Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords

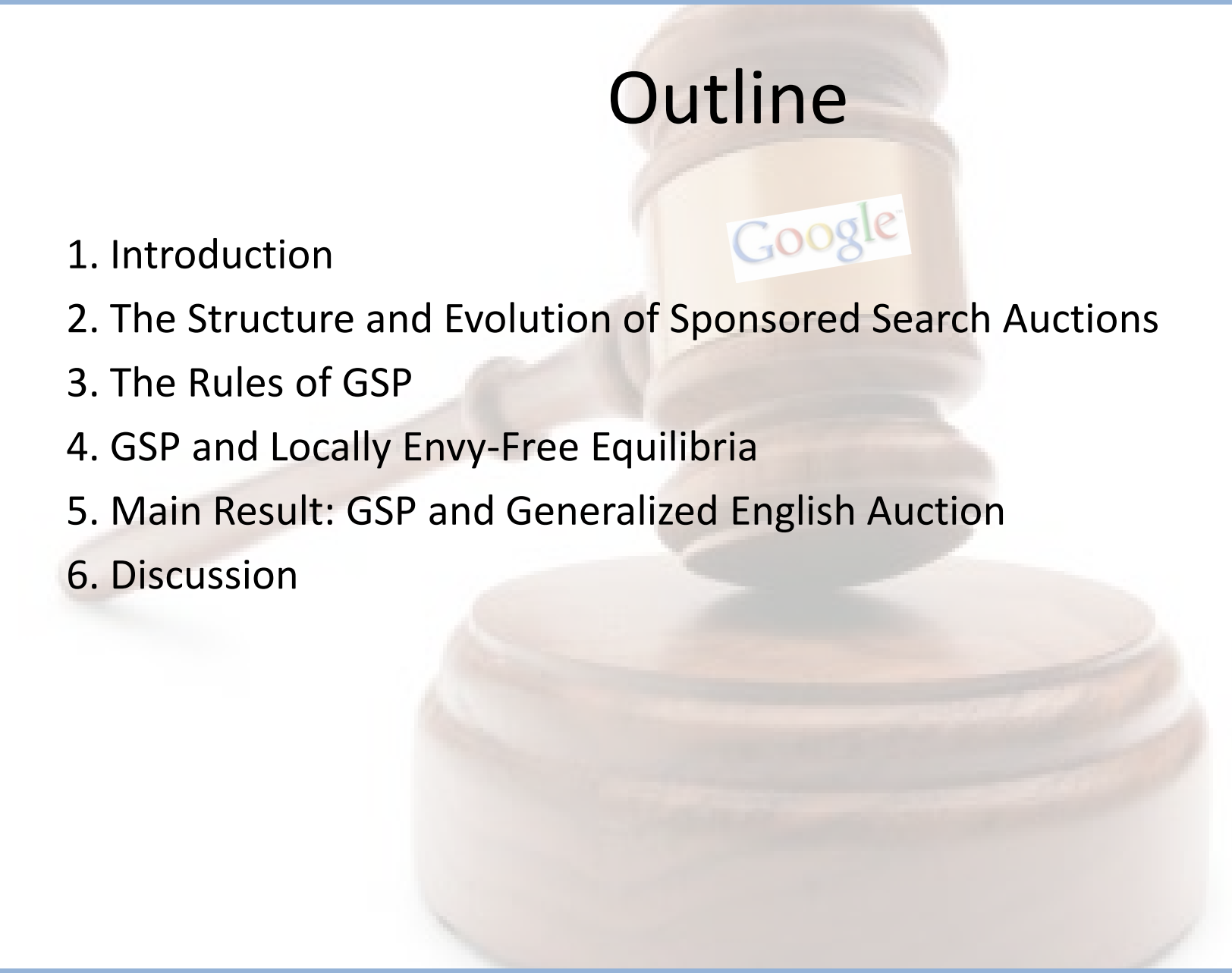
Benjamin Edelman, Michael Ostrovsky and Michael Schwarz

