Cellular Automaton Model for Mixed Traffic of Self-driving Cars and Human-driving Cars

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1 Introduction

Our project will be based on the validation and the extension of three paper [1, 2, 3]. We give a review of them in Section 2.

We will start from rebuilding the classic model and reproduce their results, which will then be compared with NaSch model, a cellular automaton model for single-lane traffic, and NGSIM trajectory data, a real-world traffic flow dataset. Beyond this point, as the intelligent self-driving system is being declared as a potential next-generation transportation solution, we are particularly interested in how self-driving cars will improve the traffic efficiency when they cooperate with other vehicles (V2V).

Here is a brief list of what we plan to address in our model:

- Accuracy and agility of autonomous vehicles, in terms of sensing and controlling.
- Cooperation between vehicles, including distance keeping, lane changing, emergency warning.
- Communication between vehicles, including information (position, velocity, acceleration) sharing.

Our model is built under the scenario of the mix of self-driving cars and human-driving cars. As one can imagine, the adoption of self-driving vehicles will be a gradual process,

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it will be important to understand the system behavior and performance with different percentages of the self-driving cars. In the subsequent text, we will use car and vehicle, self-driving and autonomous interchangeably.

2 Literature Review

Before introducing our model, we first review several classic cellular automaton models for simulating traffic.

2.1 Nagel-Schrekenberg Model

In 1992, Nagel and Schreckenberg proposed the famous NaSch model for single-lane [1], which allows heterogeneous velocities and change in velocities. In NaSch model, a road is divided into cells of length 7.5 m. This length corresponds to the typical space (vehicle length + inter-vehicle spacing) occupied by a vehicle in a dense traffic jam. Each vehicle has a velocity taken from $0, 1, 2, \cdots, v_{\max}$ where v_{max} corresponds to the maximum speed (speed limit). The rule set they introduced consists of the following four steps (in order) that are applied to all vehicles synchronously (in parallel). See Figure 2.

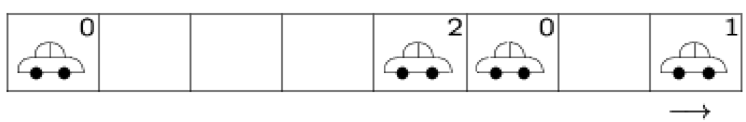

Figure 1: A typical configuration of the road

- *Acceleration*: all cars that have not already reached the maximal velocity accelerate by one unit

$$v \leftarrow \min\{v + 1, v_{\max}\}$$

- *Deceleration (safety constraint)*: if a car has d empty cells in front of it and its velocity $v > d$, then it reduces the velocity to d

$$v \leftarrow \min\{v, d\}$$

- *Deceleration (randomization)*: with probability p, the velocity is reduced by one unit

