1. Consider the sample space $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let $A = \{2, 4, 6, 8\}$, $B = \{3, 6, 9\}$, $C = \{2, 3, 5, 7\}$. Assuming that all outcomes $1, \ldots, 9$ are equally likely, write down the sets for the following outcomes and calculate their probabilities.

   (a) A and B.

   (b) C or A.

   (c) A^C (the complement of A).

   (d) B but not C.

   Next, answer the following questions.

   (e) What are the probabilities $P(A|C)$ and $P(A|B)$?

   (f) Which pairs of A, B,C are independent?

2. A traffic calming taskforce set up a speed camera at the entrance to local village Esh Losing. The speed limit at this point is 30mph.

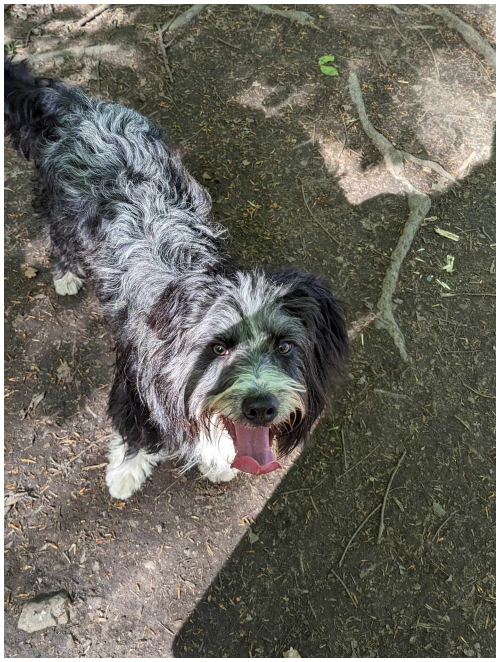
   After a month of measuring, the taskforce conclude that the speed of cars entering Esh Losing is distributed normally, with a mean of 33mph and a standard deviation of 2.8mph.

   A speed camera is then set up in secret outside Esh Losing. The camera takes a photo of the registration plate of any car travelling at more than 35mph as they enter the village, and a ticket is issued to the registered owner.

   (a) What is the probability a car entering Esh Losing gets a ticket issued?

   (b) A ticket is issued to a car's owner. What is the probability that the car was detected as travelling at more than 40mph?

3. Every week day, when I get up, I make breakfast for my dog Quiz (pictured). Quiz's breakfast is supposed to be 80g of biscuits and 150g of meat, but I'm a bit haphazard with my measures.



Assume the average amount of biscuits I give Quiz for breakfast is 80g, with a standard deviation of 5g. Assume the average weight of meat I give Quiz for breakfast is 150g, with a standard deviation of 8g. Assume the amount of biscuits and the weight of meat are independent to each other, and that each day's amounts are independent of any other day.

(a) What is the mean and standard deviation of the total weight of Quiz's daily breakfast?

(b) What is the mean and standard deviation of the total weight of Quiz's breakfast over five week days?

On weekends, my partner makes Quiz his breakfast instead. She is more reliable than me. Assume the average amount of biscuits she gives Quiz for breakfast is 80g, with a standard deviation of 2g. Assume the average weight of meat she gives Quiz for breakfast is 150g, with a standard deviation of 3g. Assume that the same independence results as above hold for these breakfasts, and that the breakfasts my partner serves Quiz are independent of the ones I serve him.

(c) What is the mean and standard deviation of the total weight of Quiz's breakfast over an entire week (five week days and two weekend days)?

(d) I decide to stop measuring out biscuits and meat independently, instead trying to measure out less meat than usual if I think I've

measured our more biscuits than usual. What effect will that have on the covariance between biscuit weight and meat weight?

4. A probability professor sets the same online quiz every year for her students. She knows (by looking at submissions from previous years), that students have a 10% probability of scoring full marks. She also knows that there is a 1% probability a student will cheat.

   The professor marks the first submission to the online quiz, and gives it full marks.

   (a) Let C be the event a student cheats, and let F be the event a student scores full marks on the online quiz. What would be a sensible probability value for $P(F|C)$, and why?

   (b) Using your value for $P(F|C)$, find $P(C|F)$.

5. (Tricky!) It is the year 2090, and some human beings are being born with the Y-gene, a genetic mutation which gives people superpowers on their 18th birthday. Professor Y is attempting to create a test that can be used to check whether someone younger than 18 has the Y gene. The test will return either a positive result (suggesting the test subject has the Y-gene), or a negative result (suggesting the test subject does not have the Y-gene).

   Only 0.01% of the population have the Y-gene. Professor Y's test is 90% accurate, meaning 90% of people with the Y-gene get a positive result, and 90% of people without the Y-gene get a negative result.

   Local youth Scat Sombers is given the test, which comes back positive.

   (a) What is the probability Scat Sombers has the Y-gene?

   (b) Professor X runs the test on Scat Sombers a second time, and receives another positive result. Assuming that the test results are independent on each other, conditional on whether or not Scat has the Y-gene, what is now the probability Scat Sombers has the Y-gene?