Table 1: Comparison of DPR, BM-25, LLM Reranking and GPR-LLM (RBF kernel, $\epsilon = 0.1$) with all-MiniLM-L6-v2 encoder across four datasets (TravelDest, POINTREC, Yelp Restaurant, TripAdvisor Hotel) at varying budget of LLM labels. Metrics reported are Precision@10, NDCG@10, Precision@30, and NDCG@30; bold values indicate the best-performing method per column. Statistically significant improvements over the best-performing baseline (paired t-test, $p < 0.05$) are indicated by an underline.

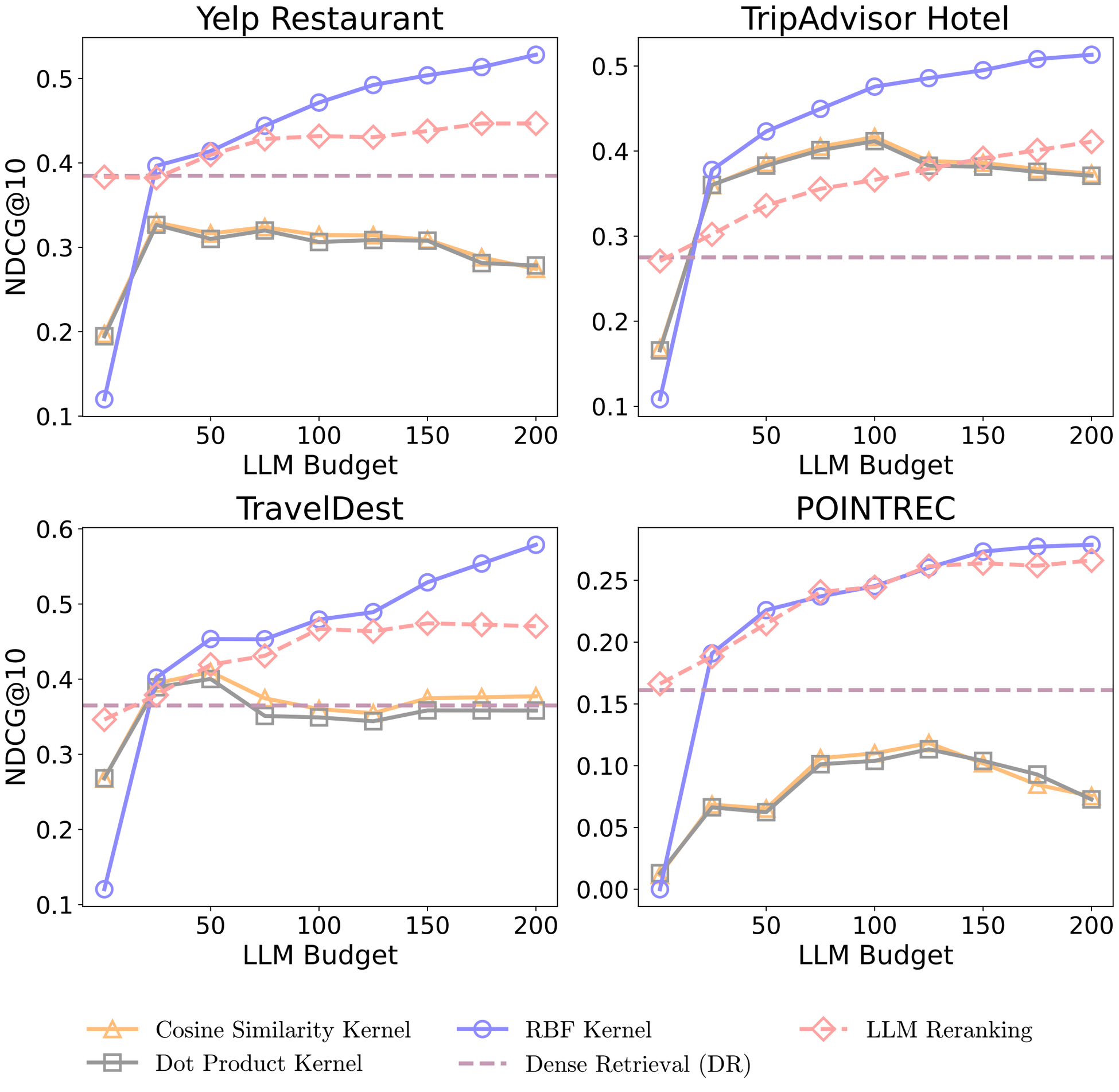
Budget	Method	TravelDest				POINTREC				Yelp Restaurant				TripAdvisor Hotel			
		P@10	NDCG@10	P@30	NDCG@30	P@10	NDCG@10	P@30	NDCG@30	P@10	NDCG@10	P@30	NDCG@30	P@10	NDCG@10	P@30	NDCG@30
N/A	DPR	0.360	0.366	0.314	0.332	0.164	0.179	0.104	0.182	0.346	0.362	0.282	0.331	0.231	0.297	0.166	0.365
	BM25	0.234	0.238	0.237	0.239	0.025	0.032	0.025	0.038	0.309	0.327	0.236	0.283	0.205	0.257	0.153	0.325
25	LLM Reranking	0.294	0.309	0.238	0.219	**0.175**	**0.206**	0.106	0.158	0.324	0.374	0.237	0.260	0.251	0.349	0.178	0.332
	GPR-LLM	**0.376**	**0.401**	**0.340**	**0.364**	0.157	0.200	**0.108**	**0.177**	**0.360**	**0.402**	**0.273**	**0.340**	**0.282**	**0.382**	**0.182**	**0.426**
50	LLM Reranking	0.356	0.371	0.248	0.281	0.196	0.234	0.105	0.208	0.386	0.426	0.254	0.332	0.289	0.397	0.169	0.417
	GPR-LLM	**0.432**	**0.445**	**0.362**	**0.389**	**0.236**	**0.262**	**0.113**	**0.222**	**0.408**	**0.451**	**0.304**	**0.377**	**0.324**	**0.439**	**0.197**	**0.482**
100	LLM Reranking	0.398	0.406	0.296	0.328	0.225	0.255	0.124	0.231	0.418	0.459	0.298	0.379	0.322	0.438	0.189	0.472
	GPR-LLM	**0.448**	**0.472**	**0.380**	**0.412**	**0.246**	**0.269**	**0.130**	**0.239**	**0.444**	**0.487**	**0.334**	**0.413**	**0.358**	**0.481**	**0.218**	**0.529**

