Supplementary Material for "On the Optimization of Adaptive Channel Contention in mmWave-Based Uplink System with Mobility and Environment Sensing"

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Particularly, let $\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b},\boldsymbol{\lambda}} \triangleq \left(\mathbf{S}_0, \mathbf{S}_1^{\mathbf{b},\boldsymbol{\lambda}}, \mathbf{S}_2^{\mathbf{b},\boldsymbol{\lambda}}, \dots\right)$ be a sequence of observed system states with the local scheduling parameters $(\mathbf{b}, \boldsymbol{\lambda})$, We have the following conclusion on the unbiased observation of the gradients.

Lemma 1. *Given the local scheduling parameters* $(\mathbf{b}, \boldsymbol{\lambda})$*,*

$$\hat{\mathsf{g}}_{\kappa,\ell}^{b}\left(\mathbf{b}, \boldsymbol{\lambda}; \boldsymbol{\tau}_{MC}^{\mathbf{b},\boldsymbol{\lambda}}\right) \triangleq \left[\left(\sum_{t\in\mathbb{N}} \gamma^t \mathsf{c}_{GS}\left(\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}}\right)\right)\left(\sum_{t\in\mathbb{N}}\sum_{k\in\mathcal{K}} \frac{\partial \log \mathbb{P}_k\left[\mathbf{S}_{t+1,k}^{\mathbf{b},\boldsymbol{\lambda}} \middle| \mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}}, \mathbf{b}, \boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}}\right)\right], \tag{1}$$

$$\hat{\mathsf{g}}_{\kappa,\ell}^{\lambda}\left(\mathbf{b}, \boldsymbol{\lambda}; \boldsymbol{\tau}_{MC}^{\mathbf{b},\boldsymbol{\lambda}}\right) \triangleq \left[\left(\sum_{t\in\mathbb{N}} \gamma^t \mathsf{c}_{GS}\left(\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}}\right)\right)\left(\sum_{t\in\mathbb{N}}\sum_{k\in\mathcal{K}} \frac{\partial \log \mathbb{P}_k\left[\mathbf{S}_{t+1,k}^{\mathbf{b},\boldsymbol{\lambda}} \middle| \mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}}, \mathbf{b}, \boldsymbol{\lambda}\right]}{\partial \lambda_{\kappa,\ell}}\right)\right], \tag{2}$$

are the unbiased estimation of $\frac{\partial \bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{b},\boldsymbol{\lambda})}{\partial b_{k,\ell}}$ *and* $\frac{\partial \bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{b},\boldsymbol{\lambda})}{\partial \lambda_{k,\ell}}$*,* $\forall k, \ell$*, respectively.*

Proof. We can reformulate the objective as

$$\bar{\mathsf{G}}\left(\mathbf{S}_0; \mathbf{b}, \boldsymbol{\lambda}\right) = \sum_{\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b},\boldsymbol{\lambda}}} \mathsf{C}\left(\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b},\boldsymbol{\lambda}}\right) \mathbb{P}[\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b},\boldsymbol{\lambda}} | \mathbf{S}_0, \mathbf{b}, \boldsymbol{\lambda}], \tag{3}$$

where $\mathsf{C}\left(\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b},\boldsymbol{\lambda}}\right) \triangleq \sum_{t=0}^{+\infty} \gamma^t \mathsf{c}_{\mathrm{GS}}\left(\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}}\right)$ defines the discounted cumulative cost over the sampled trajectory $\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b},\boldsymbol{\lambda}}$.

