Benchmarking in Clustering

K Medoids, Student T Model and DBSCAN

Math 252 Project II

By

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Outlines

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Purpose & Background

Purpose:

Benchmark clustering methods by performing experiments

Using silhouette index(SI) and adjusted random index(ARI).

Clustering methods:

K-medoids, Student-t model,

DBSCAN (density-based spatial clustering of applications with noise)

Datasets:

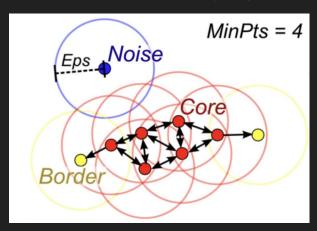
Real data (USPS MNIST handwritten digits) (-> test high dimension)

Simulated Contaminated data (-> test outliers, number of clusters and correlation).

Clustering Methods

- K-medoids: creates clusters by using a data point in the data set as the centroid.
- Student t model: is an EM algorithm which maximizes the likelihood of t distribution to model data.
- **DBSCAN**(density-based spatial clustering of applications with noise)
 - : groups points that are close to each other based on a <u>distance</u> measurement (Euclidean distance) and <u>a minimum number of points</u>. It also marks as <u>outliers</u> the points that are in low-density regions.

** All three methods are supposed to be resilient to **outliers**.

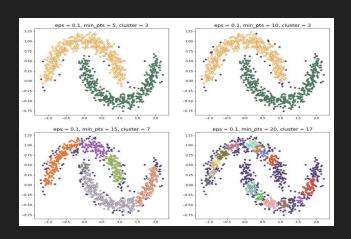


DBSCAN (Density-based Spatial clustering of applications with noise)

2 parameters: **epsilon** (radius around the point) and **k** (minimum neighbor points)

2 properties: needn't initiate the number of clusters

determine outliers (using the distance of k nearest neighbors.)



```
In R
      (fpc package)
      dbscan(data, eps, MinPts = k, ...)

Use knn to pick epsilon
      kNN(x, k, query = NULL, sort = TRUE, search =
      "kdtree", ...)
```

How we choose Optimal Number of Clusters

- K-medoids:
 - based on SI or ARI
- Student t model:
 - > based on the BIC.
- ❖ DBSCAN:
 - computed by DBSCAN algorithm automated.

I) High Dimension

Raw Data: USPS MNIST handwritten digits (70,000x784 matrix, 10 clusters)

PCA

 $\sum_{i=1}^k \lambda_i / \sum_{i=i}^p \lambda_i$

Fig.1 Percent scatter preserved formula

Limited by R, a smaller subset was used to do the experient.

 λ_{i} ordered eigenvalues of the covariance matrix of the dataset.

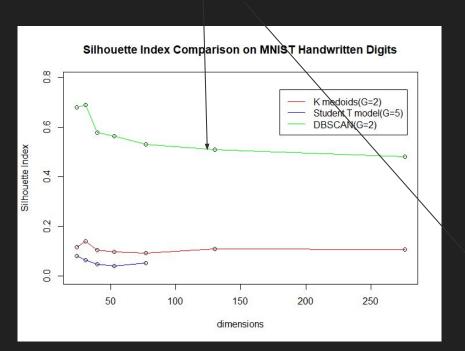
p the number of total dimensions.

k is the number of dimensions in a lower dimensional plane.

Data set preserving 70% to 99% -> subspace dims = c(24,31,40,53,77,130,276)

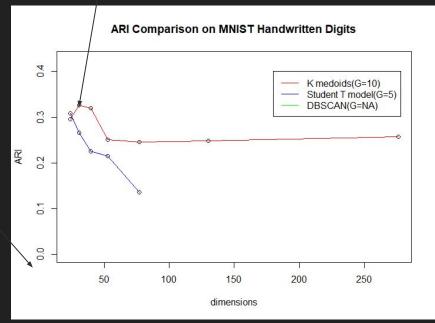
Models' Performance(High Dim)

DBSCAN considered most data points to be an outlier



K-medoids was also able to pick out 10 clusters (the

true number of labels).



** All three methods performed poorly in high dimensions.

(G are number of clusters found in the best model per method)

II) Outliers

Simulated Dataset: contaminated normal distribution

$$f(\mathbf{X}; \boldsymbol{\vartheta}) = \alpha \phi(\mathbf{X}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) + (1 - \alpha) \phi(\mathbf{X}; \boldsymbol{\mu}, \eta \boldsymbol{\Sigma}),$$

Cluster Number = 5

Means: different between clusters

Variance = 100, 1000 (low sig = 100, high sig = 1000)

Alpha = 0.05, 0.1,..., 0.3 (the percent of 'bad' data points, high sig)

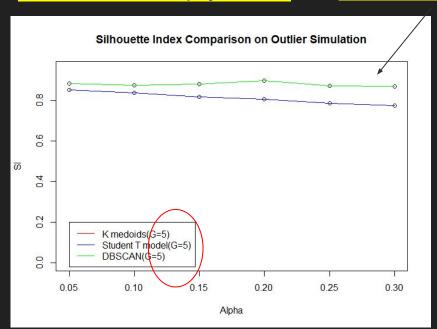
Outliers: ten times greater than the original variance.

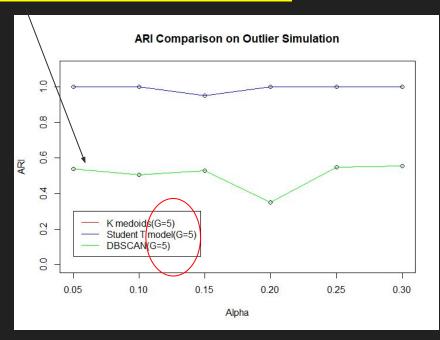
Models' Performance(Outliers)

All three methods performed well with less or more outliers

K-medoids line is overlap by Student T.

