Monty Hall Cognitive Dissonance

Samson Qian

1 Introduction

(**Scenario 1**) You are on a game show with three doors in front of you, a car behind one of them are two goats behind the others. Your goal is to win the car. You select a door at random from the three. Before revealing what is behind the door you selected, the game show host, who knows where the car and goats are, opens another door from the two you did not select with a goat behind it. The host then offers you the option to switch doors to the other remaining unopened one or keep your initial selection. Do you switch doors? Does it even matter? Does switching doors give you a boost in your chances of selecting the car?

Otherwise known as the Monty Hall Problem, this famous hypothetical scenario has baffled and confused not just regular people but also mathematicians, sparking fierce debates of logic and reasoning behind whether switching doors will increase the chances of winning the car or not. In this article, we will explore the intuition behind the Monty Hall Problem, and explain why the host's revelation of the other goat is irrelevant to making the switch decision and does not impact the odds of winning the car.

2 Monty Hall Intuition

2.1 Critics

To understand why switching doors will increase the odds of winning, we must first understand where the confusion comes from. Most critics of the Monty Hall Problem argue, in some way, shape, or form, that because only 2 doors are remaining to choose from, with one being the car and the other being the goat, switching or not does not matter and you have a $\frac{1}{2}$ probability of winning either the goat or the car.

Imagine an alternate scenario of the game where you do not select an initial door. Instead, the game show host directly reveals to you where one of the goats is, and then asks you to make your door selection. In this scenario, the critics would be correct, and there would indeed be a $\frac{1}{2}$ probability of selecting the door with the car, as you only have 2 doors to choose from, without any other options or strategies to employ.

What these critics neglect and fail to consider is the beginning of the game where you initially select a door. Although it may not seem important, the initial selection is the key reason as to why switching doors will increase winning odds.

2.2 Strategy Perspective

To understand why switching doors will increase the odds of winning, think about the Monty Hall scenario from the perspective of strategy. In essence, you only have 2 possible strategies: switch or no switch.

If you initially select a goat and switch, you are guaranteed to get the car. This is because the game show host will reveal the other goat, leaving the remaining door with the car unopened. If you do not switch, you are guaranteed to get a goat. The probability of you initially selecting a goat is $\frac{2}{3}$ as there are 2 goats with 3 doors to choose from. Alternatively, if you initially select the car and switch, you are guaranteed to get a goat, as the other 2 doors will only have goats. If you do not switch, you are guaranteed to get the car. The probability of you initially selecting the car is $\frac{1}{3}$, half the probability of selecting a goat.

Essentially, by switching doors, you are converting your initial $\frac{2}{3}$ probability of losing into a $\frac{2}{3}$ probability of winning. Since if you initially selected a goat ($\frac{2}{3}$ probability) and subsequently switch, you are guaranteed to win the car. By not switching, you are locking in your $\frac{2}{3}$ probability of losing and getting a goat. By choosing the switch strategy, you are actually hoping to choose a goat at the start, because switching will guarantee you a car. By not switching, you are hoping to choose the car on the initial pick, which is half as likely, with only $\frac{1}{3}$ probability.

Therefore, since it is more likely to select a goat initially, the best strategy is to switch doors. Because if you did indeed select a goat initially and switch doors, you are guaranteed to win the car. If you do not switch, your only hope is that you selected the car door up front, which is half as likely as selecting a goat.

3 Impacts on Human Biases and Psychology

3.1 The Goat Revelation

An alternate perspective for understanding the Monty Hall Problem involves a slightly alternated scenario but does not change the odds of the game. (**Scenario 2**) Imagine you are asked to select an initial door from the 3 doors. Instead of revealing the other goat, which we know is guaranteed to be behind 1 of those 2 other doors you did not select, the game show host asks you: Do you want to stick with the single door that you chose, or you can just take everything behind the 2 other doors you did not choose. What would you do? In this scenario, most people would now prefer to go with the other 2 doors, not knowing what's behind any of them, despite the fact that there has to be a goat behind 1 of those 2 doors. This makes more sense now because you have a $\frac{1}{3}$ probability of getting the car by sticking with your initial choice, and a $\frac{2}{3}$ probability of getting the car by taking the other 2 doors you did not select. As we will discuss, this scenario and the original Monty Hall scenario (Scenario 1), with the goat revelation, are the exact same.

