Low-Complexity LSQR-Based Linear Precoding for Massive MIMO Systems

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***Abstract*—Massive multiple-input multiple-output (MIMO) using a large number of antennas at the base station (BS) is a promising technique for the next-generation 5G wireless communications. It has been shown that linear precoding schemes can achieve near-optimal performance in massive MIMO systems. However, classical linear precoding schemes such as zero-forcing (ZF) precoding suffer from high complexity due to the fact they require the matrix inversion of a large size. In this paper, we propose a low-complexity precoding scheme based on the least square QR (LSQR) method to realize the near-optimal performance of ZF precoding without matrix inversion. We show that the proposed LSQR-based precoding can reduce the complexity of ZF precoding by about one order of magnitude. Simulation results verify that the proposed LSQR-based precoding can provide a better tradeoff between complexity and performance than the recently proposed Neumann-based precoding.**

I. Introduction

MIMO technology has played an essential role in current wireless communications systems due to it can improve the spectrum efficiency without additional requirements of bandwidth or power [1]. However, the state-of-the-art MIMO technology can not meet the exponentially increasing demand of mobile traffic due to a very limited number of antennas is usually used, e.g., only two transmit antennas are considered for the next generation wireless broadcasting standard DVB-T2 [2], and at most eight antennas can be adopted by LTE-A cellular networks [3]. Recently, massive MIMO is proposed to simultaneously improve the spectrum and energy efficiency by several orders of magnitudes by using a large number of antennas at the base station (BS) [4]. Thus, massive MIMO is considered as a promising technology for 5G wireless communications [5].

It is well known that the optimal precoding scheme for MIMO is dirty paper coding (DPC) [6], which can achieve the performance bound of channel capacity at the cost of unaffordable complexity. Other nonlinear precoding schemes that can achieve the near-optimal performance include vector perturbation (VP) [7], lattice-aided precoding [8], etc. Unfortunately, their complexity is still too high when the number of antennas becomes large. Fortunately, due to the "favorable propagation" property caused by the substantially increased number of BS antennas in massive MIMO systems, simple linear precoding schemes like zero-forcing (ZF) precoding can achieve the near-optimal performance with reduced complexity [9]. However, ZF precoding still suffers from high complexity due to the required matrix inversion of large size in massive MIMO systems, where the number of users could be also large. An approximation method based on truncated Neumann series expansion is proposed in [10] (which is called as "Neumann-based precodng" in this paper) to reduce the complexity by approximating the matrix inversion via a series of matrix-vector product, but the reduction in complexity is not obvious enough.

In this paper, we propose a low-complexity precoding scheme based on the least square QR (LSQR) method to significantly reduce the complexity of the classical ZF precoding. The proposed LSQR-based precoding iteratively computes the expected signal after precoding based on QR decomposition to avoid the undesirable matrix inversion of large size. By exploiting the asymptotically orthogonal property of massive MIMO systems [4], we can complete the iteration procedure using a simple approach. Particularly, the proposed LSQR-based precoding does not explicitly need the Gram form of channel matrix, so that the proposed LSQR-based precoding can reduce the complexity by about one order of magnitude compared with the conventional linear precoding schemes. In addition, simulation results also verify that the proposed LSQR-based precoding can approach the near-optimal performance of ZF precoding with only a small number of iterations.

The rest of the paper is organized as follows: Section II introduces the system model of massive MIMO. In Section III, we present the proposed LSQR-based precoding, and the analysis of computational complexity is provided as well. Simulation results are shown in Section IV. Finally, Section V concludes this paper.

II. System Model

We consider a typical massive MIMO system where the BS with N antennas is serving K single-antenna users [11]. Usually $N \gg K$ is assumed, e.g., $N = 128$ and $K = 16$ have been considered in [9]. In the downlink, the received signal $\mathbf{y} \in \mathbb{C}^{K\times 1}$ for K users can be expressed as

$$\mathbf{y} = \sqrt{\rho_f}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