Presented by Alberto Cipriano, Markus Schwarzmann and Simon Wernli

Auction Theory and Mechanism Design

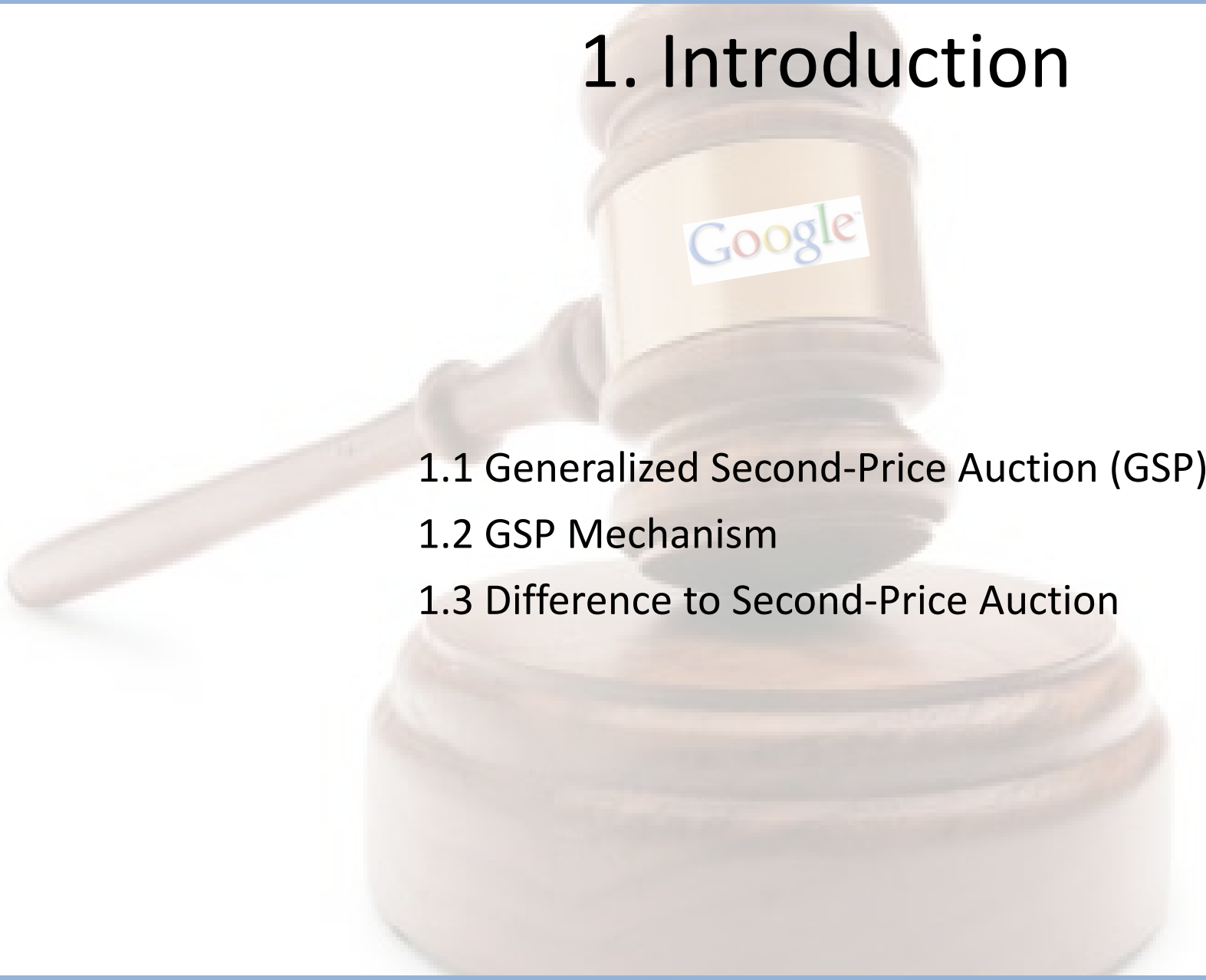
April 18, 2012

Outline



1. Introduction
2. The Structure and Evolution of Sponsored Search Auctions
3. The Rules of GSP
4. GSP and Locally Envy-Free Equilibria
5. Main Result: GSP and Generalized English Auction
6. Discussion