The partial derivative of the objective w.r.t. $b_{\kappa,\ell}$ is

$$\frac{\partial \bar{\mathsf{G}}\left(\mathbf{S}_0;\mathbf{b},\boldsymbol{\lambda}\right)}{\partial b_{\kappa,\ell}} = \sum_{\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}} \mathsf{C}\left(\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}\right) \frac{\partial \mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}} \tag{4}$$

$$= \sum_{\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}} \mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right] \left\{ \mathsf{C}\left(\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}\right) \frac{\partial \log \mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}} \right\}$$

$$= \mathbb{E}_{\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}} \left[\mathsf{C}\left(\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}\right) \frac{\partial \log \mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}} \right]. \tag{5}$$

Note that

$$\mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right] = \mathbb{P}\left[\mathbf{S}_1^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right] \mathbb{P}\left[\mathbf{S}_2^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_1^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right] ...$$

$$= \prod_{t\in\mathbb{N}} \mathbb{P}\left[\mathbf{S}_{t+1}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right], \tag{6}$$

and

$$\mathbb{P}\left[\mathbf{S}_{t+1}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right] = \prod_{k\in\mathcal{K}} \mathbb{P}\left[\mathbf{S}_{t+1,k}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right], \tag{7}$$

$\frac{\partial \log \mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}}$ in (5) can be further expanded as

$$\frac{\partial \log \mathbb{P}\left[\boldsymbol{\tau}_{\text{MC}}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_0,\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}} = \sum_{t\in\mathbb{N}} \frac{\partial \log \mathbb{P}\left[\mathbf{S}_{t+1}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}}$$

$$= \sum_{t\in\mathbb{N}} \sum_{k\in\mathcal{K}} \frac{\partial \log \mathbb{P}_k\left[\mathbf{S}_{t+1,k}^{\mathbf{b},\boldsymbol{\lambda}}|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}}. \tag{8}$$

The expectation expression for the partial derivative of the objective with respect to $\lambda_{\kappa,\ell}$, $\frac{\partial \bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{b},\boldsymbol{\lambda})}{\partial \lambda_{\kappa,\ell}}$, can be derived using the same Markov chain sampling technique.

□

Moreover,

$$\frac{\partial \log \mathbb{P}_k\left[\mathbf{S}_{t+1,k}^{\mathbf{b},\boldsymbol{\lambda}}\middle|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right]}{\partial b_{\kappa,\ell}} = \frac{\omega_{t,k}^{(1)} - \omega_{t,k}^{(2)}}{\eta_k(\boldsymbol{\theta}(\mathbf{b},\boldsymbol{\lambda}))\left(\omega_{t,k}^{(1)} - \omega_{t,k}^{(2)}\right) + \omega_{t,k}^{(2)}} \frac{\partial \eta_k(\boldsymbol{\theta}(\mathbf{b},\boldsymbol{\lambda}))}{\partial b_{\kappa,\ell}}, \tag{9}$$

$$\frac{\partial \log \mathbb{P}_k\left[\mathbf{S}_{t+1,k}^{\mathbf{b},\boldsymbol{\lambda}}\middle|\mathbf{S}_t^{\mathbf{b},\boldsymbol{\lambda}},\mathbf{b},\boldsymbol{\lambda}\right]}{\partial \lambda_{\kappa,\ell}} = \frac{\omega_{t,k}^{(1)} - \omega_{t,k}^{(2)}}{\eta_k(\boldsymbol{\theta}(\mathbf{b},\boldsymbol{\lambda}))\left(\omega_{t,k}^{(1)} - \omega_{t,k}^{(2)}\right) + \omega_{t,k}^{(2)}} \frac{\partial \eta_k(\boldsymbol{\theta}(\mathbf{b},\boldsymbol{\lambda}))}{\partial \lambda_{\kappa,\ell}}, \tag{10}$$

where $\omega_{t,k}^{(1)}$ and $\omega_{t,k}^{(2)}$ are provided as

$$\omega_{t,k}^{(1)} = \sum_{N\in\mathbb{N}} \frac{(\bar{A}_k)^N e^{-\bar{A}_k}}{N!} \left\{ \exp\left[-\frac{\mathsf{f}_{\mathtt{SNR}}(-\Delta Q_{t,k}+N)}{\bar{\Gamma}_{t,k}}\right] - \exp\left[-\frac{\mathsf{f}_{\mathtt{SNR}}(-\Delta Q_{t,k}+N+1)}{\bar{\Gamma}_{t,k}}\right] \right\}, \tag{11}$$

$$\omega_{t,k}^{(2)} = \frac{(\bar{A}_k)^{\Delta Q_{t,k}}}{\Delta Q_{t,k}!} e^{-\bar{A}_k} \mathbb{1}[\Delta Q_{t,k} \geq 0]. \tag{12}$$

