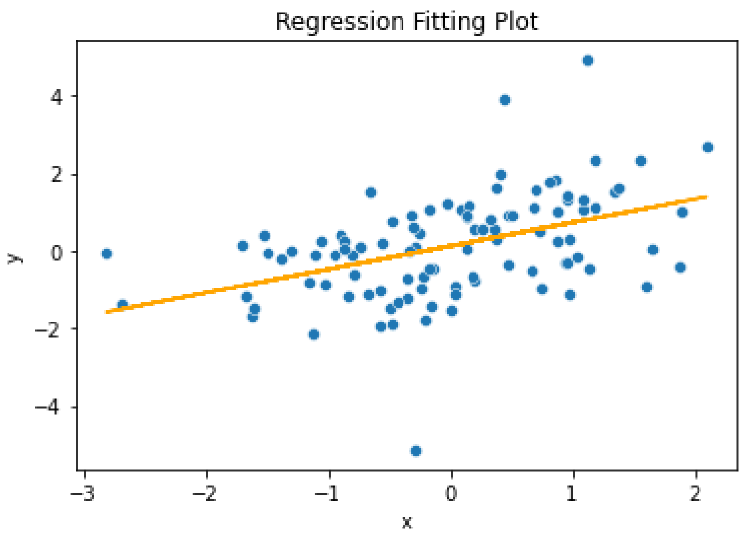
So, we can reject the H_0 at 0.1 significance level.
Therefore, we can conclude that the Skewness function in sklearn library is not unbiased, but Kurtosis function is biased.

Question 2: Least Square & MLE fitting on the Normal Distribution or T-distribution

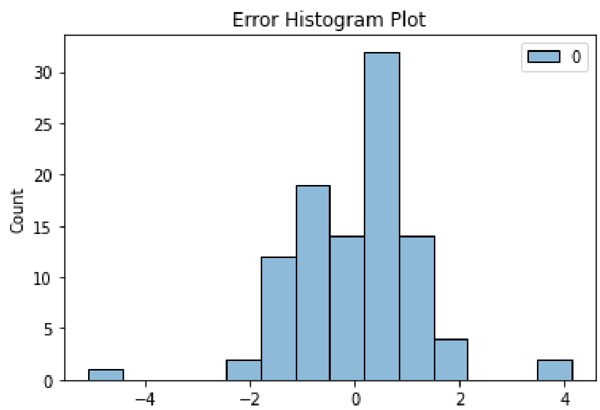
Least Sqaure Method(OLS)

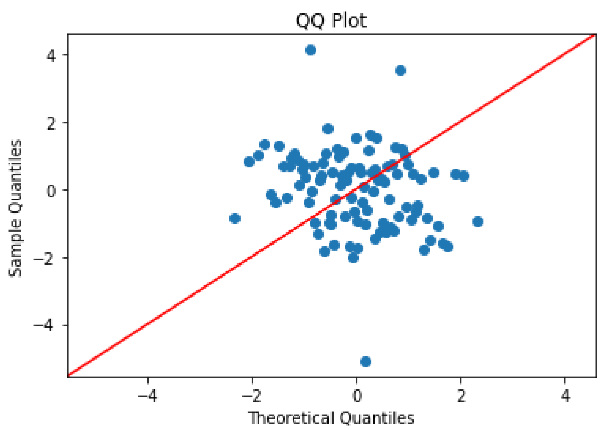
After using least square method to fit the x and y data, we get the relationship between x and y:

$$y = 0.6052x + 0.1198$$



Normality Checking (OLS)





For the first step, I made a Histogram graph for error terms, but it does not have a obvious bell form. Then, I made a QQ plot, which shows the probability relationship between error term and normal distribution. However, the error terms do not fit the line. Therefore, we can make a conclusion that the error term do meet the normality assumption.

Maximum Likelihood Assumption

Normality Assumption of Error Term

we can formulate the optimization questions into:

$$\arg max_\beta - \frac{n}{2} ln(\sigma^2 2\pi) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \beta_1 x - \beta_0 - 0)^2$$

T Distribution Assumption of Error Term

we can formulate the optimization problems into:

$$\arg max_\beta \log(\prod_{i=1}^{n} \frac{\Gamma((\nu+1)/2)}{\sigma\sqrt{\nu\pi}\,\Gamma(\nu/2)} (1 + (\frac{x-\mu}{\sigma})^2/\nu)^{-(\nu+1)/2})$$

MLE on Normal Distribution: y =0.6052x+0.1198

MLE on T Distribution: y = 0.5576x + 0.1426

Information Criteria

Method \ Criteria	R^2	AIC	BIC
Normality Assumption	0.1946	325.98	333.80
T Assumption	0.1934	319.17	329.60

Generally, the MLE estimator under the t assumption has better performance on the information Criteria.