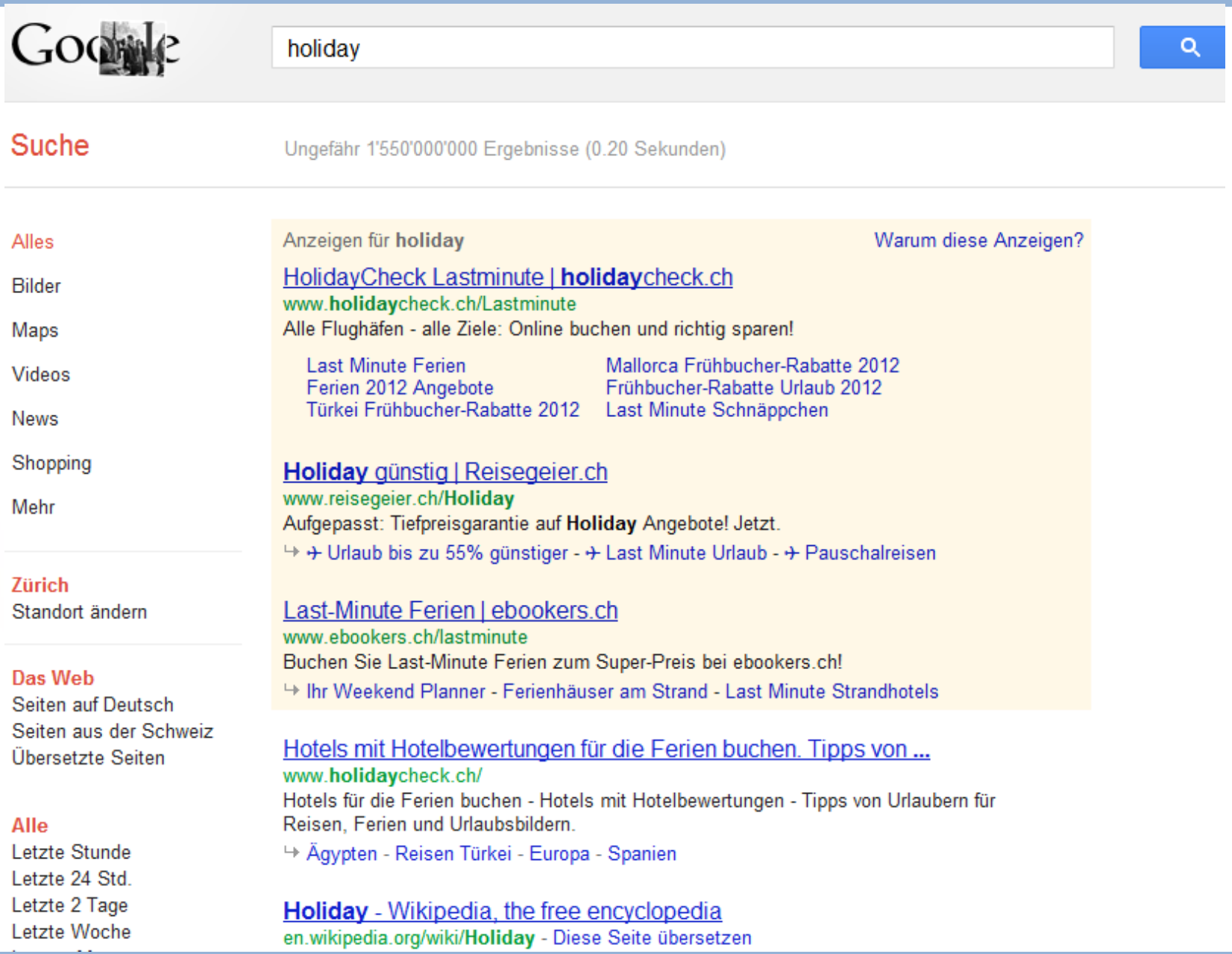
1. Introduction