Note that $\Delta Q_{t,k} = Q_{t+1,k} - Q_{t,k}$, $\mathsf{f}_{\mathtt{SNR}}(x) = \frac{1}{P_{\mathtt{UL}}}\left[2^{\left(\frac{xR_{\mathtt{pac}}}{T_{\mathtt{slot}}W}\right)} - 1\right]$, and $\bar{\Gamma}_{t,k} = \frac{1}{\sigma_{\mathtt{N}}^2}\mathbb{E}\left[\left|\mathbf{w}_{t,k}^{\mathsf{H}}\mathbf{H}_{t,k}\mathbf{f}_{t,k}\right|^2\right]$. $\frac{\partial \eta_k(\boldsymbol{\theta})}{\partial b_{\kappa,\ell}}$, and $\frac{\partial \eta_k(\boldsymbol{\theta})}{\partial \lambda_{\kappa,\ell}}$ are provided as

$$\frac{\partial \eta_k(\boldsymbol{\theta})}{\partial \lambda_{\kappa,\ell}} = \begin{cases} 0, & \mathsf{f}_{s\to\ell}\left(\mathsf{f}_{\mathtt{LS}}(\mathsf{S}_{t,\kappa})^{\mathbf{b},\boldsymbol{\lambda}}\right) \neq \ell \\ \frac{\partial \eta_k(\boldsymbol{\theta})}{\partial \theta_{\kappa,\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}}\right)}} \frac{\partial \zeta_\kappa\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}};\mathbf{b}_\kappa,\boldsymbol{\lambda}_\kappa\right)}{\partial \lambda_{\kappa,\ell}}, & \text{others} \end{cases} \tag{13}$$

$$\frac{\partial \eta_k(\boldsymbol{\theta})}{\partial b_{\kappa,\ell}} = \begin{cases} 0, & \mathsf{f}_{s\to\ell}\left(\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}}\right)\right) \neq \ell \\ \frac{\partial \eta_k(\boldsymbol{\theta})}{\partial \theta_{\kappa,\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}}\right)}} \frac{\partial \zeta_\kappa\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}};\mathbf{b}_\kappa,\boldsymbol{\lambda}_\kappa\right)}{\partial b_{\kappa,\ell}}, & \text{others} \end{cases} \tag{14}$$

where

$$\frac{\partial \eta_k(\boldsymbol{\theta})}{\partial \theta_{\kappa,s}} = \begin{cases} \mathbb{1}\left[s = \mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}}\right)\right] \mathsf{f}_{\eta_k}^{(1)}\left(\mathsf{S}_t^{\mathbf{b},\boldsymbol{\lambda}};\boldsymbol{\theta}\right), & k \neq \kappa \\ \mathbb{1}\left[s = \mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,\kappa}^{\mathbf{b},\boldsymbol{\lambda}}\right)\right] \mathsf{f}_{\eta_k}^{(2)}\left(\mathsf{S}_t^{\mathbf{b},\boldsymbol{\lambda}};\boldsymbol{\theta}\right), & k = \kappa \end{cases} \tag{15}$$

and

$$\mathsf{f}_{\eta_k}^{(1)}\left(\mathsf{S}_t^{\mathbf{b},\boldsymbol{\lambda}};\boldsymbol{\theta}\right) \triangleq \frac{-\theta_{k,\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,k}^{\mathbf{b},\boldsymbol{\lambda}}\right)}}{\left(\sum_{k'=1}^K \theta_{k',\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,k'}^{\mathbf{b},\boldsymbol{\lambda}}\right)}\right)^2}, \tag{16}$$

$$\mathsf{f}_{\eta_k}^{(2)}\left(\mathsf{S}_t^{\mathbf{b},\boldsymbol{\lambda}};\boldsymbol{\theta}\right) \triangleq \frac{\sum_{k'=1}^K \theta_{k',\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,k'}^{\mathbf{b},\boldsymbol{\lambda}}\right)} - \theta_{k,\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,k}^{\mathbf{b},\boldsymbol{\lambda}}\right)}}{\left(\sum_{k'=1}^K \theta_{k',\mathsf{f}_{\mathtt{LS}}\left(\mathsf{S}_{t,k'}^{\mathbf{b},\boldsymbol{\lambda}}\right)}\right)^2}. \tag{17}$$