$$v \leftarrow v + 1$$

- *Driving*: update the location based on the new velocity

$$x \leftarrow x + v$$

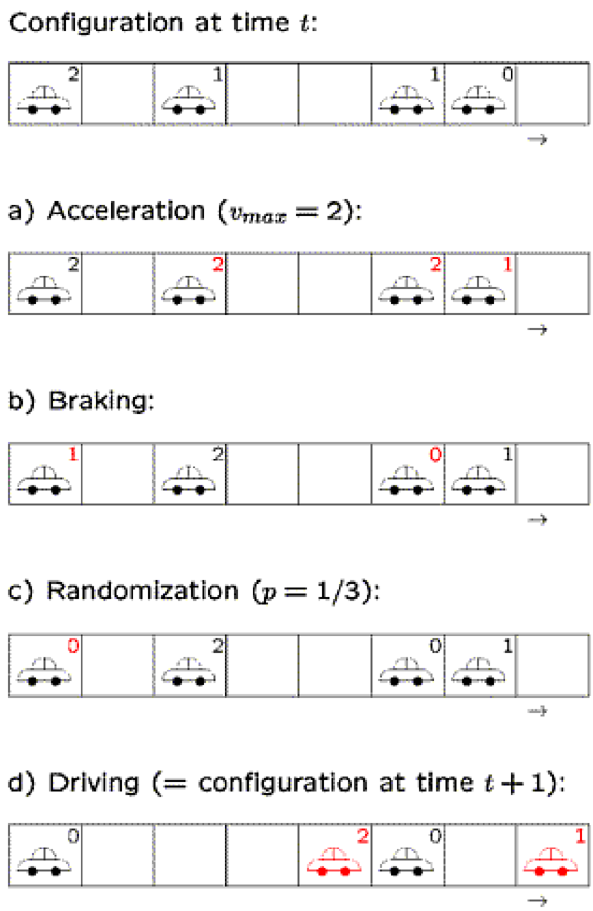

Figure 2: Step-by-step temporal evolution of Nagel-Schreckenberg model

Besides the spontaneous occurrence of congestion, NaSch model can also describe some other known facts about traffic flow, say the fundamental diagrams and the back-travelling stop-and-go waves.

2.2 Two-lane Model

When extending the single-lane traffic model to multi-lane traffic model, the passing phenomenon has to be considered. That is, when a slow vehicle is followed by a faster vehicle, it can be passed by the follower from neighboring lanes. In [2], the authors discussed two types of passing rules: symmetric and asymmetric. The symmetric model is more theoretically interesting while the asymmetric one is more realistic. The authors also pointed out that in NaSch model a vehicle only looks ahead (downstream) and a reasonable lane changing rule should include a check of sites upstream in order not to disturb the traffic of the destination lane. In summary, their lane-changing rules can be described as three steps:

1. Not enough space ahead.
2. The other lane is clear.
3. Safe after lane-changing (check forward and backward).

More specifically, the conditions are

- $d < d_s$ where d_s is the safe distance on the current lane;
- $d_{o,forward} > d_{o,s,forward}$ where $d_{o,forward}$ is the distance with the preceding vehicle on the other lane and $d_{o,s,forward}$ is the safe distance with the preceding vehicle on the other lane;
- $d_{o,backward} > d_{o,s,backward}$ where $d_{o,backward}$ is the distance with the following vehicle on the other lane and $d_{o,s,backward}$ is the safe distance with the backward vehicle on the other lane;
- $rand() < p_{change}$, i.e. if the above three conditions are met, then with a pre-defined probability, the vehicle would change to the other lane.

2.3 Realistic Multi-lane Model

In [3], the authors focused on the design of realistic multi-lane traffic rules for cellular automaton model. To meet the specific traffic rules in Germany, their passing rules are asymmetric. Modifications are made so that the results are in line with measured data. They also showed that their model can be used to simulate not only two-lane traffic but also three-lane traffic.

3 Model

3.1 Model Setup

Based on the classic models mentioned before, we build an innovative model that is more flexible and capable of characterizing the mixed traffic environment. The important features of our model are highlighted below:

- **Two-way road.** The road can be two-way with equal number of lanes on both directions.
- **Zero-length cell.** The cell has zero length and can be viewed as the center of a car. In this way, both the road length and the velocity is not necessarily the times of the cell (car) length and the car-following can be more flexible. Now the position of a car can be any integer within the range of the road length. Note that when calculating safe distance, car length and spacing should be included.
- **New cell attributes.** The state of each cell (car) consists of
  - occupancy: occupied, empty
  - car type: self-driving (autonomous), human-driving
  - velocity: updated by acceleration, real number between $v_{\max}$ and $v_{\min}$
  - acceleration: updated according to states of neighboring cars
  - direction: northward, southward

  It is worth mentioning that in classic models, the velocity of a vehicle can only be changed (increased or decreased) by a fixed value. However, considering that the self-driving vehicles have accurate sensors and in-time communication, they are more likely to have more accurate perception and control of acceleration compared to human drivers. Therefore, we introduce nonzero, variable acceleration to our model so that self-driving cars can be differentiated from human-driving cars.

3.2 Generation Rule

At the start of simulation, we need to generate cars. Specifically, the car type is assigned according to a predefined Bernoulli distribution and the initial velocity is sampled from a predefined (truncated) Gaussian distribution (approximated to an integer).

We also adopt the widely used periodic boundary condition, that is, the ends of the road are connected. When a car moves out of the north boundary, it will appear again at the south boundary. In fact, we can also assume that the arrival of cars follows Poisson distribution with parameters obtained from real-world data.

