

Introduction to Mathematics for Data Science

Personal Assignment 2

Zehao Qian

November 26, 2023

1 Question 1

1.1 Solve Matrix B

Consider the matrix $A = \begin{pmatrix} 5 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ Find a matrix B such that $AB = \begin{pmatrix} 8 & 6 \\ -1 & 9 \\ 2 & 6 \\ 2 & 0 \end{pmatrix}$.

1.2 Analytics

Due to the shape of matrix $A = [a_{ij}]$ shape is 4×3 , and the shape of matrix $AB = [a_{ik}]$ is 4×2 , we can use the matrix multiplication equation to solve for B (we can also infer that matrix B is 3 rows and 2 columns).

To solve for B , we have the matrix multiplication equation:

$$\begin{pmatrix} 5 & 2 & 3 \\ -2 & 3 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -1 & 9 \\ 2 & 6 \\ 2 & 0 \end{pmatrix}$$

Equating the corresponding elements with the given matrix AB , we get the following system

$$\text{of equations} \Rightarrow \begin{cases} 5b_{11} + 2b_{21} + 3b_{31} = 8 \\ 5b_{12} + 2b_{22} + 3b_{32} = 6 \\ -2b_{11} + 3b_{21} + b_{31} = -1 \\ -2b_{12} + 3b_{22} + b_{32} = 9 \\ 2b_{21} + 2b_{31} = 2 \\ 2b_{22} + 2b_{32} = 6 \\ b_{11} + b_{31} = 2 \\ b_{12} + b_{32} = 0 \end{cases} \quad \text{Then solve the equations and get } B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 1 & 0 \end{pmatrix}.$$

2 Question 2

2.1 Find Linear Transformation

Consider the vectors $\vec{a} = \begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$. Is there a linear transformation f :

$\mathbb{R}^3 \rightarrow \mathbb{R}^4$ such that the following conditions are satisfied? $f(\vec{a}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $f(\vec{b}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and

$f(\vec{c}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$. If there is such an f , then find its matrix. If there is not, then explain why.

2.1.1 Analytics

We can represent the matrix f as:

$$f = \begin{pmatrix} | & | & | \\ \vec{f}(\vec{a}) & \vec{f}(\vec{b}) & \vec{f}(\vec{c}) \\ | & | & | \end{pmatrix}$$

Then, set up the following system of equations:

$$\begin{pmatrix} | & | & | \\ \vec{f}(\vec{a}) & \vec{f}(\vec{b}) & \vec{f}(\vec{c}) \\ | & | & | \end{pmatrix} \begin{pmatrix} 7 & 8 & -1 \\ 1 & 1 & 0 \\ 2 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we can find the matrix f by solving this system of equations.

```
1 import numpy as np
2
3 # Define the given vectors
4 a = np.array([7, 1, 2])
5 b = np.array([8, 1, 4])
6 c = np.array([-1, 0, 2])
7
8 # Define the target matrices
9 fa_target = np.array([1, 0, 0, 0])
10 fb_target = np.array([0, 1, 0, 0])
```

```

11 fc_target = np.array([0, 0, 0, 0])
12
13 # Form the matrix equation [f] * [a, b, c] = [fa_target,
14     fb_target, fc_target]
15 A = np.vstack((a, b, c)).T # Transpose to make it 3x3
16 B_target = np.vstack((fa_target, fb_target, fc_target)).T #
17     Transpose to make it 3x4
18
19 # Solve for the matrix [f]
20 try:
21     f_matrix, residuals, rank, s = np.linalg.lstsq(A, B_target,
22         rcond=None)
23     print("Matrix [f]:")
24     print(f_matrix)
25 except np.linalg.LinAlgError:
26     print("No solution exists for the given conditions.")

```

Output Analytics: 'No solution exists for the given conditions.' It implies that there is no matrix f satisfying the specified conditions.

2.2 Another Linear Transformation

Is there a linear transformation $f: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ such that the following conditions are satisfied?

$$f(\vec{a}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, f(\vec{b}) = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \text{ and } f(\vec{c}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

2.2.1 Analytics

Bring $f(\vec{a})$, $f(\vec{b})$, $f(\vec{c})$ to the Python program we created. The output is still **'No solution exists for the given conditions.'**

3 Question 3

3.1 Find Value for x and y

Consider the linear system of equations

$$\begin{pmatrix} 6 & -4 \\ 8 & -6 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Either find a solution or explain why it does not have any solutions.

3.2 Analyics

To determine whether the linear system has a solution, we can check the consistency of the system using the augmented matrix. The augmented matrix for the given system is:

$$\begin{pmatrix} 6 & -4 & 1 \\ 8 & -6 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

4 Question 4

5 Question 5