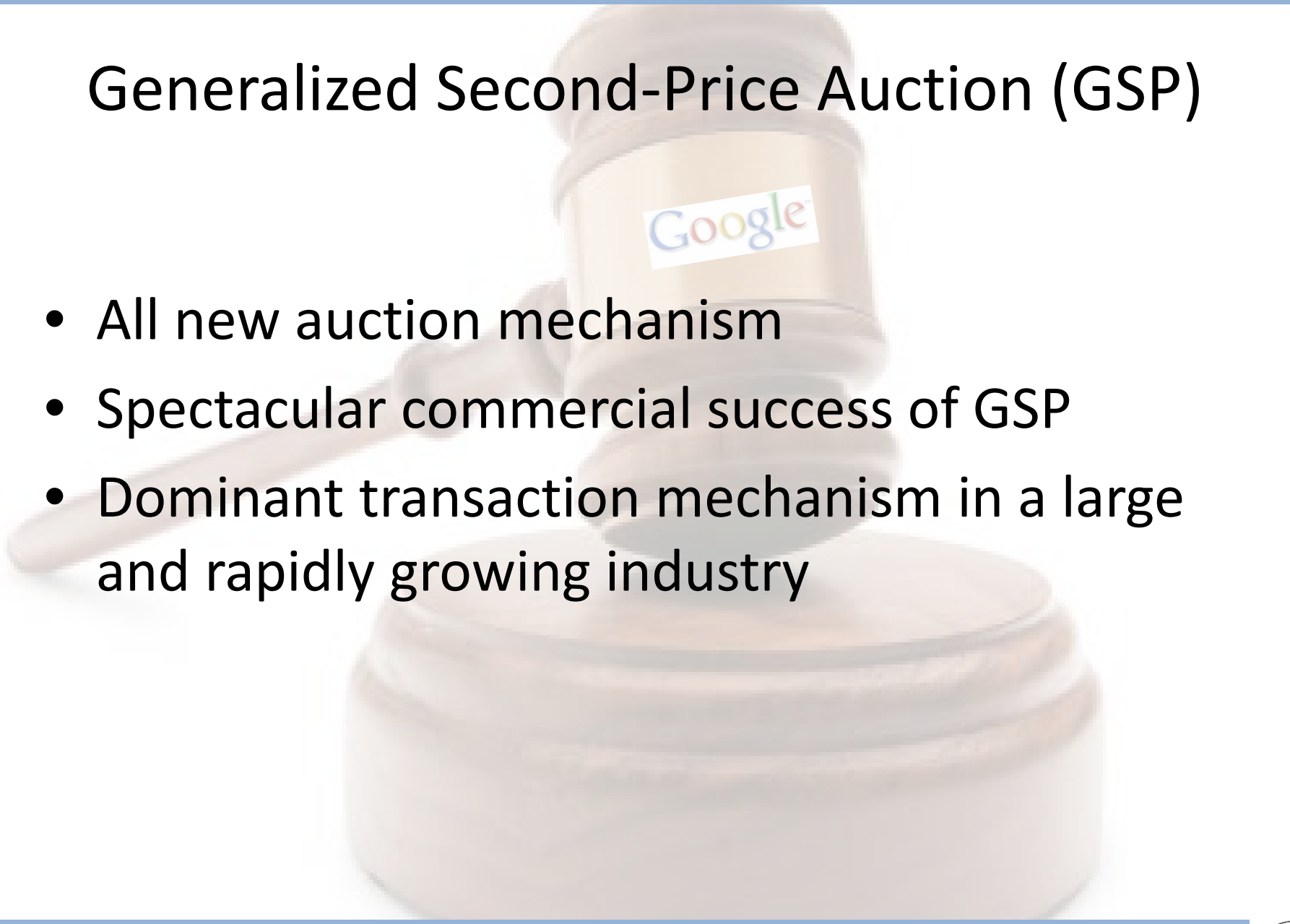
1.1 Generalized Second-Price Auction (GSP)

