

Introduction to Math for DS Group Task 3

IMDS Group 24

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1 Question 1

Are the vectors $(1, 3, 4)$, $(4, -1, 5)$ and $(5, 2, 9)$ linearly independent?

1.1 AI Bot's Answer

ChatGPT: To determine if the vectors $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 2 \\ 9 \end{bmatrix}$ are linearly independent, you can create a matrix using these vectors and perform row operations to see if the matrix reduces to the row-echelon form with non-zero rows.

Let A be the matrix with the given vectors as its columns:

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & -1 & 2 \\ 4 & 5 & 9 \end{bmatrix}$$

Now, perform row operations to get the row-echelon form:

$$\begin{bmatrix} 1 & 4 & 5 \\ 3 & -1 & 2 \\ 4 & 5 & 9 \end{bmatrix}$$

$$R_2 = R_2 - 3R_1 \rightarrow \begin{bmatrix} 1 & 4 & 5 \\ 0 & -13 & -13 \\ 4 & 5 & 9 \end{bmatrix}$$

$$R_3 = R_3 - 4R_1 \rightarrow \begin{bmatrix} 1 & 4 & 5 \\ 0 & -13 & -13 \\ 0 & -11 & -11 \end{bmatrix}$$

$$R_3 = R_3 - \frac{11}{13}R_2 \longrightarrow \begin{bmatrix} 1 & 4 & 5 \\ 0 & -13 & -13 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, we have the row-echelon form. Since there is a row of zeros, the system is linearly dependent. Specifically, the third vector can be expressed as a linear combination of the first two vectors.

2 Question 2

Is the vector $(1, 0, -1)$ in the plane spanned by the vectors $(4, 11, 7)$ and $(3, 11, 8)$?

2.1 AI Bot's Answer

ChatGPT: To determine whether the vector $(1, 0, -1)$ is in the plane spanned by the vectors $(4, 11, 7)$ and $(3, 11, 8)$, we can check if $(1, 0, -1)$ can be expressed as a linear combination of the other two vectors.

Let A be the matrix formed by the vectors $(4, 11, 7)$ and $(3, 11, 8)$, and \mathbf{v} be the vector $(1, 0, -1)$:

$$A = \begin{bmatrix} 4 & 3 \\ 11 & 11 \\ 7 & 8 \end{bmatrix}$$

We want to check if there exist scalars x and y such that $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

Solving the system of equations:

$$\begin{bmatrix} 4 & 3 \\ 11 & 11 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$