Probabilistic Graphical Model

1 Markov Network

1.1 Definition

A Markov Network (alias Markov Random Field) is defined by:

- A set of **random variables** $X = (X_1, \ldots, X_n)$
- **Undirected graph** G, with vertices corresponding to random variables.
- Non-negative **potential functions** $\{\phi_k\}$.
- Each (maximum) **clique** $c \in C$ of G is assigned with a corresponding potential function.

The joint distribution represented by a Markov network is:

$$P(X = x) = \frac{1}{Z} \prod_{c \in C} \phi_c(x_c) \tag{1}$$

- where Z normalizes the probability and is often referred to as the **partition function**.

1.2 Properties

Markov networks are often expressed as log-linear models, in which the potential functions is replaced by an exponentiated weighted sum of features:

$$P(X = x) = \frac{1}{Z} \exp\left(\sum_j w_j f_j(x) \right) \tag{2}$$

2 Markov Logical Network

(See Link)

2.1 First-order Knowledge Base

A first-order knowledge base is a set of formulas in first-order logic, constructed using four types of symbols: constants, variables, functions, and predicates. A first-order KB can also be viewed as a set of hard constraints: If a *possible world* violates even one formula, it has zero probability.

Markov logical network softens these hard constraints (formulas), and uses weights to represent how strong a constraint is.

2.2 M