As a result, the SGD method solving the problem $\mathsf{P1}$ is outlined in Algorithm 1. Finally, the following theorem establishes the convergence of Algorithm 1.

Theorem 1 (Stochastic Convergence)**.** *Let $\beta \in \mathbb{R}_+$ be a constant satisfying*

$$\left\|\bar{\mathsf{G}}(\mathsf{S}_0;\mathbf{z}) - \bar{\mathsf{G}}(\mathsf{S}_0;\mathbf{z}')\right\|_2 \leq \beta \left\|\mathbf{z} - \mathbf{z}'\right\|_2, \ \forall \mathbf{z},\mathbf{z}', \tag{20}$$

where $\mathbf{z} \triangleq (\mathbf{b},\boldsymbol{\lambda})$. Let $\nu_2^{OM} \in \mathbb{R}_+$ be a constant such that

$$\sum_{k=1}^K \sum_{\ell=1}^{|\mathcal{L}|} \mathbb{E}\left[\left(\hat{\mathsf{g}}_{\kappa,\ell}^b\left(\mathbf{b},\boldsymbol{\lambda};\boldsymbol{\tau}_{MC}^{\mathbf{b},\boldsymbol{\lambda}}\right)\right)^2 + \left(\hat{\mathsf{g}}_{\kappa,\ell}^\lambda\left(\mathbf{b},\boldsymbol{\lambda};\boldsymbol{\tau}_{MC}^{\mathbf{b},\boldsymbol{\lambda}}\right)\right)^2\right] \leq \nu_2^{OM},$$

input : $(\mathbf{b}(0), \boldsymbol{\lambda}(0))$; T_{ep}, length of each episode; step size parameter $\eta_{\texttt{LPG}}(0)$.

output: $(\mathbf{b}(\infty), \boldsymbol{\lambda}(\infty))$.

for $m = 0, 1, 2, \ldots$ **do**

  Initial global state $\mathbf{S}_0$, receive global system cost $\mathsf{c}_{\texttt{GS}}(\mathbf{S}_0)$, each agent k takes action $\theta^{\mathtt{A}}_{0,k} = \zeta_k\left(\mathbf{S}_{0,k}; \mathbf{b}(m), \boldsymbol{\lambda}(m)\right)$.

  for $t = 1$ **to** $T_{ep} + 1$ **do**

    Get global state $\mathbf{S}_t^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}$ and global system cost $\mathsf{c}_{\texttt{GS}}\left(\mathbf{S}_t^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right)$, each agent k takes action $\theta^{\mathtt{A}}_{t,k} = \zeta_k\left(\mathbf{S}_{t,k}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}; \mathbf{b}(m), \boldsymbol{\lambda}(m)\right)$.

  end

  For any $\kappa\,\ell$, caculate gradients $\hat{\mathsf{g}}^{\mathrm{b}}_{\kappa,\ell}\left(\mathbf{b}(m), \boldsymbol{\lambda}(m); \boldsymbol{\tau}_{\text{MC}}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right)$ as (1), and $\hat{\mathsf{g}}^{\lambda}_{\kappa,\ell}\left(\mathbf{b}(m), \boldsymbol{\lambda}(m); \boldsymbol{\tau}_{\text{MC}}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right)$ as (2)