To bring home this idea, consider an extreme version of the Monty Hall game. Instead of 3 doors, imagine there were now 100 doors with only 1 car behind a door and goats behind the remaining 99 doors. You are asked to select a door initially. The game show host then reveals all other goats behind 98 other doors, leaving only your door and 1 other door unopened. You are then offered the chance to switch. Would you take the switch now? Some people would still need a minute to think before making the decision, as, similar to before, there are only 2 choices. In fact, the game show host need not have opened the other 98 doors and could have just asked you if you wanted to keep the initial door you selected, or you could have everything behind the other 99 doors you did not select. In this case, where the host asks you whether you want to pick your initial door, or if you want to just take everything behind the 99 other doors you did not select, most people would not even blink an eye and choose to take the other 99 doors, even though it is already known that there are at least 98 goats in the other 99 doors. Revealing the 98 goats does not make any difference to the odds of the game, because it is already a known fact that 98 other doors have goats behind them.

3.2 Cognitive Dissonance

So what's the difference? Why does revealing a goat behind one of the other 2 doors incline people to feel there is a smaller difference in chances of winning between switching and not switching, while not revealing any doors makes people believe there is a greater probability to win with switching, whereas, in reality, these 2 scenarios are exactly the same because whether the game show host reveals the other goat or not, we already know 1 of the other 2 doors has a goat.

The Monty Hall problem is a great example to portray the effects of painful truth revelation on human cognition and behavior and the cognitive dissonance as a result. **The revelation of the goat behind the other 2 doors is irrelevant.** The revelation does not have any impact on the odds of winning or losing. It is already a given fact that 1 of the 2 other doors has a goat; revealing the goat does not make a difference. Its only purpose is to distract and scare you from making the logically and mathematically correct decision. The revelation of the goat influences people to believe that because they now only have 1 possible other unselected door that can have the car, their switch does not matter.

People are looking to avoid the goats as much as possible. Revealing the goat in one of the other 2 doors is a harsh revelation of reality that one of the 2 other doors has a goat, which biases a human's perspective and scares them into believing there is now a lower chance of the remaining door having a car. In this scenario, people are shocked by the fact that one of the 2 other doors they did not select has a goat, and falsely believe their chances of winning are smaller now by switching, now that one of the goats has been revealed. The revelation misleads them to believe that they now have a smaller chance that the car is behind 1 of those 2 doors because there is now only 1 other door remaining that can possibly have the car, whereas that was always the case, even if the goat was not revealed. The revelation of the goat influences humans to overweight the probability of the worst-case scenario: that the other unopened door also contains a goat.

In reality, however, their odds have not changed at all. The revelation of the goat simply transfers the probability of winning the car ($\frac{1}{3}$) for the revealed door to the remaining unopened door. Switching to the other door would be the exact same as taking what is behind both the other 2 doors you did not initially select, which obviously is the better decision. The revelation of one of the goats is to scare and trick you into believing there is only 1 other door remaining and that it only has a $\frac{1}{2}$ probability of having the car. The game show host's revelation of that goat does nothing else aside from creating cognitive dissonance.

Alternatively, not revealing any of the 2 other doors blinds people from the reality that there is a goat behind one of the 2 other doors. The game show host could essentially give you the options of (1) keeping the 1 door that you selected, or (2) taking both the other 2 doors you did not select. Most people in Scenario 2, where the goat is not revealed, would not think much about the worst-case scenario that the 2 other unselected doors both have goats, but would remain unbiased: that one of the 2 unselected doors has the car, and switching has doubled the chances of winning by taking the other 2 doors instead of the one initially selected. While we already know that 1 of the 2 other doors must contain a goat, most people in this scenario will still most likely go with taking the other 2 doors because they are not directly exposed to the actual revelation of the other goat. In reality, whether the goat is revealed or not behind the 2 other doors, the odds of winning and losing have not changed. The only thing that has changed or has been impacted is human cognition and human behavior. It is important to remember that in this scenario, taking the other 2 doors you did not select would be the exact same option as switching doors from your initial selection in the regular scenario.