3.3 Car-following Rule

The temporal evolution of a multi-lane traffic model includes car-following rules on current lane and lane-changing rules across lanes. We will introduce the car-following rules in this subsection and introduce the lane-changing rules in the next subsection.

The car-following rules can be decomposed into four steps.

1. Set safety constraint based on
$$d_s = t_r v$$
where d_s stands for the safe distance, t_r refers to the response time and v is the velocity. The response time is different for different types of cars, say 1s for self-driving cars and 2s for human drivers, considering that the former are more agile.

2. Update position x based on velocity v and safe distance d_s:
   - if $d - v\Delta t - s \geq d_s$, then $x \leftarrow x + v\Delta t$ where Δt is the time step for update and s refers to the pre-defined typical space (car size + spacing);
   - if $d - v\Delta t - s < d_s$, then $x \leftarrow x + \min(v\Delta t, d - s)$.

3. Update velocity v based on acceleration a:
   - if $a \geq 0$, then accelerate, $v \leftarrow \min\{v + a, v_{\max}\}$;
   - if $a < 0$, then decelerate $v \leftarrow \max\{v + a, v_{\min}\}$.

4. Update acceleration a based on types of leader-follower (the car we are focusing on is called follower and its preceding car is called leader):
   - if no car ahead (check if the cell is occupied at $x, x+1, \cdots x+l$ where l is the parameter deciding how far a car looks ahead), then accelerate to the maximum velocity $v \leftarrow v_{\max}$, $a \leftarrow 0$;
   - if the follower is a human-driving car, then $a \leftarrow \alpha(d - d_s)$;
   - if a human-driving car is followed by a self-driving car, then the follower has more flexible control on acceleration based on the difference in velocities, $a \leftarrow \alpha(d - d_s) + \beta(v_l - v_f)$ where v_l (resp. v_f) is the velocity of the leader (resp. follower);
   - if both the leader and the follower are self-driving cars, then they can communicate and share information with each other (specifically, the follower can get the real time acceleration of the leader a_l and take it into account), $a \leftarrow \alpha(d - d_s) + \beta(v_l - v_f) + \gamma a_l$.

   Here α, β and γ are adjustable parameters.

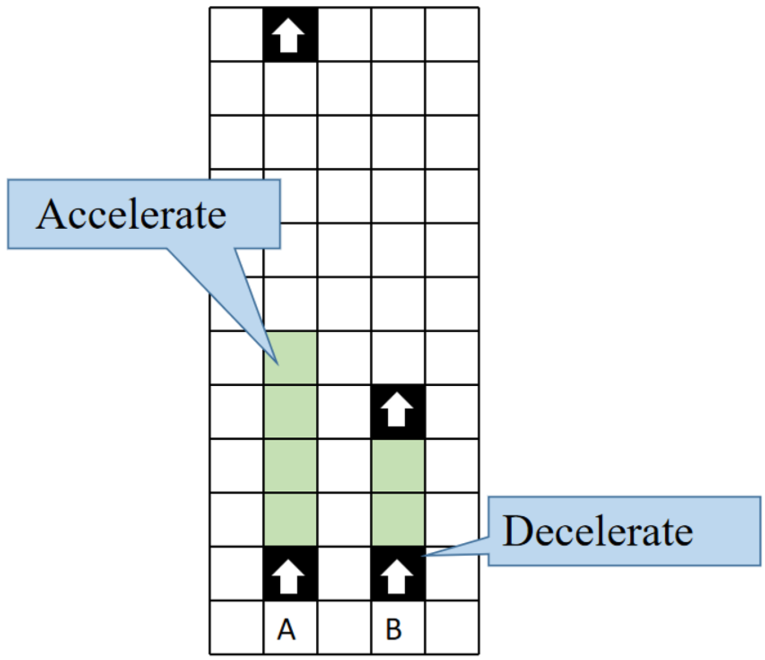

Figure 3: Illustration of car-following

3.4 Lane-changing Rule

Since our model is out of theoretical interest, we consider a symmetric passing here, i.e. there is no difference between passing from the right lane and passing from the left lane. We adopt the conditions (lane-changing incentive and safety constraint) proposed in the two-lane model [2] with some modifications:

- The safe distance d_s, $d_{o,s,forward}$ and $d_{o,s,backward}$ are also determined by the car types.
- If the car is on a middle lane, then if the conditions are met and both neighbouring lanes are safe, it changes to either lane with equal probability $p_{change}/2$.

