15.467 Asset Management, Lifecycle Investing, and Retirement Finance

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1. Portfolio Optimization with Human Capital Constraint

(a) Using portfolio optimization theory, we have

$$\mu_p = \mu^T w$$
$$\sigma_p^2 = w^T C w$$

Where C is the covariance matrix.

To find the optimal portfolio of risky assets only, we need to maximize

$$Sharpe\ ratio_p = \frac{\mu_p - r_f}{\sigma_p}$$

Using Excel Solver, we have the optimal Combination of Risky Assets given no constraints is

Sector fund	**Weight**
Energy	9.58%
Entertainment	14.08%
Financial	14.90%
Food	17.03%
Healthcare	15.33%
Retail	19.16%
Technology	9.93%

(b) We have: Total amount of investment is \$3 million. The human capital has the same mean and standard deviation as the financial sector stock. We invest 1/3 of the total money (\$1 million) in human capital.

Because Jane's human capital is 100% correlated to the Financial Sector's return, we can firstly combine the human capital and Financial Sector's return to a larger category "H+F" with restriction that "H+F" > 33.33% of the total investment. After we have portfolio weights calculated by Excel Solver with target mean of 10% and minimizing standard deviation, we then subtract 33.33% from "H+F" to get the true percentage of investment in the Financial Sector.

Therefore, my final answers are:

Asset / Sector	**Total weight**	**Dollar investment (US$ million)**
Risk-free asset	0.0000%	0.0000

Energy	0.0000%	0.0000
Entertainment	0.0000%	0.0000
Financial	15.5949%	0.4678
Food	0.0000%	0.0000
Healthcare	0.5359%	0.0161
Retail	0.0000%	0.0000
Technology	50.5359%	1.5161
Human capital	33.3333%	1.0000

(c) Similarly, using the same method, we first combine John's human capital and the Healthcare Sector to a larger category "H+H" with restriction that "H+H" > 33.33% of the total investment. After we calculated the portfolio weights using Excel Solver with target mean return 5% and minimizing standard deviation, we subtract 33.33% from the larger category "H+H" to get the true percentage of investment in healthcare sector.

Therefore, my final answers are:

Asset / Sector	**Total weight**	**Dollar investment (US$ million)**
Risk-free asset	36.9400%	1.1082
Energy	3.3706%	0.1011
Entertainment	4.9373%	0.1481
Financial	5.2243%	0.1567
Food	5.9816%	0.1794
Healthcare	0.0000%	0.0000
Retail	6.7213%	0.2016
Technology	3.4916%	0.1047
Human capital	33.3333%	1.0000

2. Option Pricing

(a) Price

Option	Strike	Price	Lot size
Call	4300	224.1	200
Put	3900	320.7	100

Price = 2call – 1put = 127.5

Total amount that the client receives = 127.5000*100*100=1,275,000

(b) Notional value of call = 4000*200*100 = $80,000,000

(c) Using the Excel file "BSM Option Pricing", we have the **implied volatility** as:

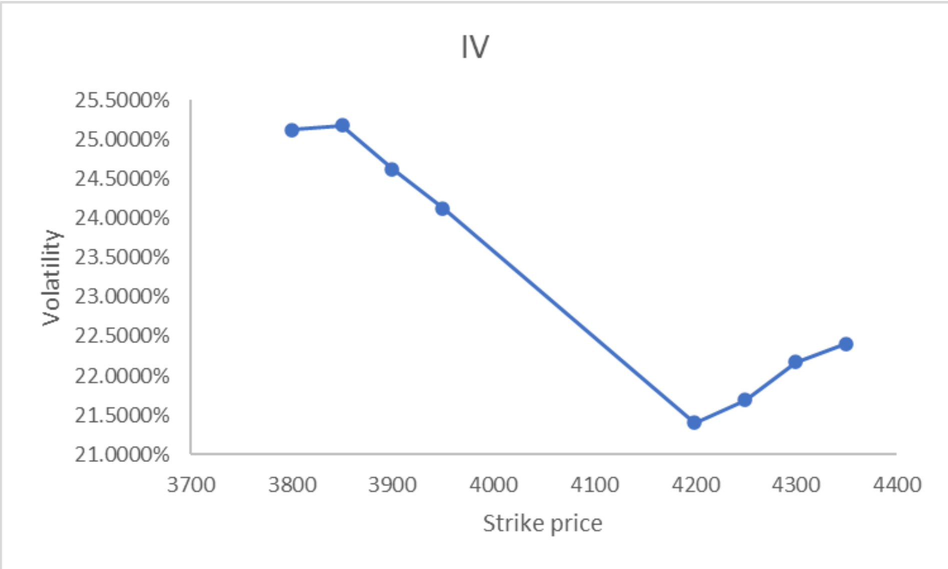
Option	BSM Option Price	Implied Volatility
Call	214.1182	22.1740%
Put	357.6957	24.4991%

Volatility smile/smirk:

Using excel and ask price, we can calculate IV as follows:

Strike Price	IV	Ask price
3800	25.1244%	283.4
3850	25.1801%	307.0
3900	24.6207%	322.5
3950	24.1287%	339.8
4200	21.4021%	245.8
4250	21.6929%	233.1
4300	22.1740%	224.1
4350	22.4059%	212.2

We can also draw graph:



From the graph, we can see that it is a volatility smirk because the curve is weighted to one side.

Why does this phenomenon happen?

Volatility smirk occurs when investors purchase put options to compensate for the risk associated with the security because they perceive market concerns. For S&P500, put is quite expensive because people would like to hedge downside market risk. Therefore, the IV curve is volatility smirk.

(d) For the quoted price, we use the BSM pricing model in the Excel.

Therefore, we have our answers as follows:

Option	Strike Price	Quoted Price	Volatility
Call	4300	214.1182	21.50%
Put	3900	357.6957	27.00%

If we quote the price as a structure based on a, we have 214.1182*2 - 357.6957 = 70.5407

(e) Using excel, we know

Option	Strike Price	Quoted Price	Volatility	Delta
Call	4300	214.1182	21.50%	0.3895
Put	3900	357.6957	27.00%	-0.3896

Therefore, the aggregate delta for the structure from the client side = 0.3895*2 - (-0.3896) = 1.1686

Therefore, the delta for the structure from sell-side = -1.1686. To hedge this position, I need to buy SPX futures.

Assume the delta of 1 SPX is 1.

We know that

delta * number of lots * lot size * underlying price = number of futures * multiplier * underlying price

Therefore, we know that the total number of SPX contract = $\frac{\mathbf{1.1686*100*100}}{\mathbf{50}} = \mathbf{233.72}$ **contracts in total of the SPX futures.**