1.2 GSP Mechanism

1.3 Difference to Second-Price Auction


Google
holiday
Suche
Ungefähr 1'550'000'000 Ergebnisse (0.20 Sekunden)
Alles
Bilder
Maps
Videos
News
Shopping
Mehr
Zürich
Standort ändern
Das Web
Seiten auf Deutsch
Seiten aus der Schweiz
Übersetzte Seiten
Alle
Letzte Stunde
Letzte 24 Std.
Letzte 2 Tage
Letzte Woche
Anzeigen für holiday
Warum diese Anzeigen?
HolidayCheck Lastminute | holidaycheck.ch
www.holidaycheck.ch/Lastminute
Alle Flughäfen - alle Ziele: Online buchen und richtig sparen!
Last Minute Ferien
Ferien 2012 Angebote
Türkei Frühbucher-Rabatte 2012
Mallorca Frühbucher-Rabatte 2012
Frühbucher-Rabatte Urlaub 2012
Last Minute Schnäppchen
Holiday günstig | Reisegeier.ch
www.reisegeier.ch/Holiday
Aufgepasst: Tiefpreisgarantie auf Holiday Angebote! Jetzt.
↳ ✈ Urlaub bis zu 55% günstiger - ✈ Last Minute Urlaub - ✈ Pauschalreisen
Last-Minute Ferien | ebookers.ch
www.ebookers.ch/lastminute
Buchen Sie Last-Minute Ferien zum Super-Preis bei ebookers.ch!
↳ Ihr Weekend Planner - Ferienhäuser am Strand - Last Minute Strandhotels
Hotels mit Hotelbewertungen für die Ferien buchen. Tipps von ...
www.holidaycheck.ch/
Hotels für die Ferien buchen - Hotels mit Hotelbewertungen - Tipps von Urlaubern für Reisen, Ferien und Urlaubsbildern.
↳ Ägypten - Reisen Türkei - Europa - Spanien
Holiday - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/Holiday - Diese Seite übersetzen

Generalized Second-Price Auction (GSP)