Although most people can come to understand the logic described above, there is still a slight feeling of uneasiness and less comfortability with switching doors in Scenario 1 than there is in Scenario 2. This hesitation and uneasiness is the cognitive dissonance that the host's revelation of the goat creates. It creates fear and worry in the player's judgment that now, instead of 2 possible doors to win the car, they only have 1 possible door to win the car by switching. Most are not even cognizant of how the goat revelation influences them. Whereas the odds of winning in Scenario 1 are identical to those in Scenario 2, except that no fear or distractions are created in Scenario 2, so the player does not experience any cognitive dissonance in their decision-making of switching doors.

3.3 Lessons in Trading and Financial Markets

Many of the lessons from Monty Hall can be applied to trading and financial markets. Cognitive dissonance is a large aspect of emotional trading and is one of the greatest drivers of capital loss. The period from 2023 to 2024 is a great example for demonstrating the effects of harsh truth revelation. For two consecutive years, there was a boom in technology, consumer spending, employment, and economic activity and expansion, although coupled with signs of increased **inflation**, such as rising energy prices, consumer debt, and negative consumer sentiment. When inflationary macro data such as NFP/ADP, CPI/PPI, etc were released, many were overcome with fear of Fed rate hikes and sold off their long positions and became bearish, overlooking the strengths of the economy. The revelation of the macro data led many to focus heavily on the down-sides of the economy and the worst-case scenario, and become bearish in the markets and subsequently lose their capital through buying Put options or taking short positions. Whereas in reality, despite the inflationary signs, the economy as a whole was still very strong and business and technology were growing at a solid pace. The strength of the economy clearly outweighed the risks of rising inflation, but the revelations of macro data, just like the revelation of the goat, scared and tricked many traders to becoming bearish and overlooking the strengths, just like how many people are tricked into not switching doors.

Ultimately, the lesson is to not let painful truth revelations make you underestimate the positives and up-sides. To be successful in the markets, it is essential to always be unbiased, have a holistic view, and be aware of both the negatives and the positives, carefully weighing them to make decisions. Revelations of facts you already know and have accounted for should not influence your decision making.

4 Conclusion

The crux of the Monty Hall problem is that you are more likely to select a goat on your first choice and that you are given the option to turn your $\frac{2}{3}$ losing probability into a $\frac{2}{3}$ winning probability by switching because if you initially select a goat and switch, you are guaranteed to win the car.

The probability of winning the car is equivalent in both scenarios, whether the host reveals the other goat or not. It is already a given that 1 of the other 2 doors you did not select contains a goat. The revelation of the other goat just confirms this fact; it does **not** impact the odds of the game in any way. If the other goat was not revealed, it would be immediately obvious to most that switching (taking the other 2 doors you did not initially select) is the correct decision because picking the other 2 doors doubles your chances of winning the car, even though it is still true that one of those doors has a goat. Therefore, one should make the same decision of switching doors in the original Scenario 1, where the other goat is revealed, as they would in Scenario 2, where the other goat is not revealed.

The direct revelation of unfavorable and painful truths creates cognitive dissonance and distracts from the optimal thing to do, as evidenced by the reactions to the Monty Hall problem. The revelation of painful truths biases people by making them overestimate the probability of the worst-case scenario.

While the Monty Hall problem may seem just like another probability puzzle, the implications of the problem are much more significant. The most important lesson from the Monty Hall problem is not to let insignificant things in life affect you, or at least be aware of the insignificant events that constantly occur. There may be adversities that scare or shock you, just like the goat revelation, but do not actually change the circumstances of things. You must be constantly wary of these distractions and not let them create cognitive dissonance or deter you from making the most optimal and logical decisions, despite the emotions you may feel.

5 Monty Hall Extended

5.1 Four Doors

To test your understanding and intuition of the Monty Hall scenario, consider an extended version of the original problem: Imagine there are 4 doors, with 3 goats and 1 car behind them. You initially select a door. The game show host then reveals a goat behind 1 of the 3 other doors you did not select. You are then offered the opportunity to either keep your initial door or switch to 1 of the 2 other unopened doors. Do you make the switch from your initial door? What are the probabilities of winning in both cases?

