CV Homework 6

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Introduction

Optical flow, according to the definition in the lecture notes, is a vector field in the image that represents a projection of the image motion field, along the direction of image gradient[1]. And the optical flow can be solved under some additional assumptions. This homework is to implement one of the optical flow estimating methods and evaluate the results.

Theories for the algorithm

According to the lecture notes, the motion field is defined as the 2D vector field of velocities of the image points, included by relative 3D motin between the view camera and the observed scene. It can also be interpreted as the projection of 3D velocity field on the image plane.[1] And to derive the relationship between the image points and the motion field we need to make an assumption

$$\frac{dI}{dt} = 0 \tag{1}$$

where we see $I(c, r, t)$ the intensity as a function of image coordinates c, r and time frame t. And the assumption we made is basicly that the intensity of a specific 3D point's projection on 2D image plane is time invariant. Then we have

$$\frac{dI}{dt} = \frac{\delta I}{\delta c}\frac{dc}{dt} + \frac{\delta I}{\delta r}\frac{dr}{dt} + \frac{\delta I}{\delta t} = 0 \tag{2}$$

where $\frac{\delta I}{\delta c}$ and $\frac{\delta I}{\delta r}$ represent the spatial intensity gradient, $\frac{\delta I}{\delta t}$ represents the temporal intensity gradient. And $\begin{bmatrix} \frac{dc}{dt} \\ \frac{dr}{dt} \end{bmatrix}$ is the motion vector $\begin{bmatrix} v_c \\ v_r \end{bmatrix}$. Thus we can rewrite Eq.2 as

$$(\triangledown I)^T v + I_t = 0 \tag{3}$$

where $\triangledown I$ is called image intensity gradient. we can easily find that Eq.3 can only determine motion flow component in the direction of the intensity gradient. And this component is called optical flow. However only using the Eq.3 we can't reconstruct the optical flow because we have only 1 equation with 2 unknowns. Then we need more constraints to solve the optical flow.

The assumption I use in this homework is called image brightness change constancy assumption. The main idea of this assumption is that while objects are in motion, we can assume both spatial and temporal image brightness change to be constant, i.e., image brightness change rates in c, r, and t are constant.[1]

$$\frac{d^2 I}{dtdc} = 0, \frac{d^2 I}{dtdr} = 0, \frac{d^2 I}{dtdt} = 0 \tag{4}$$

Then for Eq.3 we can have

$$v_c I_c + v_r I_r + I_t = 0$$
$$v_c I_{cc} + v_r I_{rc} + I_{tc} = 0$$
$$v_c I_{cr} + v_r I_{rr} + I_{tr} = 0$$
$$v_c I_{ct} + v_r I_{rt} + I_{tt} = 0$$

(5)

And we can rewrite Eq.5 in matrix form

$$Av = b \qquad (6)$$

where $A = \begin{bmatrix} I_c & I_r \\ I_{cc} & I_{cr} \\ I_{rc} & I_{rr} \\ I_{tc} & Itr \end{bmatrix}, b = - \begin{bmatrix} I_t \\ I_{ct} \\ I_{rt} \\ I_{tt} \end{bmatrix}$ We can solve optical flow vector v using

least squares method by minimizing $\|Av - b\|_2^2$, then

$$x = (A^T A)^{-1} A^T b \qquad (7)$$

To implement the method mentioned above, we need to compute the iamge derivatives first. And normally we have two methods. The fisrt one is called numerical method. This method is simple but very sensitive to the noise[1]. The second method is called surface fitting method, and it is also the method I use for this homework. We chose a $5 \times 5 \times 5$ window, and we can see the points in this window locate on a hypersurface $I(c, r, t)$. And by least squares method, we can get the surface and reduce the influence of noise.

$$\begin{aligned} I(c, r, t) =& a_1 + a_2 c + a_3 r + a_4 t + a_5 c^2 + a_6 cr \\ &+ a_7 r^2 + a_8 rt + a_9 t^2 + a_{10} ct + a_{11} c^3 + a_{12} c^2 r \\ &+ a_{13} cr^2 + a_{14} r^3 + a_{15} r^2 t + a_{16} rt^2 + a_{17} t^3 \\ &+ a_{18} c^2 t + a_{19} ct^2 + a_{20} crt \end{aligned} \qquad (8)$$

And we ca rewrite Eq.8 in matrix form

$$Da = J \qquad (9)$$

2

where $a = (a_1, a_2, \ldots, a_{20})^T$, $D =$

$$
\begin{bmatrix}
1 & c_1 & r_1 & t_1 & \ldots & c_1 r_1 t_1 \\
1 & c_2 & r_1 & t_1 & \ldots & c_2 r_1 t_1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
1 & c_k & r_1 & t_1 & \ldots & c_k r_1 t_1 \\
1 & c_1 & r_2 & t_1 & \ldots & c_1 r_2 t_1 \\
1 & c_2 & r_2 & t_1 & \ldots & c_2 r_2 t_1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
1 & c_k & r_2 & t_1 & \ldots & c_k r_2 t_1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
1 & c_1 & r_k & t_k & \ldots & c_1 r_k t_k \\
1 & c_2 & r_k & t_k & \ldots & c_2 r_k t_k \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
1 & c_k & r_k & t_k & \ldots & c_k r_k t_k
\end{bmatrix}
$$

and $J = \begin{bmatrix} I(c_1, r_1, t_1) \\ I(c_2, r_1, t_1) \\ \vdots \\ I(c_k, r_k, t_k) \end{bmatrix}$ We can use least squares method to solve a by mini-

mizing $\|Da - J\|^2$. And

$$a = (D^T D)^{-1} D^T J \tag{10}$$

By using the relative coordinates instead of actual coordinates we can make the D matrix same for all the points.

Results
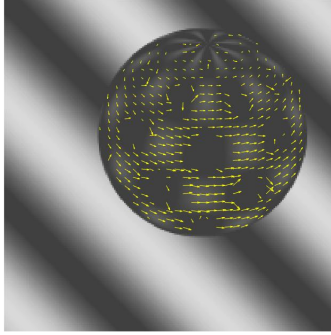
The results are as following

Figure 1: optical flow of sequence 1

This is the optical flow of sequence 1. Form figure.1 we can find that for most of the points the optical flows seem correct, however there are still some points with incorrect optical flow. I think this is because the optical flow is the motion's component that parallel with the image gradient, so if the iamge spatial gradient is influenced by iamge noise, the optical flow will als be influenced.
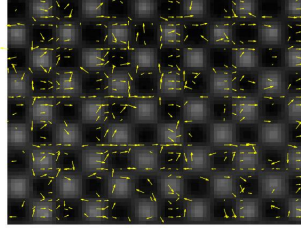
The result of the second sequence is as following

Figure 2: optical flow of sequence 2

We can find that the result contains much more error than the first sequence.

Summary and conclusion

In this homework, we made two important assumptions, the first one is the iamge brightness constancy assumption and the second one is the image brightness change constancy assumption. And based on these assumptions we can set up linear equations to solve the optical flow. And during constructing the linear system, we applied surface fitting method to compute the intensity gradients. From the results we can find that for the first sequence, this method works well, however for the second sequence, the result contains much error.

References

[1] Qiang Ji. RPI ECSE 6650 Computer Vision, Lecture Notes: Motion. URL:https://www.ecse.rpi.edu/q̃ji/CV/motion2.pdf. Last visited on 2019/11/26.