Introduction to Statistics for DS
Week 3

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1 QA

The total weight of Quiz's daily breakfast is the sum of the weights of biscuits and meat.

- Mean of biscuits = 80g
- Mean of meat = 150g
- Total mean weight = Mean of biscuits + Mean of meat = 80g + 150g = 230g

Since the amounts are independent, the variances add:

- $Variance\ of\ biscuits = (Standard\ deviation\ of\ biscuits)^2 = 5^2g$
- $Variance\ of\ meat = (Standard\ deviation\ of\ meat)^2 = 8^2g$

$$Total\ variance\ of\ the\ daily\ breakfast = Variance\ of\ biscuits + Variance\ of\ meat = 5^2g + 8^2g = 89g$$

So, the standard deviation of the daily breakfast is the square root of the total variance: $\sqrt{89}g$

2 QB

$$5\ Days\ Mean = 5 * Totalmeanweight$$

$$5\ Days\ standard\ deviation = 25 * Total\ variance\ of\ the\ daily\ breakfast$$

Prove That:

$$Var(aX) = a^2Var(X)$$

1. Start with the definition of variance: $Var(aX) = E[(aX - E[aX])^2]$
2. Expand the square and use the linearity of expectations: $Var(aX) = E[(a^2X^2 - 2aXE[aX] + (E[aX])^2)]$
3. Use the linearity of expectations to separate the terms: $Var(aX) = a^2E[X^2] - 2aE[X]E[aX] + (E[aX])^2$
4. Since "a" is a constant, E[aX] = aE[X]: $Var(aX) = a^2E[X^2] - 2a^2E[X]E[X] + (aE[X])^2$
5. Simplify: $Var(aX) = a^2E[X^2] - 2a^2(E[X])^2 + a^2(E[X])^2$
6. Combine like terms: $Var(aX) = a^2E[X^2] - a^2(E[X])^2$
7. Notice that a^2 is a constant, so you can factor it out: $Var(aX) = a^2(E[X^2] - (E[X])^2)$
8. Recall that $Var(X) = E[X^2] - (E[X])^2$: $Var(aX) = a^2Var(X)$

Important Conclusion:

1. $E(X + Y) = E(X) + E(Y)$
2. $E(aX) = aE(X)$
3. $Var(aX) = a^2Var(X)$
4. $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$
5. $Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$

3 QC

Given:

- $\lambda_{weekday\ biscuits} = 80$, $\sigma_{weekday\ biscuits} = 5$
- $\lambda_{weekday\ meat} = 150$, $\sigma_{weekday\ meat} = 8$
- $\lambda_{weekends\ biscuits} = 80$, $\sigma_{weekends\ biscuits} = 2$
- $\lambda_{weekends\ biscuits} = 150$, $\sigma_{weekends\ biscuits} = 3$

4 QD: Effect on Covariance Between Biscuit Weight and Meat Weight

The covariance between two random variables X and Y is defined as:

$$Cov(X, Y) = E[(X - E[X]) * (Y - E[Y])]$$

The covariance between biscuit weight and meat weight measures the joint variability of these two variables. If you change your measurement method and start measuring less meat when you think you've measured more biscuits, it means that the two variables are no longer independent. This change in measurement approach may lead to a decrease in their covariance.

The effect on the covariance between biscuit weight and meat weight would depend on how you adapt your measurements based on your perception of biscuit measurements. If your adjustments make the two variables less correlated, the covariance will decrease. If your adjustments make them more correlated, the covariance will increase. The exact effect on the covariance would require a detailed analysis of the measurement adjustments you make.