4 Simulation

Based on the model introduced in this paper, we build a program in Python 3.7 with simple graphical user interface (GUI) as shown in Figure 5. The red vehicles are autonomous vehicles, while the blue vehicles are human-driving. We monitor the macroscale parameters, i.e. average speed, average passing time, flow, and density, in the middle of the GUI. Some important parameters are displayed on the right side, for example, lane number, road length, density, and ratio of self-driving vehicles (Auto Ratio as shown in Figure 5).

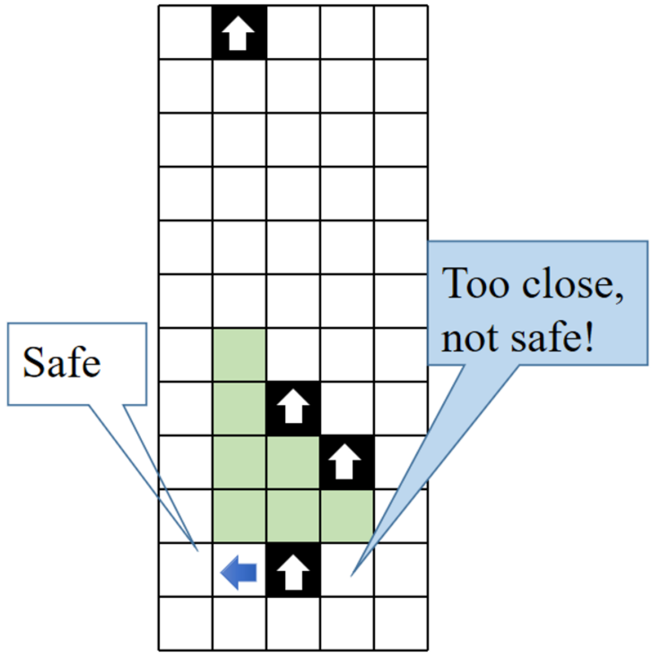

Figure 4: Illustration of lane-changing

We select traffic density and ratio of self-driving vehicles as independent variables in our study as summarized in Table 1. Other parameters are summarized in Table 2.

Table 1: Independent variables.

Independent Variables	**Value**	**Notes**
traffic density	$(0, \infty)$	The program will initiate vehicles on the roads randomly based on the density.
auto ratio	$[0, 1]$	The ratio of self-driving vehicles

5 Results

5.1 Comparison with Nagel-Schrekenber Model

We first compare our model with Nagel-Schrekenber model as described in [1]. We set the lane number of our model to be 2, i.e. one lane for each direction. The ratio of self-driving vehicles is set to be zero. We run the model from a small traffic density and gradually in-

