Causal Inference

Lecture #3

Juraj Medzihorsky

SGIA & RMC Durham University

6 February 2024

Synthetic Control

Comparative politics and the synthetic control method

A Abadie, A Diamond... - American Journal of ..., 2015 - Wiley Online Library

- ... Moreover, in contrast to regression analysis techniques, the synthetic control method makes
- ... of interest and the synthetic control. We apply the synthetic control method to estimate the ...
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Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program

A Abadie, A Diamond, J Hainmueller - Journal of the American ..., 2010 - Taylor & Francis

- ... the application of **synthetic control** methods to comparative ... relative to a comparable **synthetic control** region. We estimate ... units, the potential applicability of **synthetic control** methods to ...
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Assessing economic liberalization episodes: A **synthetic control** approach A Billmeier. T Nannicini - Review of Economics and Statistics. 2013 - direct.mit.edu

- ... Within the synthetic control framework, we ask whether liberalizing the economy in year T ...
- —the synthetic control. The comparison economies that form the synthetic control unit are ...
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Synth: An R package for **synthetic control** methods in comparative case studies

A Abadie, A Diamond, J Hainmueller - Journal of Statistical ..., 2011 - papers.ssrn.com

- ... The weights that define the synthetic control unit are chosen such that the synthetic control
- ... The goal of the synthetic control method is to construct a synthetic control group that yields a ...
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Synthetic	Control: Motivation
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- that level of aggregation leads to two major challenges:
 - 1. Only a small number of cases (often just 1 unit) experience the treatment, and
 - We have reason to believe that the parallel trends assumption of the difference-in-differences framework is unreasonable
- Now we find ourselves in the world of what is often referred to as a "comparative case study"

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Qualitative case studies

- Identify two or more instances of a phenomenon
- Inductive reasoning about similarities and differences between cases that shed light on causal effects

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Quantitative case studies

- Identify an exogenous event that puts a small number of aggregate units into one group and a small number into another group
- Employ causal inference techniques within the potential outcomes framework

Researchers performing comparative case studies often (but not always)

- Do not have the type of longitudinal data that we have when we say that we have "panel" data
 - The same quantitative measurements on the same units at regular time intervals

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What can we do?

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- If we are confident that the cases are simply not comparable, and therefore that the parallel trends assumption of difference in differences will not hold...
- Why not try to combine information across units that did not receive the treatment to construct a more reasonable counterfactual for the treated unit(s)?
- And if the number of cases is too small for standard statistical inference, simulate to get measures of uncertainty

Synthetic Control Method: Implementation

- Let $X_1=(Z_1,\bar{Y}_1^{K_1},\ldots,\bar{Y}_1^{K_M})'$ be a $(k\times 1)$ vector of pre-intervention characteristics.
 - The vector X_1 contains all relevant available predictors of the outcome variable
- Similarly, X_0 is a $(k \times J)$ matrix which contains the same variables for the unaffected units

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 - The vector X_1 contains all relevant available predictors of the outcome variable
- Similarly, X_0 is a $(k \times J)$ matrix which contains the same variables for the unaffected units
- The vector W^* is chosen to minimize the following expression:

$$\sum_{m=1}^{k} v_m (X_{1m} - X_{0m} W)^2,$$

where

- $w_j \ge 0$ for $j=2,\ldots,J+1$ (only positive or zero weight can be applied to a case in the donor pool)
- ullet v_m is a weight that captures the relative importance of the mth variable (covariate or pretreatment outcome) in measuring the distance between treated and control units

Synthetic Control Method: Implementation

With these two components in mind:

- 1. W: the vector of weights, w_2, \ldots, w_{J+1} applied to each unit in the donor pool
- 2. V: the diagonal matrix whose diagonal is v_1, \ldots, v_k , the weights applied to each covariate

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there are a variety of ways one could choose the synthetic control, $W^*(V)$

The most common method in the literature for choosing V seems to be the choice that minimizes the mean-squared prediction error for the pre-intervention periods:

$$\sum_{t=1}^{T_0} \left(Y_{1t} - \sum_{j=2}^{J+1} w_j^*(V) Y_{jt} \right)^2$$

The Application: California's Proposition 99

In 1988, California first passed comprehensive tobacco control legislation:

- increased cigarette tax by 25 cents/pack
- earmarked tax revenues to health and anti-smoking budgets
- funded anti-smoking media campaigns
- spurred clean-air ordinances throughout the state
- produced more than \$100 million per year in anti-tobacco projects

