

3 level models - further insight

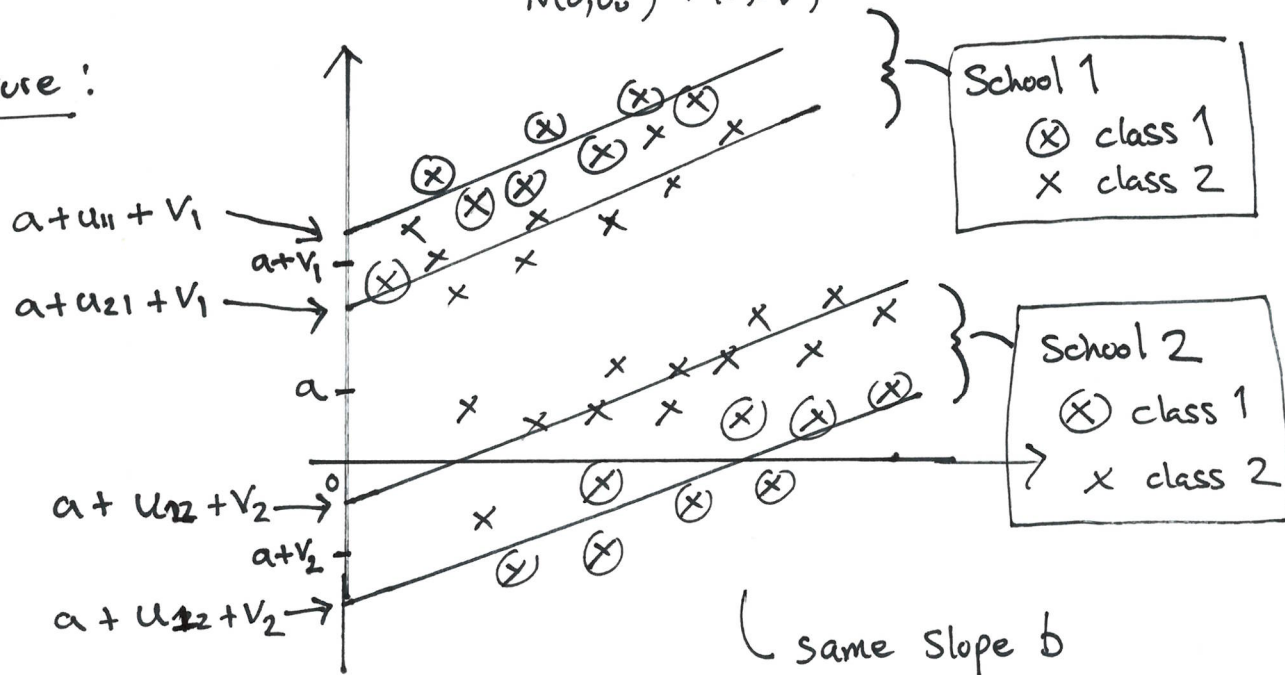
Response: y_{ijk} e.g. pupils (i) inside classes (j) inside schools (k)

Consider a single covariate x_{ijk} [at the pupil level]

Intercept only + covariate

$$\text{Model: } y_{ijk} = a + bx_{ijk} + u_{jk} + v_k + \epsilon_{ijk} \quad \begin{matrix} N(0, \sigma_u^2) & N(0, \sigma_v^2) & N(0, \sigma_\epsilon^2) \end{matrix}$$

Picture:



Building up the model

We have $y_{ijk} = a_{jk} + bx_{ijk} + \epsilon_{ijk}$.

↑ allow different intercept for each class-school combination

Now $a_{jk} = a_k + u_{jk}$, $u_{jk} \sim N(0, \sigma_u^2)$

↑ school level intercept.

Finally $a_k = a + v_k$, $v_k \sim N(0, \sigma_v^2)$

↑ 'overall average' intercept

Hence, $a_{jk} = (a + v_k) + u_{jk}$

Can do the same for the slope term, replace b with b_{jk} .
Notation gets messy!