Introduction to Math for DS Group Task 2

IMDS Group 24
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1 Question 1

If $A = \begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 2 & 0 & 2 \\ 6 & 0 & 3 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. What is the product AB?

```python
import numpy as np

# Define the matrices A and B
A = np.array([
    [1, 0, 4, 1],
    [0, 2, 0, 2],
    [6, 0, 3, 11]
])

B = np.array([
    [7, -1, 2],
    [1, 1, 0],
    [2, 0, 1]
])

# Calculate the product of A and B
AB_product = np.dot(A, B)
AB_product
```

Return ValueError: shapes (3,4) and (3,3) not aligned: 4 (dim 1) != 3 (dim 0)

Analytics: The product of two matrices AB is undefined if the number of columns in the first matrix A does not match the number of rows in the second matrix B. In this case, matrix A has 4 columns, while matrix B has 3 rows, so their product cannot be computed.

Matrix multiplication requires that the number of columns in the first matrix be equal to the number of rows in the second matrix. If there's a third matrix C that should be involved to make the multiplication possible, please provide it, otherwise matrix A and B as given cannot be multiplied.

2 Question 2

What is the dimension of the span of the vectors $(5, 7, 9, 0)$, $(2, 5, 0, 1)$, $(0, 0, 0, 1)$ and $(7, 12, 9, 3)$?

```python
import numpy as np

# Define the vectors
vectors = np.array([
    [5, 7, 9, 0],
    [2, 5, 0, 1],
    [0, 0, 0, 1],
    [7, 12, 9, 3]
])

# Using numpy to find the rank of the matrix composed of the
  given vectors
rank_of_matrix = np.linalg.matrix_rank(vectors)
rank_of_matrix
```

Analytics: The four vectors $(5, 7, 9, 0)$, $(2, 5, 0, 1)$, $(0, 0, 0, 1)$, and $(7, 12, 9, 3)$ are actually linearly related, because the dimensions of the space they stretch are 3, not 4. This means that of the four vectors, at least one can be linearly represented by the other three. To find a set of linearly independent vectors, we need to remove at least one of the vectors so that the rank of the remaining set of vectors equals the number of vectors. In this example, since any three of these four vectors can form a basis of a stretched space, a linearly independent set of vectors can be obtained by removing any one of them.