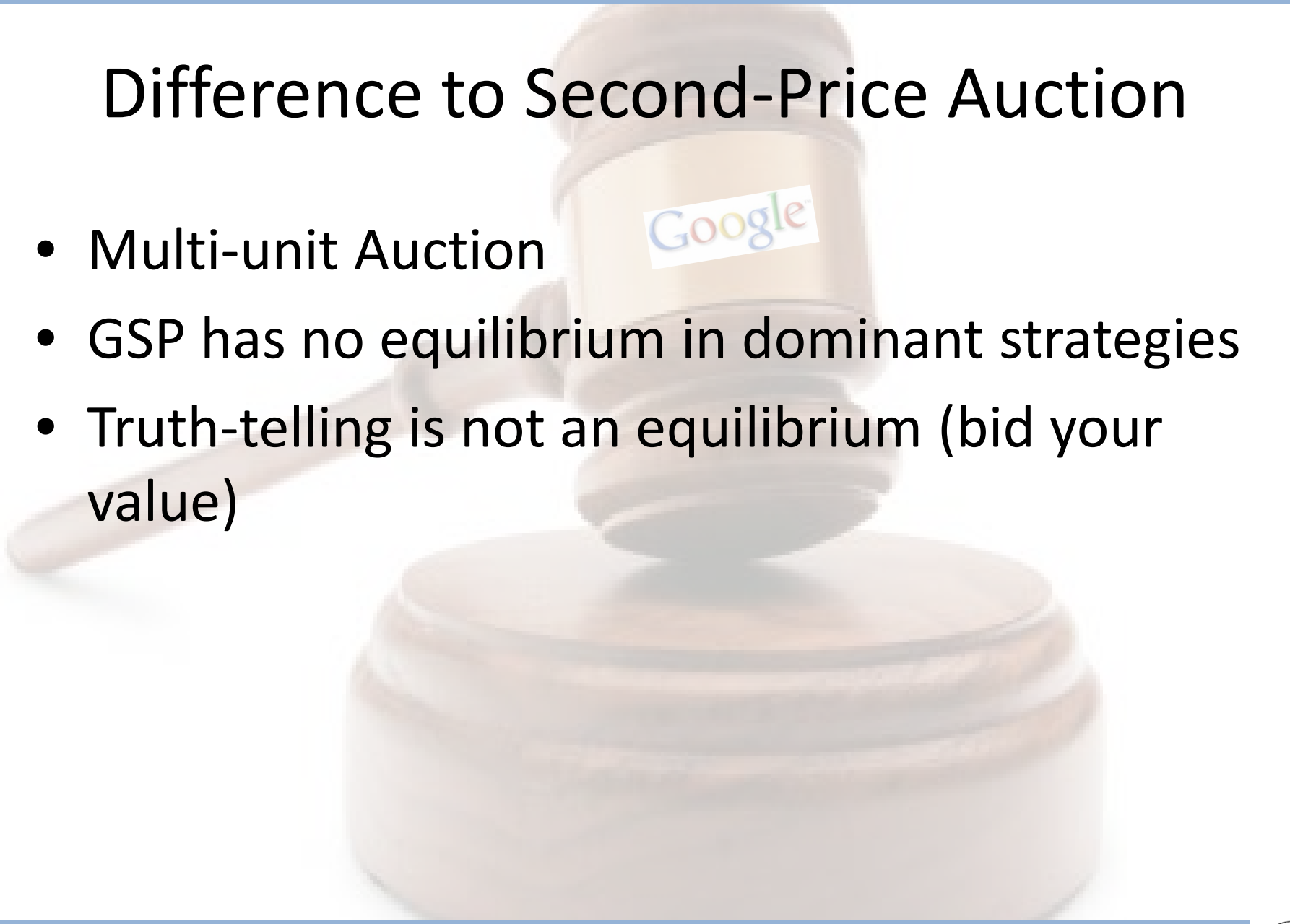


- All new auction mechanism
- Spectacular commercial success of GSP
- Dominant transaction mechanism in a large and rapidly growing industry

GSP Mechanism

- Enter a Keyword
- Auction
- Result Page
- Payoff

Difference to Second-Price Auction



- Multi-unit Auction
- GSP has no equilibrium in dominant strategies
- Truth-telling is not an equilibrium (bid your value)

2. The Structure and Evolution of Sponsored Search Auction

2.1 Features of the Market for Internet Advertising

2.2 Evolution of the Systems

Features of the Market for Internet Advertising

- Bids can be changed at any time
- Ad services are perishable
- No «unit» of Internet advertisement
- Click-Through Rates (CTR)

Evolution of the Systems

- Early Internet Advertising 1994
- Generalized First-Price Auction 1997
- Generalized Second-Price Auction 2002

3. The Rules of GSP

3.1 The Rules

3.2 Payments under GSP and VCG

3.3 Conclusions of “The Rules of GSP”

The Rules

- N objects (positions)
- K bidders (advertisers)
- b_k bid of k
- α_i number of clicks per period in position i
- s_k value per click to bidder k
- $p^{(k)}$ payment of bidder k

} $\alpha_i s_k - p^{(k)}$

- Risk-Neutrality, positions are labeled in descending order, only one position per bidder
- $b^{(j)}$ bid of the j-th highest bidder
- $g(j)$ identity of the j-th highest bidder

} e.g. Top position goes to bidder $g(1)$ with highest bid $b^{(1)}$

Payments under GSP and VCG

- Generalized Second-Price (GSP) Auction

$$p^{(i)} = \alpha_i b^{(i+1)} \text{ for } i \in \{1,\ldots,\min\{N,K\}\}$$

$$p^{(K)} = 0 \text{ for } N \geq K$$

- Vickrey-Clarke-Groves (VCG) Mechanism

$$p^{V,(i)} = (\alpha_i - \alpha_{i+1}) b^{(i+1)} + p^{V,(i+1)} \text{ for } i < \min\{N,K\}$$

$$p^{(K)} = 0 \text{ for } N \geq K, \ \alpha_N b^{(N+1)} \text{ otherwise}$$

Conclusions of "The Rules of GSP"

- "If all advertisers were to bid the same amounts under the two mechanisms, then each advertiser's payment would be at least as large under GSP as under VCG."

The bidder who gets the last position pays in both cases the same.

For all others $i < \min\{N, K\}$:

$$p^{V,(i)} - p^{V,(i+1)} = (\alpha_i - \alpha_{i+1}) b^{(i+1)} \leq \alpha_i b^{(i+1)} - \alpha_{i+1} b^{(i+2)} = p^{(i)} - p^{(i+1)}$$

- "Truth-telling is a dominant strategy under VCG."

Property of VCG mechanism.

Conclusions of "The Rules of GSP"

- "Truth-telling is not a dominant strategy under GSP."