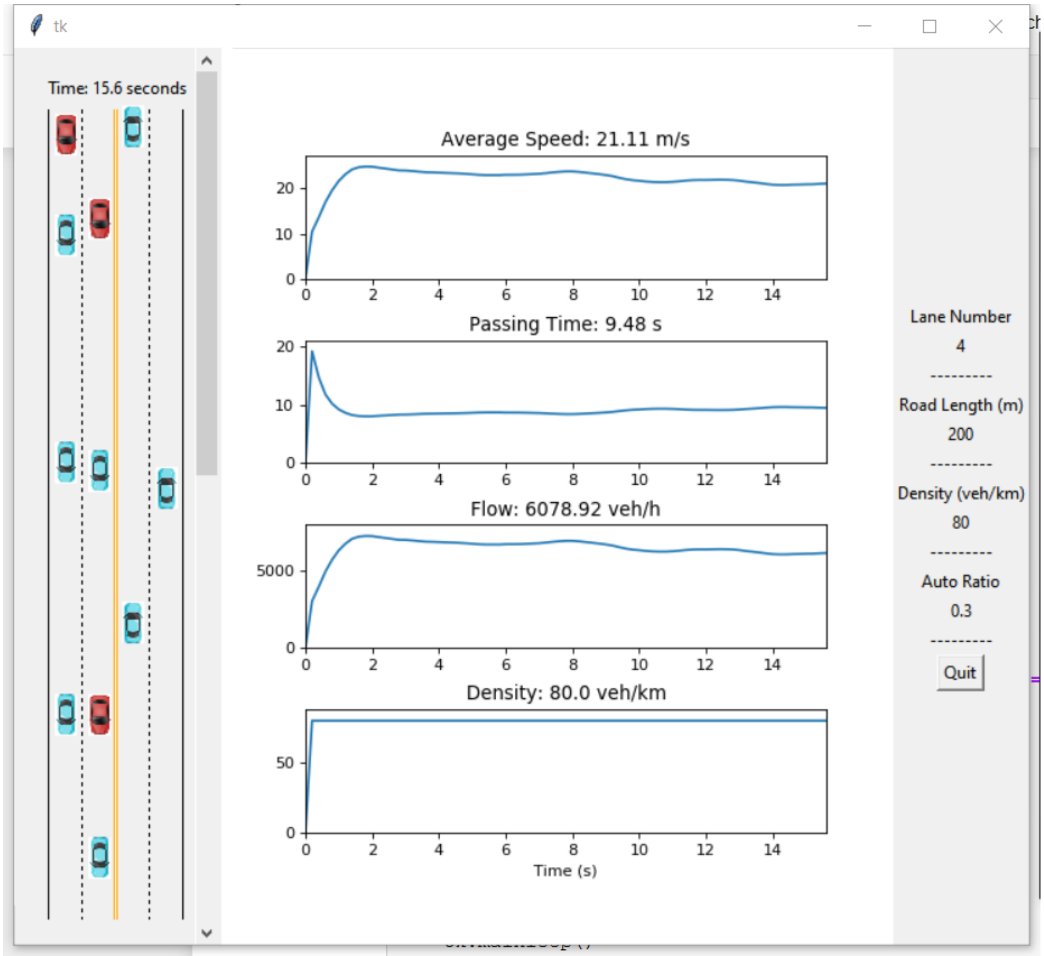

Figure 5: User interface of the program

crease the traffic density until the flow becomes zero. As shown in Fig. 6, the left side is the result of Nagel-Schrekenber model presented in [1] and the right side is what we obtain with our model. They exhibit a similar pattern that the flow increases with the traffic density when the traffic density is relatively low, but then this trend turns down as the traffic density exceeds a certain level. There is a concave of the curve in our model at a traffic density around 50 veh/mile. This can happen because of the random seed and convergence threshold we select.

5.2 Comparison with Real-world Data

We compare our model with an NGSIM trajectory dataset (available in the week 1 course material).As can be seen in Figure 7, in terms of the trends, our model matches well with the data for low and high densities. However, we do see a significant difference between the model and the real-world data for densities around 50 to 200 veh/mile. As a preliminary interpretation, we consider it is because of the intrinsic property of the cellular automata that it is discrete. Therefore, when matching with real-world data, discrepancy is

Table 2: Parameters.

Parameter	**Value**	**Notes**
road length	1 km	Vehicles will run recursively with this area
lane number	2/4/6...	Even number
v_{max}	30 m/s	Maximum speed of any type of vehicles
v_{min}	0	Minimum speed of any type of vehicles
v_{ini}	$\sim N(20,5)$	Initial speed of the vehicles, follow Gaussion distribution
t_h	2 s	Reaction time of human driver
t_a	1 s	Reaction time of self-driving vehicles
safe margin	10 feet	Minimum distance between vehicles
α	1	Coefficient of acceleration related to distance
β	1	Coefficient of acceleration related to speed difference
γ	1	Coefficient of acceleration related to acceleration of the front car
p_c	0.5	Lane changing possibility
dt	0.2 s	Time step
space resolution	1 m	The vehicles in the system can move in a precision of 1 meter

unavoidable. We need more future work to understand such a discrepancy and further improve the model to match the reality.