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where ρ_f denotes the downlink signal-to-noise ratio (SNR), $\mathbf{H} \in \mathbb{C}^{K\times N}$ is the downlink channel matrix, $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the signal vector after precoding, and $\mathbf{n} \in \mathbb{C}^{K\times 1}$ following the distribution $\mathcal{CN}(0,1)$ denotes the noise. Additionally, $\mathbf{H}^H = [\mathbf{h}_1^H, \mathbf{h}_2^H, \cdots, \mathbf{h}_K^H]$, where $\mathbf{h}_k \in \mathbb{C}^{1\times N}$ is the channel vector for user k, which can be modeled as an independent and identically distributed (i.i.d.) random vector. For massive MIMO systems, linear precoding is usually considered, so we have

$$\mathbf{x} = \mathbf{P}\mathbf{s}, \tag{2}$$

where $\mathbf{P} \in \mathbb{C}^{N\times K}$ is the precoding matrix, and $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the transmitted signal vector for K users. In addition, the BS has the following power constraint:

$$\mathrm{E}[||\mathbf{x}||_2^2] \le K. \tag{3}$$

In this paper, we assume that channel state information (CSI) is known at the BS, which is a common assumption in massive MIMO systems [4], [12]–[14] and can be achieved by using the time-domain and/or frequency-domain training pilot [15]–[21]. More specifically, in time division duplex (TDD) massive MIMO systems, the BS estimates the uplink channel using the pilots that users send in the uplink, and then the downlink CSI can be easily obtained by using existing channel reciprocity in TDD systems.

When the number of BS antennas N tends to infinity in massive MIMO systems, we have

$$\lim_{N\to\infty} \frac{\mathbf{h}_k\mathbf{h}_k^H}{N} \overset{a.s.}{\to} 1 \text{ and } \lim_{N\to\infty} \frac{\mathbf{h}_k\mathbf{h}_j^H}{N} \overset{a.s.}{\to} 0, \tag{4}$$

where $\overset{a.s.}{\to}$ denotes almost sure convergence. Eq. (4) implies the well-known "favorable propagation" property of massive MIMO channels [4] (i.e., different channels among users are asymptotically orthogonal), which will be used by the proposed LSQR-based precoding scheme in the next section.

III. Proposed LSQR-Based Precoding

In this section, we first briefly review the conventional ZF precoding. Next, the low-complexity LSQR-based precoding scheme is proposed. Finally, analysis of computational complexity is provided to show the advantage of the proposed scheme over conventional solutions.

A. Conventional ZF Precoding

According to [9], the conventional ZF precoding matrix $\mathbf{P}_{\text{ZF}}$ can be expressed as

$$\mathbf{P}_{\text{ZF}} = \beta_{\text{ZF}}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1} = \beta_{\text{ZF}}\mathbf{H}^H\mathbf{G}^{-1}, \tag{5}$$

where $\mathbf{G} = \mathbf{H}\mathbf{H}^H$, and β_{ZF} denotes the power normalization factor. Based on (3), β_{ZF} can be selected as

$$\beta_{\text{ZF}} = \sqrt{\frac{K}{\text{tr}(\mathbf{G}^{-1})}}. \tag{6}$$

Combining (2) and (5), we can rewrite the transmitted signal $\mathbf{x}$ as

$$\mathbf{x} = \beta_{\text{ZF}}\mathbf{H}^H\mathbf{G}^{-1}\mathbf{s} = \beta_{\text{ZF}}\mathbf{H}^H\mathbf{t}, \tag{7}$$

where

$$\mathbf{G}^{-1}\mathbf{s} = \mathbf{t}. \tag{8}$$

Based on (1), (2) and (5), we can use the matrix $\mathbf{W} = \mathbf{H}\mathbf{P}_{\text{ZF}}$ to present the equivalent channel. Since we have $|w_{ki}|^2 = 0$ for $k \ne i$, the signal-to-interference-plus-noise ratio (SINR) for user k can be computed as

$$\gamma_k = \frac{\frac{\rho_f}{K}|w_{kk}|^2}{\frac{\rho_f}{K}\sum_{i\ne k}^{K}|w_{ki}|^2 + 1} = \frac{\rho_f}{K}|w_{kk}|^2 = \frac{\rho_f}{\text{tr}(\mathbf{G}^{-1})}, \tag{9}$$

where w_{ki} denotes the element of $\mathbf{W}$ in the kth row and the ith column. Furthermore, the sum rate C_{ZF} achieved by ZF precoding is [22]

$$C_{\text{ZF}} = \sum_{i=1}^{K}\log_2(1+\gamma_i) = K\log_2\left(1 + \frac{\rho_f}{\text{tr}(\mathbf{G}^{-1})}\right). \tag{10}$$

As shown in (5), a matrix inversion of large size is required by ZF precoding, whose complexity grows with cubic of $\min(N, K)$. In massive MIMO systems, both N and K could be large, which means that the resulting high complexity is unexpected.