Table 2: Comparison of DPR, BM-25, LLM Reranking and GPR-LLM (RBF kernel, $\epsilon = 0.1$) with msmarco-distilbert-base-tas-b encoder across four datasets (TravelDest, POINTREC, Yelp Restaurant, TripAdvisor Hotel) at varying budget of LLM labels. Metrics reported are Precision@10, NDCG@10, Precision@30, and NDCG@30; bold values indicate the best-performing method per column. Statistically significant improvements over the best-performing baseline (paired t-test, $p < 0.05$) are indicated by an underline.

Budget	Method	TravelDest				POINTREC				Yelp Restaurant				TripAdvisor Hotel			
		P@10	NDCG@10	P@30	NDCG@30	P@10	NDCG@10	P@30	NDCG@30	P@10	NDCG@10	P@30	NDCG@30	P@10	NDCG@10	P@30	NDCG@30
N/A	DPR	0.358	0.365	0.318	0.333	0.143	0.161	0.094	0.165	0.363	0.385	0.294	0.351	0.224	0.275	0.165	0.349
	BM25	0.234	0.238	0.237	0.239	0.025	0.032	0.025	0.038	0.309	0.327	0.236	0.283	0.205	0.257	0.153	0.325
25	LLM Reranking	0.344	0.379	0.313	0.339	0.154	0.188	0.094	0.178	0.340	0.382	**0.289**	**0.355**	0.227	0.302	0.165	0.369
	GPR-LLM	**0.370**	**0.402**	**0.325**	**0.356**	**0.159**	**0.192**	**0.098**	**0.183**	**0.359**	**0.397**	0.272	0.343	**0.286**	**0.378**	**0.177**	**0.419**
50	LLM Reranking	0.380	0.419	0.298	0.339	0.171	0.215	0.098	0.193	0.368	0.410	0.282	0.355	0.251	0.336	0.166	0.389
	GPR-LLM	**0.416**	**0.453**	**0.348**	**0.386**	**0.176**	**0.217**	**0.103**	**0.197**	**0.376**	**0.414**	**0.286**	**0.361**	**0.325**	**0.423**	**0.199**	**0.470**
100	LLM Reranking	0.428	0.467	0.323	0.371	0.214	0.245	0.102	0.200	0.391	0.432	0.285	0.366	0.273	0.366	0.170	0.408
	GPR-LLM	**0.444**	**0.479**	**0.344**	**0.389**	**0.215**	**0.246**	**0.121**	**0.227**	**0.430**	**0.472**	**0.311**	**0.398**	**0.360**	**0.476**	**0.225**	**0.535**

Table 3: Per-query time complexity and latency for different retrieval methods. Let N be the number of passages, D the embedding dimension, R the LLM budget, and C_{LLM} the cost of a single LLM scoring call. For GPR, the time complexity includes: kernel matrix computation $\mathcal{O}(R^2D)$, matrix inversion $\mathcal{O}(R^3)$, and inference over N passages $\mathcal{O}(NRD)$. We report empirical per-query latency (in seconds) under the following setup: $R = 50$, $N = 100{,}000$, using `MiniLM` embeddings. Latency values include 95% confidence intervals in $[\cdot]$.
System specifications: CPU—Intel(R) Core(TM) i7-14700HX; GPU—NVIDIA GeForce RTX 4070 Laptop GPU; average CPU utilization during measurement: $\sim$5%.

Method	**Per-query Complexity**	**Latency (sec)**
DPR	$O(ND)$	0.165 [0.161, 0.168]
LLM Rerank	$O(ND + R \cdot C_{\text{LLM}})$	0.678 [0.671, 0.685]
GPR-LLM	$O(ND + R \cdot C_{\text{LLM}} + R^2D + R^3 + NRD)$	Dot Product: 0.782 [0.769, 0.795] Cosine Similarity: 0.774 [0.762, 0.787] RBF Kernel: 0.754 [0.730, 0.780]


Yelp Restaurant
TripAdvisor Hotel
TravelDest
POINTREC
NDCG@10
LLM Budget
0.1
0.2
0.3
0.4
0.5
0.6
0.00
0.05
0.10
0.15
0.20
0.25
50
100
150
200
Cosine Similarity Kernel
Dot Product Kernel
RBF Kernel
Dense Retrieval (DR)
LLM Reranking