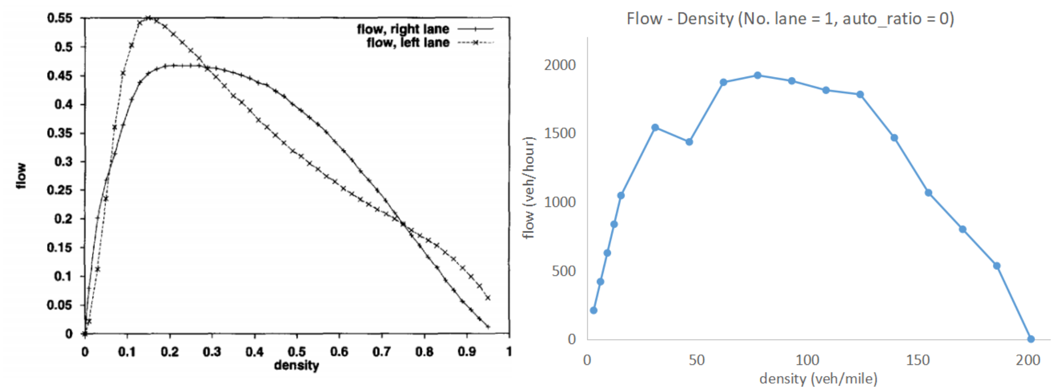

Figure 6: Compare model in this paper with Nagel-Schrekenber model

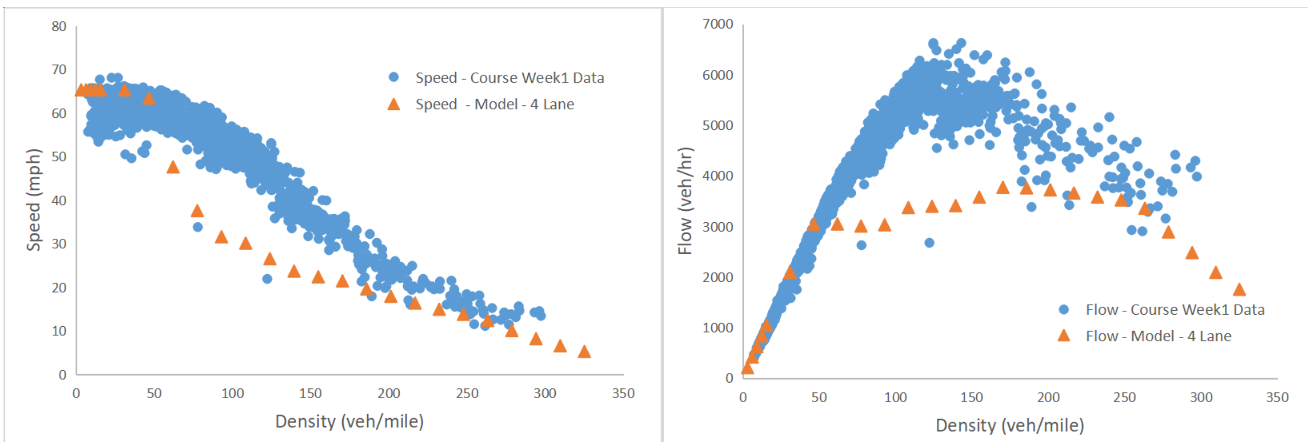

Figure 7: Compare model (4 lane, 0 self-driving ratio) with NGSIM trajectory data

5.3 Impact of the Ratio of Self-driving Vehicles

The most important part of this research is to understand how the mix of self-driving and human-driving vehicles impact the traffic flow. This is done by varying the ratio of self-driving vehicles in different conditions of traffic density or lane numbers.

Fig. 8 shows how the average passing time is related to the ratio of self-driving vehicles. The average passing time is calculated by dividing the road length with average speed. For all of the three traffic densities we select, the average passing time are reduced with the increase of ratio of self-driving vehicles, though the degree of reduction varies in different density settings. For the low density case (traffic density = 100 veh/km), including more self-driving vehicles has limited improvement to the average passing time]. For the high density case (traffic density = 300 veh/km), the avarage passing time drops dramatically from 140s to 40s. This demonstrates the significant impact of reduced reaction time and increased car following agility of self-driving vehicles.

