



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

DDL: 14:00 Thursday of the fifth academic week(March 20th).

The homework contains 4 questions and the score is 100 in total.

1. (25 marks) Given an affine dynamical system:

$$a_{n+1} = ra_n + b, \quad (1)$$

- (a) Derive an explicit solution to this affine dynamical system.

Hint: Consider when $r \neq 0$ and $r = 0$, respectively.

- (b) Discuss all possible cases of equilibrium points.

- (c) Assume $r \neq 1$. What kind of equilibrium point is $b/(1 - r)$?

2. (25 marks) The Fibonacci sequence begins with the numbers 0 and 1. The third number in the sequence is the first two numbers added together ($0 + 1 = 1$). The fourth number in the sequence is the second and third numbers added together ($1 + 1 = 2$). Each successive number is the addition (the sum) of the previous two numbers in the sequence.

$$F_{n+1} = F_n + F_{n-1}, \quad n = 1, 2, \dots \quad (2)$$

- (a) Derive an explicit solution to this second order discrete dynamical system.

Hint: Using the matrix method or quadratic root method.

3. (25 marks) Not every matrix can be diagonalized. Given a 2-by-2 matrix A that has only one eigenvalue λ_1 . The Putzer's algorithm can be applied to compute A^n :

$$A^n = \lambda_1^n I + n\lambda_1^{n-1}(A - \lambda_1 I), \quad n \geq 1. \quad (3)$$

- (a) Use the Putzer's algorithm to compute A^n , where $A = [1, 1; 0, 1]$.

4. (25 marks) Given G_n for all $n \in \mathbb{N}$

$$G_0 = 1, \quad G_1 = 3, \quad G_{n+2} = 2G_{n+1} - 2G_n, \quad (4)$$

(a) Find a matrix 2-by-2 A such that

$$A [G_{n+1} \ G_n]^T = [G_{n+2} \ G_{n+1}]^T$$

(b) Show that the matrix A is diagonalizable.

(c) Use (a) and (b) to find G_n .

Hint: You can reasonably use any AI tools to assist you in completing your homework.

Attention: You should send your homework to jzlisustc@gmail.com to keep record.