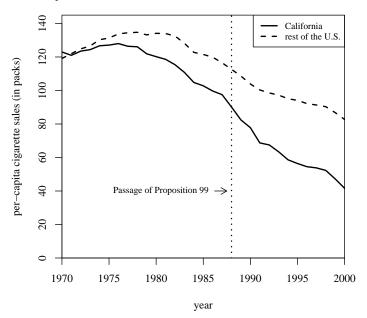
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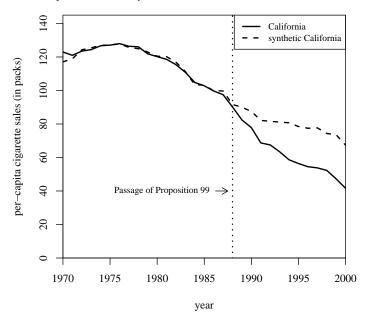
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Other states that subsequently passed control programs are excluded from donor pool of controls (AK, AZ, FL, HA, MA, MD, MI, NJ, NY, OR, WA, DC)

Cigarette Consumption: CA and the Rest of the U.S.



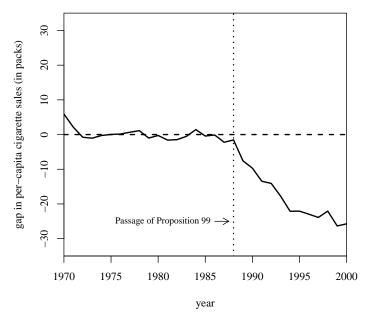
Cigarette Consumption: CA and synthetic CA



Predictor Means: Actual vs. Synthetic California

	California		Average of
Variables	Real	Synthetic	38 control states
Ln(GDP per capita)	10.08	9.86	9.86
Percent aged 15-24	17.40	17.40	17.29
Retail price	89.42	89.41	87.27
Beer consumption per capita	24.28	24.20	23.75
Cigarette sales per capita 1988	90.10	91.62	114.20
Cigarette sales per capita 1980	120.20	120.43	136.58
Cigarette sales per capita 1975	127.10	126.99	132.81

Note: All variables except lagged cigarette sales are averaged for the 1980-1988 period (beer consumption is averaged 1984-1988).



Positive State Weights in the Synthetic CA

State	Weight
Colorado	0.164
Connecticut	0.069
Montana	0.199
Nevada	0.234
Utah	0.334

Inference	
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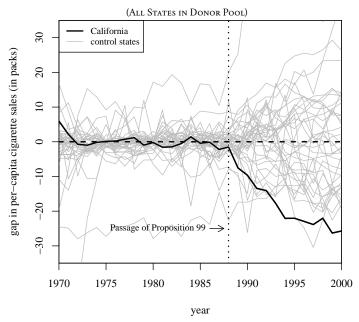
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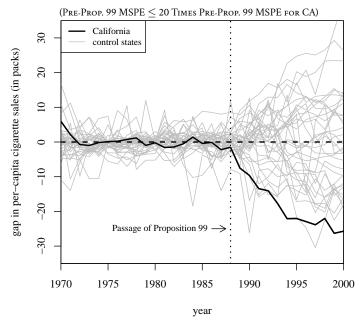
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- This produces valid inference regardless of the number of available comparison units, time periods, and whether the data are individual or aggregate

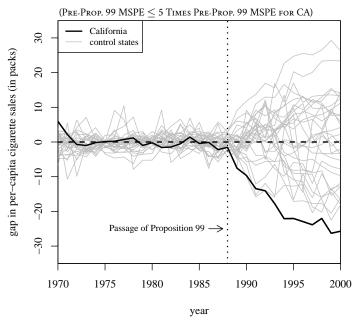
Smoking Gap for CA and 38 control states



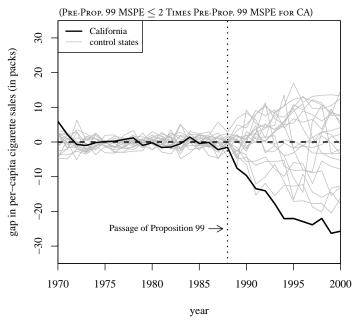
Smoking Gap for CA and 34 control states



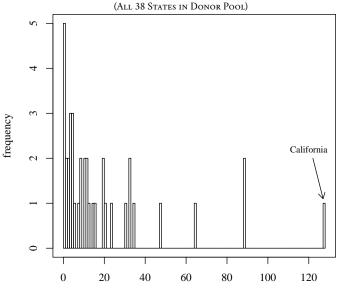
Smoking Gap for CA and 29 control states



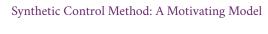
Smoking Gap for CA and 19 control states



Ratio Post-Prop. 99 MSPE to Pre-Prop. 99 MSPE



post/pre-Proposition 99 mean squared prediction error



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- We aim to estimate the effect of the intervention on the treated unit $(\alpha_{1T_0+1},\ldots\alpha_{1T})$, where $\alpha_{1t}=Y_{1t}^I-Y_{1t}^N=Y_{1t}-Y_{1t}^N$ for $t>T_0$.