We further change the lane number to validate the above result. When the traffic density is fixed for a specific length, increasing the lane number can be thought of equivalent to decreasing traffic density. A similar trend is observed in Figure 9 that average passing time is reduced with higher ratio of self-driving vehicles.

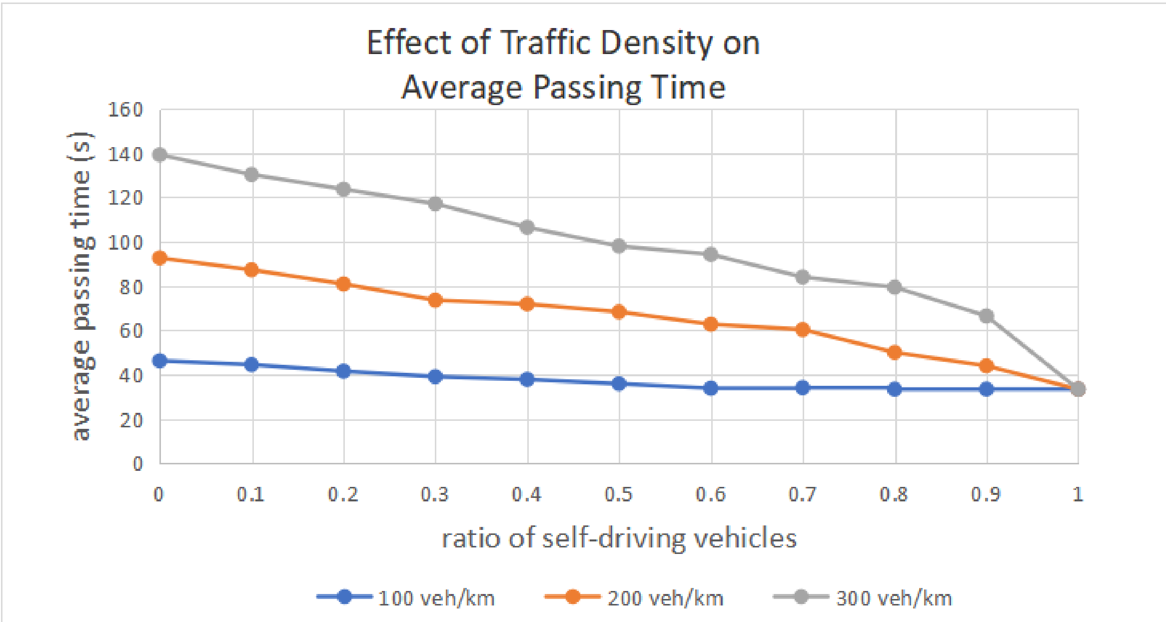

Figure 8: Relationship between the ratio of self-driving vehicles and the average passing time under different traffic densities