Counter-example:

three bidders with values \$10, \$4, \$2

two positions with 200, 199 clicks per hour

advertisers bid thruthfully

$\rightarrow$ bidder 1's payoff: $(\$10 - \$4) * 200 = \$1'200$

if he bids only \$3 $\rightarrow$ second position

$\rightarrow$ bidder 1's payoff: $(\$10 - \$2) * 199 = \$1'592$

4. GSP and Locally Envy-Free Equilibria

4.1 Assumptions and Restrictions

4.2 Locally Envy-Free equilibrium

4.3 First Theorem

Assumptions and Restrictions

- Sponsored search auctions as continuous time or infinitely repeated games.
- Focus on simple strategies and rest points.
- Therefore impose three assumptions and restrictions
  - All values are common knowledge
  - Stable bids must be best responses to each other
  - Consider simple strategies beyond simple best responses
    - e.g. Forcing out the advertiser with position immediately above

Locally Envy-Free Equilibrium

- "in a locally envy-free equilibrium, for any $i \leq \min\{N+1, K\}$, $\alpha_i s_{g(i)} - p^{(i)} \geq \alpha_{i-1} s_{g(i)} - p^{(i-1)}$."
- "The outcome of any locally envy-free equilibrium of auction Γ is a stable assignment."
- "If the number of advertisers is greater than the number of available positions, then any stable assignment is an outcome of a locally envy-free equilibrium of auction Γ."

First Theorem

- Construct a locally envy-free equilibrium of game Γ with following properties $\left(p^{(i-1)} \geq (\alpha_{i-1} - \alpha_i) s_{g(i)} + p^{(i)} \geq (\alpha_{i-1} - \alpha_i) s_{g(i)} + p^{V,(i)} = p^{V,(i-1)}\right)$
  - Payments are the same as in the dominant-strategy equilibrium of VCG
  - Worst for search engine and best for advertisers $\left(p^{(i-1)} = p^{V,(i-1)}\right)$
- "Strategy profile B^* is a locally envy-free equilibrium of game Γ. In this equilibrium, each advertiser's position and payment are equal to those in the dominant-strategy equilibrium of the game induced by VCG. In any other locally envy-free equilibrium of game Γ, the total revenue of the seller is at least as high as in B^*."

5. Main Result: GSP and Generalized English Auction

GSP and Generalized English Auction

- A clock showing the current price: 1¢,2¢,3¢…
- A bid is the price at the time of dropping out
- Payments computed according to GSP rules (pay price where last advertiser dropped out: second price!)
- Private information of bidders private value
- # of slots: $N \geq 2$
- # of advertisers $K \geq N+1$

GSP and Generalized English Auction

Generalized English Auction with GSP rules: Example

- 2 slots, 3 players (valuations: $10, $4, $2)
- Clock starts: 1¢,2¢,3¢…
- Player 3 drops out at $2 – gets no slot, pays nothing!

- Player 2 has α_1: 200 clicks in slot 1, α_2: 100 clicks in slot 2. If he drops out now (at ≥$2.01), he gets 100 clicks at $2 (profit: ($4-$2)*100 = $200).
- If player 1 drops out before him, player 2 would get position 1 at that price.
- Player 2 stays till he is indifferent between both positions 1 and 2:
- (4-p)*200 ≥ $200 ➔ drop out at $3 (bid ≠ valuation)!
- Players don't need to know others valuation – see only earlier drop out!

GSP and Generalized English Auction

THEOREM 2: In the unique perfect Bayesian equilibrium…

$$p_k(i,h,s_k) = s_k - \frac{\alpha_i}{\alpha_{i-1}}(s_k - b_{i+1})$$

p_k: Moment of dropping out of player k dependent on
i: # of remaining advertisers (= position of slot),
h: the history of the game
s_k: valuation for one click for player k
a_i: expected number of clicks in position I
a_{i-1}: expected # of clicks in position i-1, i.e. if he stays in the game for the next position above
b_{i+1}: the last bid in the history where somebody else dropped out and got position i+1

➔**position and payoff equals VCG-equilibrium (ex post)**

GSP and Generalized English Auction

THEOREM 2: In the unique perfect Bayesian equilibrium…

$$p_k(i,h,s_k) = s_k - \frac{\alpha_i}{\alpha_{i-1}}(s_k - b_{i+1})$$

In our Example: $3 = 4 - \frac{100}{200}(\$4 - \$2)$

➔**position and payoff equals VCG-equilibrium (ex post)**

Main Result: GSP and Generalized English Auction

English auction that corresponds to GSP has

- unique (ex post) equilibrium
- same payoffs to all players as the dominant strategy equilibrium of VCG

6. Discussion