B. LSQR-Based Precoding

To reduce the complexity for precoding, we propose to use LSQR method to avoid the complicated matrix inversion of large size in ZF precoding. The LSQR algorithm is a numerical iterative method based on QR decomposition to monotonically reduce the norm of residual $\mathbf{r}_k = \mathbf{b} - \mathbf{A}\mathbf{x}_k$ in the least square problem $\mathbf{A}\mathbf{x} = \mathbf{b}$ of large size [23]. The proposed LSQR-based precoding based on LSQR algorithm to iteratively realize ZF precoding without directly computing the matrix inversion is specified in **Algorithm 1**. Specifically, we initialize the solution vector together with the involved parameters in steps 1-3, i.e., we set the initial solution as a zero vector since no prior information of the final solution is available. Then, in steps 4-13, the solution is iteratively computed, where the iteration procedure can be divided into three parts:

1) *Steps 4-7:* Bi-diagonalizing the matrix $\mathbf{H}\mathbf{H}^H$;
2) *Steps 8-11:* Constructing and applying rotations;
3) *Steps 12-13:* Updating the solution.

According to (4), when the number of BS antennas tends to infinity, the Gram form of $\mathbf{H}$ is diagonal dominant, i.e.

$$\lim_{N\to\infty} \mathbf{H}\mathbf{H}^H\mathbf{x} \overset{a.s.}{\to} \mathbf{D}\mathbf{x} \approx N\mathbf{I}_K\mathbf{x}, \tag{11}$$

where $\mathbf{D} = \text{diag}(\mathbf{h}_1\mathbf{h}_1^H, \mathbf{h}_2\mathbf{h}_2^H, \cdots, \mathbf{h}_K\mathbf{h}_K^H)$, and $\mathbf{I}_K$ is the identity matrix of size $K \times K$. So we can simply compute $\mathbf{H}\mathbf{H}^H$ in steps 2, 4, 6 by only taking the diagonal component of $\mathbf{H}\mathbf{H}^H$ into account. This approximation can further reduce the complexity of LSQR-based precoding, since we only need to compute and store the matrix $\mathbf{D}$, which stays unchanged during the whole procedure. However, such reduction of computational complexity is achieved at the cost of increased approximation error. Based on (4) and the related discussion

Input: 1) The downlink channel matrix $\mathbf{H}$;
2) the transmitted signal vector for K users $\mathbf{s}$;
3) iteration number k.
Output: signal vector $\mathbf{x}$ after precoding
Initialize:
$[N,K] = \text{size}(\mathbf{H})$; $\mathbf{x}_0 = \text{zeros}(K,1)$; $\beta = \text{norm}(\mathbf{s})$;
$\mathbf{u} = \frac{1}{\beta}\mathbf{s}$; $\mathbf{v} = \mathbf{H}\mathbf{H}^H\mathbf{u}$; $\alpha = \text{norm}(\mathbf{v})$;
$\mathbf{v} = \frac{1}{\alpha}\mathbf{v}$; $\mathbf{w} = \mathbf{v}$; $\rho = \alpha$; $\phi = \beta$;
iteration
for $i = 1:k$
$\mathbf{u} = \mathbf{H}\mathbf{H}^H\mathbf{v} - \alpha\mathbf{u}$;
$\beta = \text{norm}(\mathbf{u})$; $\mathbf{u} = \frac{1}{\beta}\mathbf{s}$;
$\mathbf{v} = \mathbf{H}\mathbf{H}^H\mathbf{u} - \alpha\mathbf{v}$;
$\alpha = \text{norm}(\mathbf{v})$; $\mathbf{v} = \frac{1}{\alpha}\mathbf{v}$;
$\xi = \text{norm}([\rho,\beta])$;
$c = \frac{\rho}{\xi}$; $s = \frac{\beta}{\rho}$;
$\theta = s\alpha$; $\rho = -c\alpha$;
$\tau = c\phi$; $\phi = s\phi$
$\mathbf{x} = \mathbf{x} + (\frac{\tau}{\xi})\mathbf{w}$;
$\mathbf{w} = \mathbf{v} + (\frac{\theta}{\xi})\mathbf{w}$;
end

Algorithm 1: LSQR-Based Precoding

in [24], the approximation in (11) becomes more accurate when N is sufficiently large, while it suffers from some performance loss when the number of BS antennas can not be too large in practical massive MIMO systems. In addition, another advantage of LSQR-based precoding is that it has higher regularity than the commonly used Cholesky-based approach to solve matrix inversion. This advantage enables more efficient hardware design for practical systems [25].