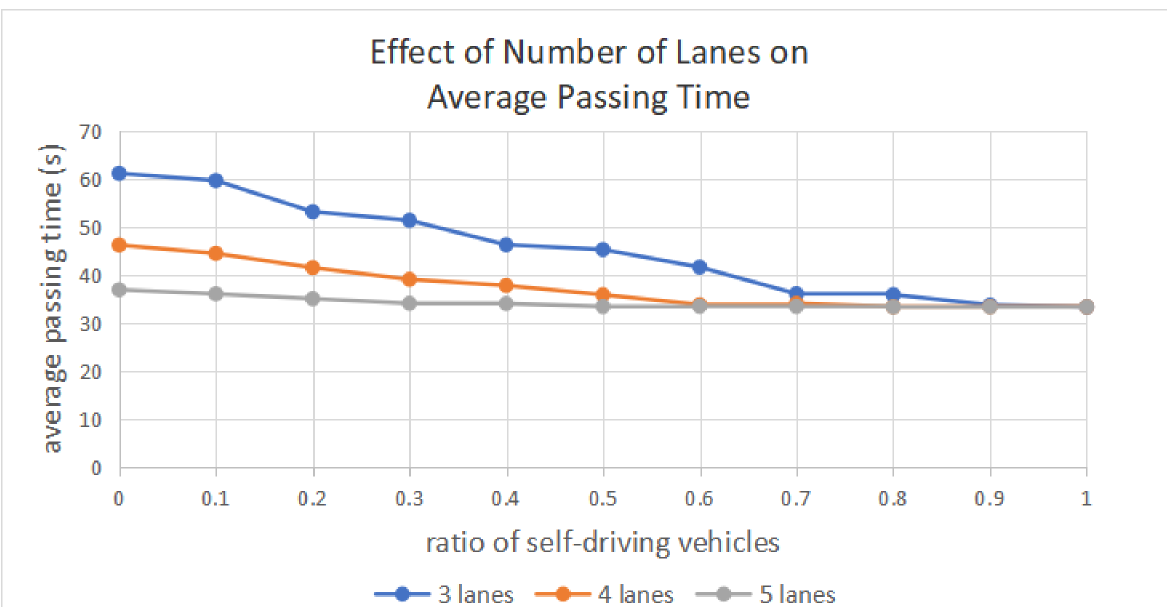

Figure 9: Relationship between the ratio of self-driving vehicles and the average passing time under different lane numbers

6 Conclusion

In this project, we establish a framework of cellular automaton model for simulating the multi-lane mixed traffic on highway with both self-driving cars and human-driving cars as well as studying the effect of self-driving cars on traffic flow.

Unlike normal human drivers, self-driving cars have their own special characteristics: automation and cooperation. Considering this, we not only introduce the car type to the state of a cell but also design more general and thoughtful temporal-evolution rules reflecting such characteristics. Specifically, compared to classic models, we introduce response time and variable acceleration so that the safe distance and updating rules are targeted for different types.

To analyze the effect of self-driving cars on traffic flow, simulations are run under different sets of key parameters (variables). Our findings are concluded below:

- self-driving cars can improve traffic conditions by speeding up traffic, increasing highway capacity and alleviating traffic jam;
- more lanes and smaller density can speed up traffic;
- self-driving vehicles can reduce the impact of number of lanes and traffic density.

We also plot the fundamental diagrams based on the simulation results and compare them to NaSch model and real-world data. They are matched at large, but the discrepancy needs further investigation and explanation. Our model can be better calibrated to describe the realistic macroscopic feature.

7 Future Work

After analyzing the results we have obtain from the model, we establish a deeper understanding of cellular automata and its application in traffic system. We propose the following work to be conducted in the future to further improve the model:

- **Improve space precision**. Traditional cellular automaton model uses one cell to represent an object. If such method is adopted for traffic flow system, a car can only moves in a value of the multiple of integer of the car size (0/5/10m, if the car size is 5m). This will seriously hurt the precision of the model. Though in our model, a precision of 1m is achieved by introducing the safe margin, we can still see the impact of discreteness, for example, we set the time step dt to be 0.2s, and suppose there are two vehicles with speed of 8m/s and 9m/s respectively, the ideal movement for them will be 1.6m and 1.8m. However, because of the discreteness property, we can only move them the same number of cells (1 cell is the floor value is taken, 2 if the ceiling value is taken). Therefore, it is necessary to further improve the space precision. We also need to pay attention to the fact that increasing the space precision will take away more computer memories and slow down the computation speed. A well balance of these issues is desired.

- **Introduce new rules for self-driving vehicles**. In our current model, we majorly introduce higher agility to self-driving vehicles regarding car following process. Other characteristics of the self-driving vehicles could be included to further explore the potential of self-driving. What come to our mind now are 1) Platooning of self-driving vehicles, during which a smaller distance is possible. 2) Communication between self-driving vehicles when performing the lane-changing. With communication, the lane-changing could be performed in less restrict conditions and hence people can more efficiently get to a faster lane. 3) Communication with infrastructures (V2I), say information sharing with traffic lights at the intersection. 4) Dedicated lanes for self-driving, which is possible when the percentage of self-driving becomes high and the effectiveness needs examination.

- **Introduce trucks**. What we have studied here are all regular family cars. However, trucks are almost everywhere in the transportation system, and by no mean we should neglect them. Introducing trucks to the model means less space is available for acceleration and lane changing, even the safe distance has to be increased. The function of platooning can also be included to trucks.

References

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