DBSCAN finds outliers and omits them from clusters





III) Number of Clusters

Simulated Dataset: contaminated normal distribution

$$f(\mathbf{X}; \boldsymbol{\vartheta}) = \alpha \phi(\mathbf{X}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) + (1 - \alpha) \phi(\mathbf{X}; \boldsymbol{\mu}, \eta \boldsymbol{\Sigma}),$$

Cluster Number = 5,10,...,30

Variables Number = 10

Means: different between clusters

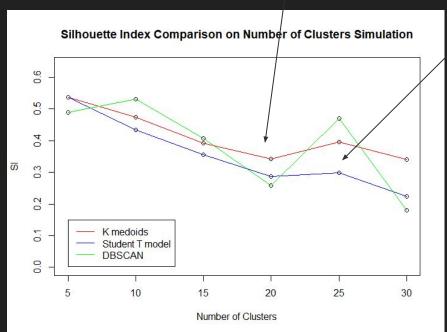
Variance = 100 (for most points)

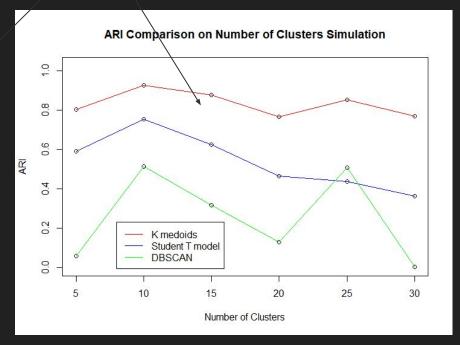
Alpha = 0.05 (the percent of 'bad' data points, sig = 1000)

Models' Performance (Cluster Number)

K medoids performed better than the student t. (no outliers, var stable)

Downward sloping pattern — the more classes, the worse the performance is.





IV) Correlation

Simulated Dataset: contaminated normal distribution

$$f(\mathbf{x}; \boldsymbol{\vartheta}) = \alpha \phi(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) + (1 - \alpha) \phi(\mathbf{x}; \boldsymbol{\mu}, \eta \boldsymbol{\Sigma}),$$

Cluster Number =5

Correlations = 0, 0.2,..., 1

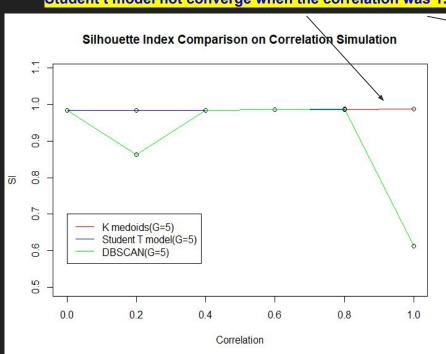
Variance = 1

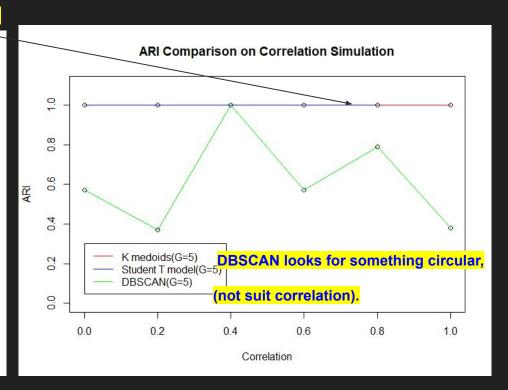
Means: different between clusters

Models' Performance(Correlation Far Clusters)

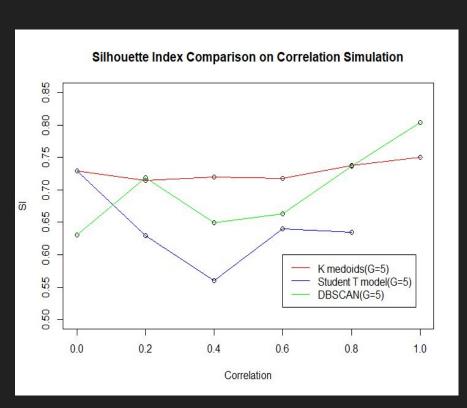
The SI and ARI values for k medoids and student t are close to 1 (not actually 1).

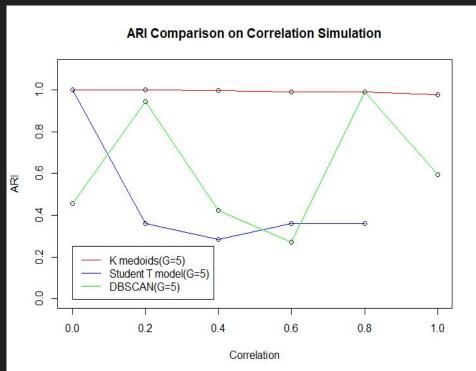
Student t model not converge when the correlation was 1.





Models' Performance(Correlation Close Clusters)





Summary

- * K medoids and student t model seems to be more similar to each other than DBSCAN.
- * K medoids and student t performed well with outliers and correlation but poorly with high dimensions.
- DBSCAN performed well with high dimensions and outliers but poorly with correlation.

	Dimensions	Outliers	Number of Clusters (more clusters)	Correlation
K Medoids	X	√	X	✓
Student T Model	X	√	X	Δ
DBSCAN	Δ	✓	X	X

Bibliography

- [1] Math 252 Course slides (by Dr. Tortora)
- [2] https://rdrr.io/cran/dbscan/man/kNN.html
- [3] https://medium.com/@agarwalvibhor84/lets-cluster-data-points-using-dbscan