C. Complexity Analysis

In this subsection, we will numerically analyze the lower computational complexity of proposed LSQR-based precoding. As the computational complexity is dominated by multiplications, we evaluate the complexity in number of complex multiplications. In each iteration, the corresponding times of complex multiplication involved in the three steps as mentioned are: 1) $2NK + 3K$ for step 1, since we need to compute the product of a matrix and a vector; 2) $\mathcal{O}(1)$ for step 2, as there is only scalar operations; 3) $2K$ for step 3, where we need to update two vectors, respectively. Thus, the total complexity is $i(2NK + 5K) + 4K$, where i denotes the iteration number. The complexity comparison between the recently proposed Neumann-based precoding and our proposed LSQR-based precoding is shown in Table I below.

From Table I, we can conclude that when $i > 2$, the complexity of Neumann-based precoding is still $\mathcal{O}(K^3)$, which is of the same order as ZF precoding with exact matrix inversion. By contrast, the proposed LSQR-based precoding enjoys the reduced complexity $\mathcal{O}(K^2)$ for any number of iteration. What's more, the Neumann-based precoding needs to computed the Gram form of the channel matrix in advance, which is of high complexity NK^2 and needs at least K^2 storage space. Specially, when N grows large and $N \gg K$ in massive MIMO systems, the term NK^2 takes the dominant complexity, which is difficult to reduce because it is a premise to use Neumann-based precoding. In comparison, the LSQR-based precoding computes the Gram form by the product of a matrix and a vector, which has much smaller complexity NK and only needs a $K \times 1$ vector storage space. Thus, we can conclude that LSQR-based precoding can reduce the complexity of linear precoding by about one order of magnitude compared with conventional schemes and avoids large storage space.

TABLE I
COMPUTATIONAL COMPLEXITY

Iteration number	Neumann-based precoding [10]	LSQR-based precoding
$i = 2$	$NK^2 + 3K^2 - K$	$5NK + 14K$
$i = 3$	$K^3 + NK^2 + K^2$	$7NK + 19K$
$i = 4$	$2K^3 + NK^2$	$9NK + 24K$
$i = 5$	$3K^3 + NK^2 - K^2$	$11NK + 29K$

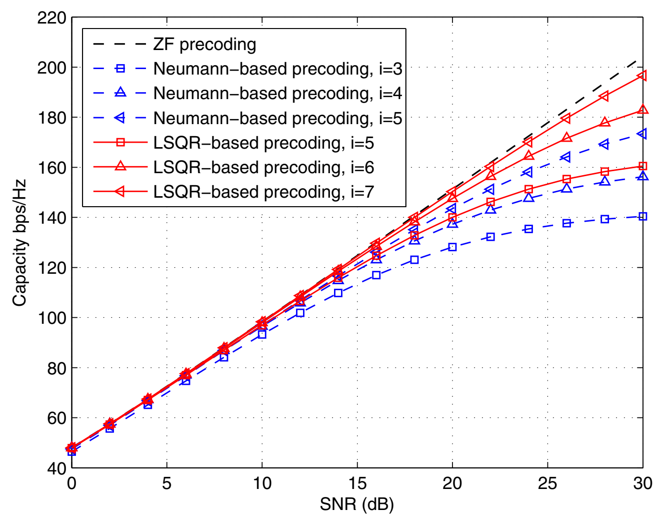

Fig. 1. Capacity comparison in a 128×16 massive MIMO system.

IV. SIMULATION RESULTS

In this section, we present some simulation results of the achievable capacity and bit error rate (BER) to illustrate the performance of LSQR-based precoding. Here, we consider two typical massive MIMO systems with $N \times K = 128 \times 16$ and $N \times K = 256 \times 16$ in Rayleigh fading channels, respectively.

