

# Introduction to Mathematics for Data Science

## Personal Assignment 2

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November 26, 2023

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### 1 Question 1

#### 1.1 Solve Matrix B

Consider the matrix  $A = \begin{pmatrix} 5 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$  Find a matrix  $B$  such that  $AB = \begin{pmatrix} 8 & 6 \\ -1 & 9 \\ 2 & 6 \\ 2 & 0 \end{pmatrix}$ .

#### 1.2 Analytics

Due to the shape of matrix  $A = [a_{ij}]$  shape is  $4 \times 3$ , and the shape of matrix  $AB = [a_{ik}]$  is  $4 \times 2$ , we can use the matrix multiplication equation to solve for  $B$  (we can also infer that matrix  $B$  is 3 rows and 2 columns).

To solve for  $B$ , we have the matrix multiplication equation:

$$\begin{pmatrix} 5 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -1 & 9 \\ 2 & 6 \\ 2 & 0 \end{pmatrix}$$

Equating the corresponding elements with the given matrix  $AB$ , we get the following system

$$\text{of equations} \Rightarrow \begin{cases} 5b_{11} + 2b_{21} + 3b_{31} = 8 \\ 5b_{12} + 2b_{22} + 3b_{32} = 6 \\ -2b_{11} + 3b_{21} + b_{31} = -1 \\ -2b_{12} + 3b_{22} + b_{32} = 9 \\ 2b_{21} + 2b_{31} = 2 \\ 2b_{22} + 2b_{32} = 6 \\ b_{11} + b_{31} = 2 \\ b_{12} + b_{32} = 0 \end{cases} \quad \text{Then solve the equations and get } B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 1 & 0 \end{pmatrix}.$$

## 2 Question 2

### 2.1 Find Linear Transformation

Consider the vectors  $\vec{a} = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$ ,  $\vec{b} = \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$ ,  $\vec{c} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ . Is there a linear transformation  $f$ :

$\mathbb{R}^3 \rightarrow \mathbb{R}^4$  such that the following conditions are satisfied?  $f(\vec{a}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ,  $f(\vec{b}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$  and

$f(\vec{c}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ . If there is such an  $f$ , then find its matrix. If there is not, then explain why.

#### 2.1.1 Analytics

We can represent the matrix  $f$  as:

$$f = \begin{pmatrix} | & | & | \\ \vec{f}(\vec{a}) & \vec{f}(\vec{b}) & \vec{f}(\vec{c}) \\ | & | & | \end{pmatrix}$$

Then, set up the following system of equations:

$$\begin{pmatrix} | & | & | \\ \vec{f}(\vec{a}) & \vec{f}(\vec{b}) & \vec{f}(\vec{c}) \\ | & | & | \end{pmatrix} \begin{pmatrix} 7 & 8 & -1 \\ 1 & 1 & 0 \\ 2 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we can find the matrix  $f$  by solving this system of equations.

```
1 import numpy as np
2
3 # Define the given vectors
4 a = np.array([7, 1, 2])
5 b = np.array([8, 1, 4])
6 c = np.array([-1, 0, 2])
7
8 # Define the target matrices
9 fa_target = np.array([1, 0, 0, 0])
10 fb_target = np.array([0, 1, 0, 0])
```

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11 fc_target = np.array([0, 0, 0, 0])
12
13 # Form the matrix equation [f] * [a, b, c] = [fa_target,
14     fb_target, fc_target]
15 A = np.vstack((a, b, c)).T # Transpose to make it 3x3
16 B_target = np.vstack((fa_target, fb_target, fc_target)).T #
17     Transpose to make it 3x4
18
19 # Solve for the matrix [f]
20 try:
21     f_matrix, residuals, rank, s = np.linalg.lstsq(A, B_target,
22         rcond=None)
23     print("Matrix [f]:")
24     print(f_matrix)
25 except np.linalg.LinAlgError:
26     print("No solution exists for the given conditions.")

```

**Output Analytics:** 'No solution exists for the given conditions.' It implies that there is no matrix  $f$  satisfying the specified conditions.

### 2.1.2 Another Linear Transformation

Is there a linear transformation  $f: \mathbb{R}^3 \rightarrow \mathbb{R}^4$  such that the following conditions are satisfied?

$$f(\vec{a}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, f(\vec{b}) = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \text{ and } f(\vec{c}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

### 3 Question 3

#### 4 Question 4

## 5 Question 5