Comparing the parameters, under t distribution assumption, linear regression has a smaller slope but a larger intercept.

- From the plot, we can find that there are 3 outliers on the upper right side of the scatter plot. The normal distribution has a thinner tail than t distribution. Normality assumption is more sensitive to outliers, which may led to a decrease in the goodness of fit. For t-distribution, it tends to be less sensitive to outliers, which leads that the fitted line has a better performance.

- We can make the conclusion that for breaking the normality assumption, t distribution assumption is a better choice, since it has a better fitting performance and a higher tolerant level for outliers.

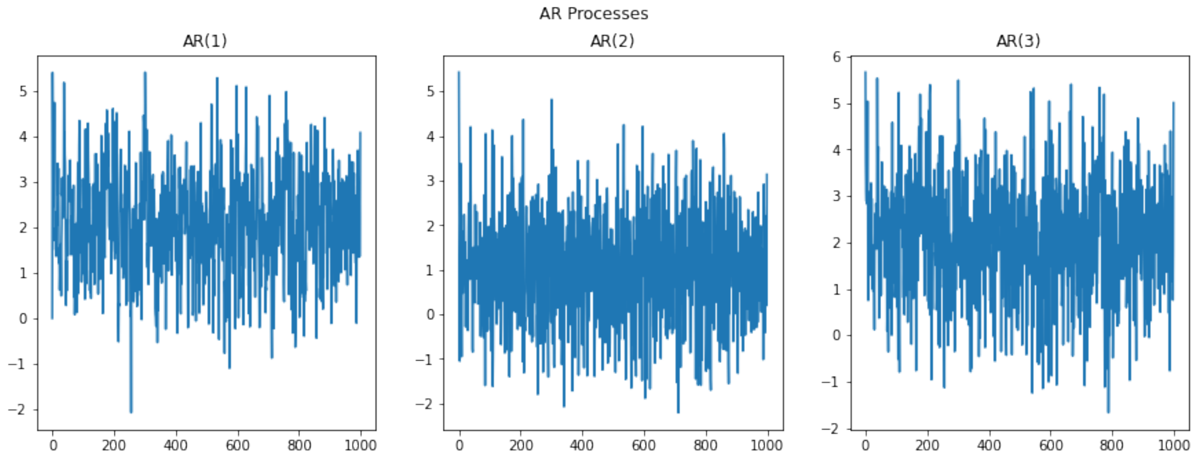
Question 3: AR(q) and MA(q) Processes Simulation

My simulation are based on the following functions:

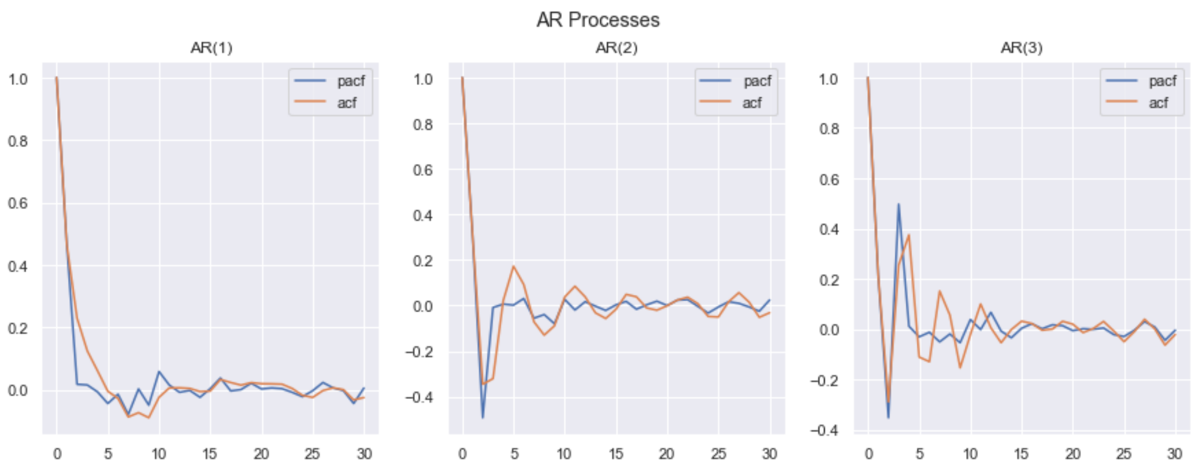
AR(q) Processes

$$AR(1) : y_t = 1 + 0.5 \times y_{t-1} + \epsilon_t$$
$$AR(2) : y_t = 1 + 0.5 \times y_{t-1} - 0.5 \times y_{t-2} + \epsilon_t$$
$$AR(3) : y_t = 1 + 0.5 \times y_{t-1} - 0.5 \times y_{t-2} + +0.5 \times y_{t-3} + \epsilon_t$$