Fig. 1 and Fig. 2 compare the achievable capacity against SNR when different precoding schemes are adopted in $N \times K = 128 \times 16$ and $N \times K = 256 \times 16$ massive MIMO systems, respectively. We can observe that as the number of BS antennas N increases, the performance of all adopted linear precoding schemes improves, since more BS antennas provide more degrees of freedom, and different channel vectors are more likely to be orthogonal [9]. Compared with the recently proposed Neumann-based precoding, the proposed LSQR-based precoding generally requires more iterations to achieve

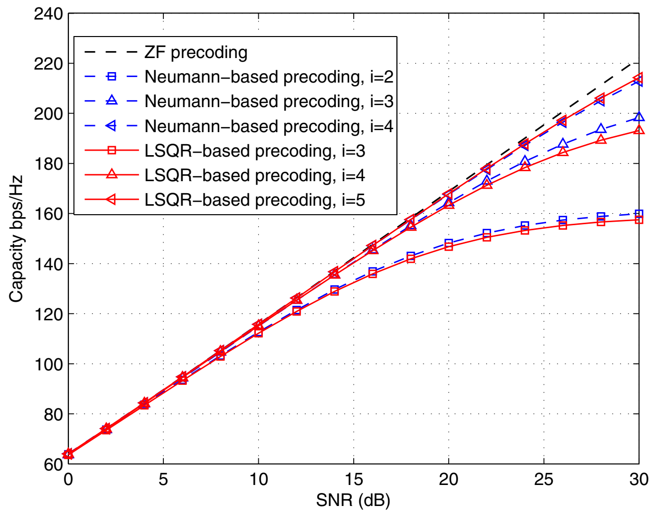

Fig. 2. Capacity comparison in a 256×16 massive MIMO system.

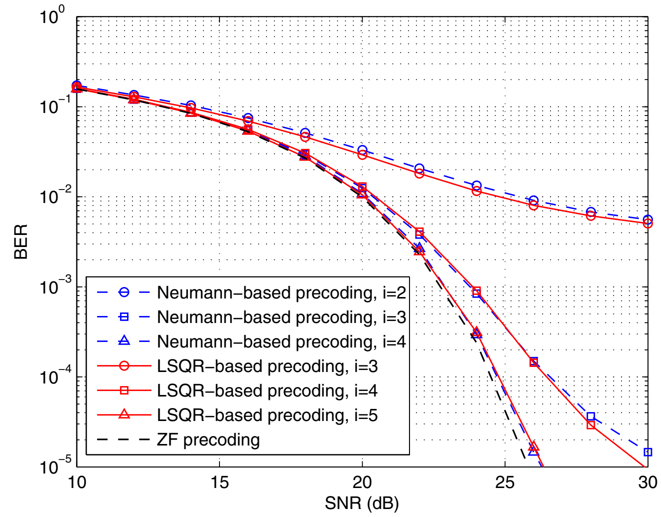

Fig. 4. BER comparison in a 256×16 massive MIMO system in Rayleigh fading channel.

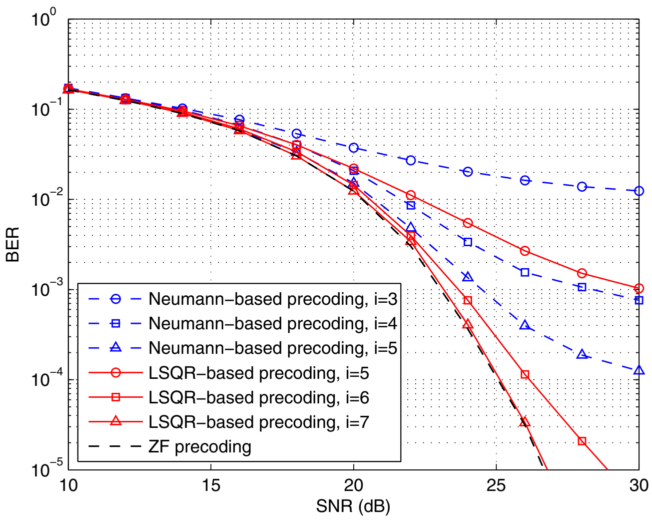

Fig. 3. BER comparison in a 128×16 massive MIMO system in Rayleigh fading channel.

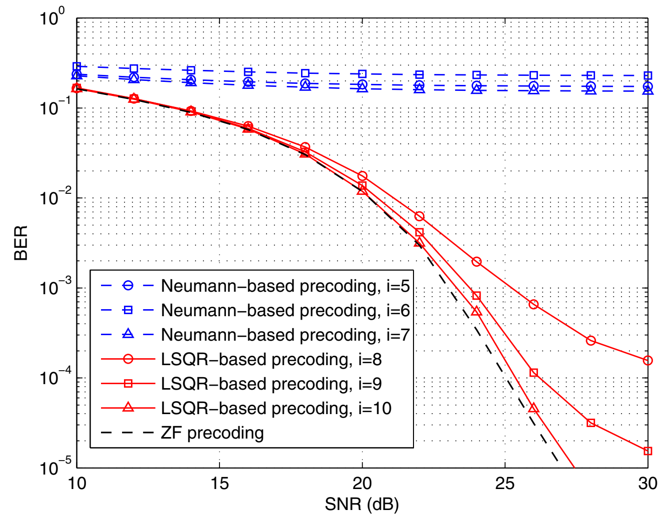

Fig. 5. BER comparison in a 128×16 massive MIMO system in spatial correlated channel.

