南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

DDL: 14:00 Thursday of the sixteenth academic week (June 5th) .

The homework contains 4 questions and the score is 100 in total.

1. (25 marks) Certain stars, whose light and radial velocities undergo periodic vibrations, are thought to be pulsating. It is hypothesized that the period t of pulsation depends on the star's radius r, its mass m, and the gravitational constant G. Express t as a product of m, r, and G, so the equation becomes:

$$t = m^a r^b G^c$$

2. (25 marks) In checking the dimensions of an equation, you should note that derivatives also possess dimensions. For example, the dimension of ds/dt is LT^{-1} and the dimension of d^2s/dt^2 is LT^{-2}, where s denotes distance and t denotes time. Determine whether the equation

$$\frac{dE}{dt} = \left[mr^2 \left(\frac{d^2\theta}{dt^2} \right) mgr \sin\theta \right] \frac{d\theta}{dt}$$

for the time rate of change of total energy E in a pendulum system with damping force is dimensionally compatible.

3. (25 marks) The lift force F on a missile depends on its length r, velocity v, diameter δ, and initial angle θ with the horizon; it also depends on the density ρ, viscosity μ, gravity g, and speed of sound s of the air.

   Show that

$$F = \rho v^2 r^2 h \left(\frac{\delta}{r}, \theta, \frac{\mu}{\rho v r}, \frac{s}{v}, \frac{rg}{v^2} \right)$$

4. (25 marks) It is desired to study the velocity v of a fluid flowing in a smooth open channel. Assume that

$$v = f(r, \rho, \mu, \sigma, g)$$

where r is the characteristic length of the channel cross-sectional area divided by the wetted perimeter, ρ is the fluid density, μ is the fluid viscosity, σ is the surface tension, and g is the acceleration of gravity.

(a) (5 marks) Describe the appropriate pair of shape factors r_1 and r_2.

(b) (5 marks) Show that

$$\frac{v^2}{gr} = H\left(\frac{\rho v r}{\mu}, \frac{\rho v^2 r}{\sigma}, r_1, r_2\right)$$

Discuss the design conditions required of the model.

(c) (5 marks) Will it be practical to use the same fluid in the model and the prototype?

(d) (5 marks) Suppose the surface tension σ is ignored and the design conditions are satisfied. If $r_m = r/n$, what is the equation for the velocity of the prototype? When is the equation compatible with the design conditions?

(e) (5 marks) What is the equation for the velocity v if gravity is ignored? What if viscosity is ignored? What fluid would you use if you were to ignore viscosity?

Hint: You can reasonably use any AI tools to assist you in completing your homework. Attention: Please submit ONLY the PDF of your homework to

jzlisustc@gmail.com to keep record.