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- We can always write $Y_{it}^N = \delta_t + \nu_{it}$, where
 - $\delta_t \Rightarrow \text{Time FE}$
 - $\nu_{it} \Rightarrow$ Zero mean in each time period
- Suppose that ν_{it} is given by the following model:

$$\nu_{it} = Z_i \theta_t + \lambda_t \mu_i + \varepsilon_{it},$$

- Z_i is a $(1 \times r) \Rightarrow$ Exogenous covariates
- $\theta_t \Rightarrow$ Coefficients, constant across units
- $\lambda_t \Rightarrow \text{DiD}$ assumes this to be time-constant
- μ_i is a $(F \times 1) \Rightarrow \lambda_t \cdot \mu_i = \text{Time-varying unit FE}$
- $\varepsilon_{it} \Rightarrow$ Usual i.i.d. error term
- Specification allows heterogeneous responses to multiple unobserved factors
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Synthetic Control Method: A Motivating Model	
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- The vector Z_i may contain pre- and post-intervention values of time-varying variables, as long as they are not affected by the intervention
- For example, if T = 2, $T_0 = 1$, $Z_i = (Z_{i1}, Z_{i2})$,

$$\theta_1 = \left(\begin{array}{c} \beta \\ 0 \end{array} \right) \quad \text{and} \quad \theta_2 = \left(\begin{array}{c} 0 \\ \beta \end{array} \right),$$

then $Z_i\theta_t = Z_{it}\beta$

• Let $W=(w_2,\ldots,w_{J+1})'$ with $w_j\geq 0$ for $j=2,\ldots,J+1$ and $w_2+\cdots+w_{J+1}=1$. Each value of W represents a potential synthetic control

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$$\sum_{j=2}^{J+1} w_j^* Z_j = Z_1, \sum_{j=2}^{J+1} w_j^* \bar{Y}_j^{K_1} = \bar{Y}_1^{K_1}, \cdots, \sum_{j=2}^{J+1} w_j^* \bar{Y}_j^{K_M} = \bar{Y}_1^{K_M}$$

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• Then, for $t \in \{T_0 + 1, \dots, T\}$, an unbiased estimator of α_{1t} is given by:

$$\widehat{\alpha}_{1t} = Y_{1t} - \sum_{j=2}^{J+1} w_j^* Y_{jt}$$

A Design-Based Perspective on Synthetic Control Methods

Lea Bottmer* Guido Imbens† Jann Spiess‡ Merrill Warnick§

First version: November 2020 Current version: January 2021

Abstract

Since their introduction in Abadie and Gardeazabal (2003), Synthetic Control (SC) methods have quickly become one of the leading methods for estimating causal effects in observational studies with panel data. Formal discussions often motivate SC methods by the assumption that the potential outcomes were generated by a factor model. Here we study SC methods from a design-based perspective, assuming a model for the selection of the treated unit(s), e.g., random selection as guaranteed in a randomized experiment. We show that SC methods offer benefits even in settings with randomized assignment, and that the design perspective offers new insights into SC methods for observational data. A first insight is that the standard SC estimator is not unbiased under random assignment. We propose a simple modification of the SC estimator that guarantees unbiasedness in this setting and derive its exact, randomization-based, finite sample variance. We also propose an unbiased estimator for this variance. We show in settings with real data that under random assignment this Modified Unbiased Synthetic Control (MUSC) estimator can have a root mean-squared error (RMSE) that is substantially lower than that of the difference-in-means estimator. We show that such an improvement is weakly guaranteed if the treated period is similar to the other periods, for example, if the treated period was randomly selected. The improvement is most likely to be substantial if the number of pre-treatment periods is large relative to the number of control units.

Keywords: Randomization, Synthetic Controls, Panel Data, Causal Effects

SCM: Summary

- Panel data is useful in that it allows us to get to conditional independence via selection on unobservables
- Difference-in-differences designs allow us to make standard statistical inferences when we are willing to make the parallel trends assumption
- Synthetic control methods can be seen as an extension of the difference-in-differences logic for situations in which
 - Both treatment and control are measured at a high level of aggregation
 - There is a small number of, and usually only 1, treated unit
 - We have a small N with respect to potential control units
- Synthetic control methods use measures on the non-treated donor pool units to construct counterfactuals for treated units based on a weighted average
- Methods of inference for synthetic control are non-standard, but will almost surely involve some type of resampling/simulation methods



Thank you!

Appendix