

Probabilistic Graphical Model

1 Markov Network

1.1 Definition

A Markov Network (alias Markov Random Field) is defined by:

- A set of **random variables** $X = (X_1, \dots, X_n)$
- **Undirected graph** G , with vertices corresponding to random variables.
- Non-negative **potential functions** $\{\phi_k\}$.
- Each (maximum) **clique** $c \in C$ of G is assigned with a corresponding potential function.

The joint distribution represented by a Markov network is:

$$P(X = x) = \frac{1}{Z} \prod_{c \in C} \phi_c(x_c) \quad (1)$$

- where Z normalizes the probability and is often referred to as the **partition function**.

1.2 Properties

Markov networks are often expressed as log-linear models, in which the potential functions is replaced by an exponentiated weighted sum of features:

$$P(X = x) = \frac{1}{Z} \exp\left(\sum_j w_j f_j(x)\right) \quad (2)$$

2 Markov Logical Network

(See [Link](#))

2.1 First-order Knowledge Base

A first-order knowledge base is a set of formulas in first-order logic, constructed using four types of symbols: constants, variables, functions, and predicates. A first-order KB can also be viewed as a set of hard constraints: If a *possible world* violates even one formula, it has zero probability.

Markov logical network softens these hard constraints (formulas), and uses weights to represent how strong a constraint is.

2.2 Definition

A Markov logic network L consists of a set of pairs (F_i, w_i) . F_i is a first-order logic formula and weight $w_i \in \mathbb{R}$ denotes its importance.

- Nodes: For each predicates in L (0/1 random variables)
- Factors: Each factor represents a formula F_i in L . It contains a weight w_F and a factor function $f_F : \bar{v}_F \rightarrow \{0, 1\}$, where \bar{v}_F is the clique in Markov network.
- The joint probability is:

$$P(X = x) = \frac{1}{Z} \exp\left(\sum_{F \in \mathcal{F}} w_F f_F(x(\bar{v}_F))\right) \quad (3)$$

3 Bayesian Network