similar performance. However, LSQR-based precoding still has a much lower complexity according to Table I, and the reduction in complexity is more obvious when i grows large. Moreover, the proposed LSQR-based precoding can approach the performance of the classical ZF precoding with a small number of iterations, which verifies the near-optimal performance of the proposed scheme.

The BER performance comparison is presented in Fig. 3 and Fig. 4. The modulation scheme is 64QAM. From Fig. 3, we can see that when the iteration number is smaller (e.g. $i = 3$ in Neumann-based precoding and $i = 5$ in LSQR-based precoding), both precoding schemes suffer from an obvious BER error floor, which indicates that we should adopt more iterations to guarantee the performance. When more antennas are equipped at BS (see Fig. 4), though the performance of Neumann-based improves, it still needs at least 4 iterations to achieve the near-optimal performance. Given similar BER performance (e.g., $i = 3$ in Neumann-based precoding and $i = 4$ in LSQR-based precoding in Fig. 4), in spite that LSQR-based precoding requires more iterations, the overall complexity of LSQR-based precoding is much lower than that of Neumann-based precoding according to the analysis in Section III-C. Additionally, LSQR-based precoding can achieve the near-optimal performance of ZF precoding within acceptable number of iterations (e.g., $i = 7$ in Fig. 3 and $i = 5$ in Fig. 4).

Next, we will give the BER performance comparison under spatial correlation MIMO channels, which play a crucial role in realistic MIMO systems. The exponential correlation channel model according to [26] is adopted here, where ξ

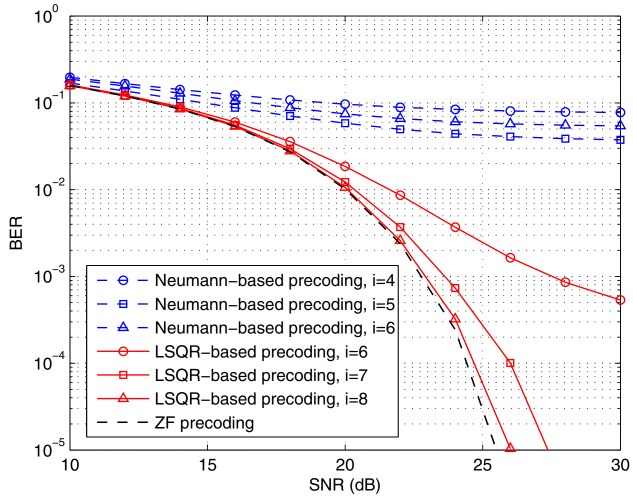

Fig. 6. BER comparison in a 256×16 massive MIMO system in spatial correlated channel.

$(0 \leq \xi \leq 2)$ denotes the antenna correlation factor in two adjacent antennas. Here, we set $\xi = 0.25$. Fig. 5 and Fig. 6 show the BER results in $N \times K = 128 \times 16$ and $N \times K = 256 \times 16$ massive MIMO systems, respectively. We can find the performance degradation for all three linear precoding schemes in spatial correlation channels, which is consistent with the results in [26]. The BER error floor for Neumann-based precoding is severe, while LSQR-based precoding still shows a similar good convergent trend as that in uncorrelated channels, since it can also approach the performance of ZF precoding by increasing the number of iterations. Thus, we can conclude that the proposed LSQR-based precoding is more robust to channel correlation than the existing Neumann-based precoding.

V. Conclusions

In this paper, we proposed an LSQR-based precoding to significantly reduce the complexity of the classical ZF precoding by exploiting the LSQR algorithm. We have shown that LSQR-based precoding reduces the complexity from $\mathcal{O}(K^3)$ to $\mathcal{O}(K^2)$, and it requires less storage space. Simulation results have verified that the proposed scheme can achieve the near-optimal performance with only a small number of iterations, and is more robust to spatial correlation channels compared with existing Neumann-based precoding scheme.

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