AR(q) Processes Time Series Graphs



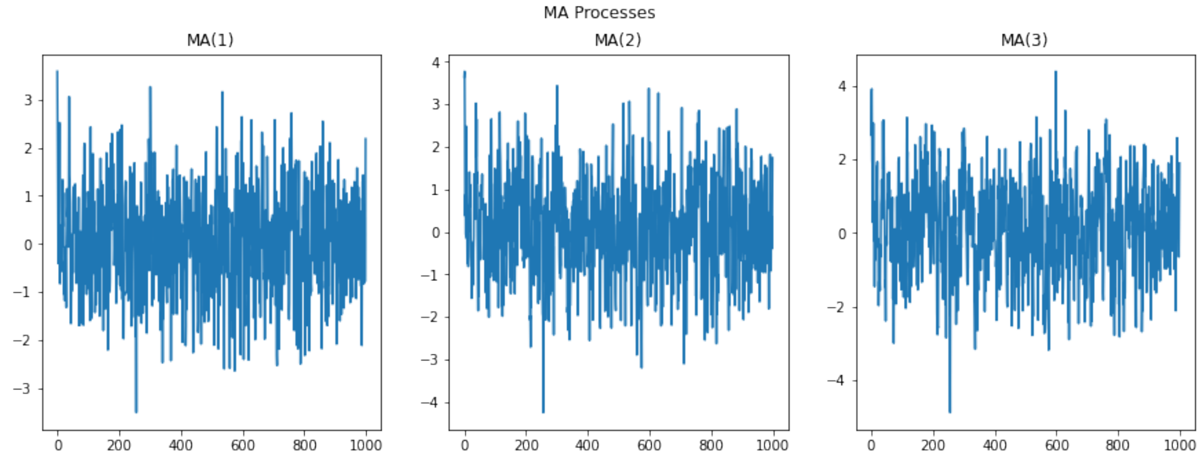
AR(q) Autocovariance and Partial Autocovariance Graphs



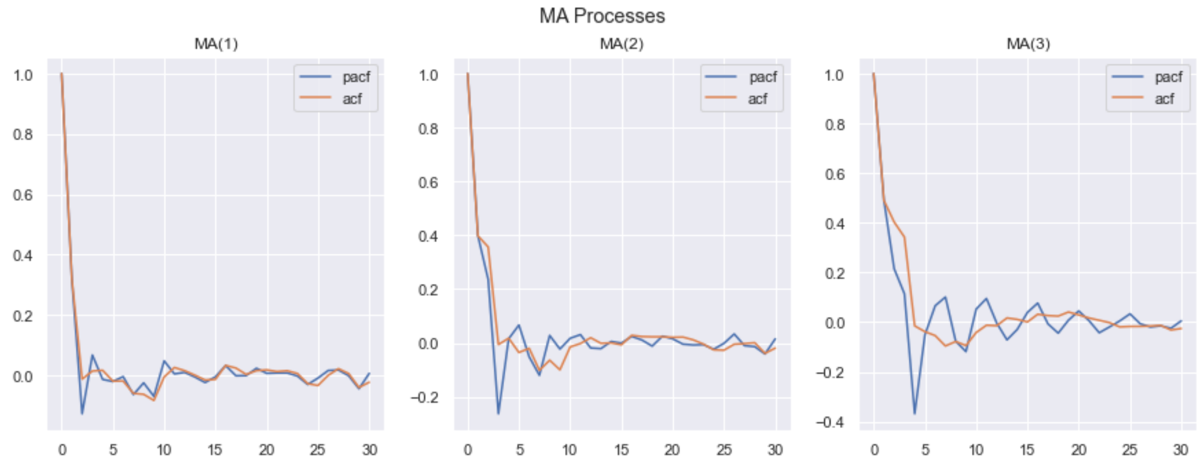
MA(q) Processes

$$MA(1) : y_t = \epsilon_t + 0.4 \times \epsilon_{t-1}$$
$$MA(2) : y_t = \epsilon_t + 0.4 \times \epsilon_{t-1} + 0.5 \times \epsilon_{t-2}$$
$$MA(3) : y_t = \epsilon_t + 0.4 \times \epsilon_{t-1} + 0.5 \times \epsilon_{t-2} + 0.6 \times \epsilon_{t-3}$$

MA(q) Processes Time Series Graphs



MA(q) Autocovariance and Partial Autocovariance Graphs



According to above graphs, we have found two rules to distinguish types and orders:

- **Orders:** No matter whether it is MA Processes or AR Processes, the Processes with higher order have more fluctuant ACF and PACF. Therefore, we can judge the order of Processes through the fluctuantion of line plots.
- **Types:** For MA(q) Processes, PACF is generally more flexible than ACF. For AR(q) Processes, ACF is generally more flexible than PACF. Therefore, we can know the type of Processes through the difference of flexibility between AR and MA Processes.