Solution: We know that there is a $\frac{1}{4}$ probability of selecting the car on the first choice. Therefore, the probability that the car is in 1 of the other 3 doors you did not select is $\frac{3}{4}$. Because the host reveals a goat in 1 of the 3 other doors, there are now only 2 other doors remaining that can possibly have the car if you switch. But remember, if you switch, you only get to choose 1 of the 2 doors. The $\frac{3}{4}$ probability that the car is in 1 of the 3 other doors you did not select does not change, even if the host reveals a goat because it is a fact that there are at least 2 goats in the 3 doors you did not select. Therefore, each of the 2 unopened doors you did not select has a $\frac{3}{4} * \frac{1}{2} = \frac{3}{8}$ probability of having the car, which is greater than the $\frac{2}{8}$ probability of the door you initially chose.

Therefore, you should make the switch and randomly pick 1 of the 2 unopened doors. This will increase your chances of winning the car by $\frac{1}{8} = 12.5\%$.

5.2 Infinite Doors

The previous four-door scenario can be generalized and extended to n total doors with 1 car and $n-1$ goats, where you select an initial door, a goat is revealed from the other $n-1$ doors, and you are allowed to switch to one of the other $n-2$ unopened doors you did not select.

There is a $f_{\text{stay}}(n) = \frac{1}{n}$ probability of choosing the car on the first door, and a $f_{\text{switch}}(n) = \frac{n-1}{n}$ probability of the car being in the other doors you did not select. One of the goats is eliminated from the unselected $n-1$ doors, bringing the probability of each of the remaining $n-2$ unselected doors containing the car to:

$$f_{\text{switch}}(n) = \frac{n-1}{n} * \frac{1}{n-2} = \frac{n-1}{n^2-2n} \tag{1}$$

It can be shown that $f_{\text{switch}}(n) > f_{\text{stay}}(n)$ for all $\forall n \in \mathbb{R}$, $n > 2$. Let $h(n) = f_{\text{switch}}(n) - f_{\text{stay}}(n)$:

$$h(n) = \frac{n-1}{n^2-2n} - \frac{1}{n} = \frac{n(n-1) - (n^2-2n)}{n(n^2-2n)} = \frac{n^2-n-n^2+2n}{n(n^2-2n)} = \frac{n}{n(n^2-2n)} = \frac{1}{n(n-2)} \tag{2}$$

$$h(n) > 0, \{\forall n \in \mathbb{R},\, n > 2\} \tag{3}$$

$$f_{\text{switch}}(n) > f_{\text{stay}}(n), \{\forall n \in \mathbb{R},\, n > 2\} \tag{4}$$

For $n > 2$, $h(n)$ is always positive. Therefore, for any finite number of doors > 2, there is always some finite increase in the probability of winning by switching to another door you did not initially select, so you should always switch doors no matter how many doors there are. Intuitively, this makes sense, because as the number of doors, n, gets larger, your probability of selecting the door with the car on your first pick decreases. Therefore, as the number of doors gets large, you might as well switch to another door you did not select, as there is a very high probability the single door you initially chose does not have the car. However, simultaneously, the probability of your switch containing the car also rapidly diminishes. The difference in winning probability of switching is larger and more significant for 2 or 3 doors but becomes very minuscule for larger numbers of doors. This can be shown by showing that $h(n)$ is strictly decreasing for $n > 2$:

$$h'(n) = -\frac{2(n-1)}{n^2(x-2)^2} \tag{5}$$

$$\because 2(n-1) > 0, \{\forall n \in \mathbb{R},\, n > 2\} \tag{6}$$
$$\because n^2(x-2)^2 > 0, \{\forall n \in \mathbb{R},\, n > 2\} \tag{7}$$
$$\therefore h'(n) < 0, \{\forall n \in \mathbb{R},\, n > 2\} \tag{8}$$

Therefore, the marginal increase in the probability of winning decreases as the number of doors gets large. One can compute the second derivative, $h"(n)$, to know how quickly the probability decreases.

So is there a number of doors where the switch does not matter? It can be shown that:

$$\lim_{n\to\infty} h(n) = \lim_{n\to\infty} \frac{1}{n(n-2)} = 0 \tag{9}$$

Therefore, if there are an infinite number of doors to choose from, staying with your initial door or switching to another door makes no difference on the probability of winning the car.