  For any κ, ℓ, update parameters as

$$b_{\kappa,\ell}(m+1) = b_{\kappa,\ell}(m) - \eta_{\texttt{LPG}}(m)\hat{\mathsf{g}}^{\mathrm{b}}_{\kappa,\ell}\left(\mathbf{b}(m), \boldsymbol{\lambda}(m); \boldsymbol{\tau}_{\text{MC}}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right). \tag{18}$$

$$\lambda_{\kappa,\ell}(m+1) = \lambda_{\kappa,\ell}(m) - \eta_{\texttt{LPG}}(m)\hat{\mathsf{g}}^{\lambda}_{\kappa,\ell}\left(\mathbf{b}(m), \boldsymbol{\lambda}(m); \boldsymbol{\tau}_{\text{MC}}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right), \tag{19}$$

  where $\eta_{\texttt{LPG}}(m) = \frac{\eta_{\texttt{LPG}}(0)}{m+1}$.

end

Algorithm 1: SGD-Base Algorithm

We have

$$\mathbb{E}\left[\min_{m\in[\![0:M]\!]} \eta_{LPG}(m)\left\|\nabla\bar{\mathsf{G}}\left(\mathbf{S}_0; \mathbf{b}(m), \boldsymbol{\lambda}(m)\right)\right\|_2^2\right]$$
$$\leq \frac{1}{M+1}\mathbb{E}\left[\bar{\mathsf{G}}\left(\mathbf{S}_0; \mathbf{b}(0), \boldsymbol{\lambda}(0)\right) - \bar{\mathsf{G}}\left(\mathbf{S}_0; \mathbf{b}^\star, \boldsymbol{\lambda}^\star\right) + \frac{\pi^2}{12}\nu_2^{OM}\right].$$

Proof. Lemma 3.4 in [1] states that if $\bar{\mathsf{G}}(\mathbf{S}_0; \mathbf{z})$ is β-smooth, the following inequality holds

$$\begin{aligned}&\Big|\bar{\mathsf{G}}(\mathbf{S}_0; \mathbf{z}(m+1)) - \bar{\mathsf{G}}(\mathbf{S}_0; \mathbf{z}(m))\\ &\qquad - \nabla\bar{\mathsf{G}}(\mathbf{S}_0; \mathbf{z}(m))^\top(\mathbf{z}(m+1) - \mathbf{z}(m))\Big|\\ &\leq \frac{\beta}{2}||\mathbf{z}(m+1) - \mathbf{z}(m))||_2^2,\ \forall m \in [\![0:M]\!].\end{aligned} \tag{21}$$

Denote

$$\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m)) \triangleq \mathsf{vec}\Big[\left(\hat{\mathrm{g}}^{\mathrm{b}}_{\kappa,\ell}\left(\mathbf{b}(m),\boldsymbol{\lambda}(m);\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right)\right)_{k,\ell}, \\ \left(\hat{\mathrm{g}}^{\lambda}_{\kappa,\ell}\left(\mathbf{b}(m),\boldsymbol{\lambda}(m);\boldsymbol{\tau}_{\mathrm{MC}}^{\mathbf{b}(m),\boldsymbol{\lambda}(m)}\right)\right)_{k,\ell}\Big].$$

By substituting $\mathbf{z}(m+1)-\mathbf{z}(m)$ with $-\eta_{\mathrm{LPG}}(m)\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))$ in (21), the inequality can be rewritten as

$$\begin{aligned}&\left|\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m+1))-\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))+\eta_{\mathrm{LPG}}(m)\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))^{\top}\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\right|\\ &\le\frac{\beta}{2}\eta^2_{\mathrm{LPG}}(m)||\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))||_2^2,\ \forall m\in [\![0:M]\!]\,.\end{aligned}\tag{22}$$

Rearranging the formulas, we arrive at the following conclusions.

$$\begin{aligned}&\eta_{\mathrm{LPG}}(m)\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))^{\top}\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\\ &\le\left[\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m+1))-\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\right]*(-1)\\ &\quad+\frac{\beta}{2}\eta^2_{\mathrm{LPG}}(m)||\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))||_2^2,\ \forall m\in [\![0:M]\!]\,.\end{aligned}\tag{23}$$

By taking expectations on both sides of the inequality, we derive the following results.

$$\begin{aligned}&\mathbb{E}\left[\eta_{\mathrm{LPG}}(m)\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))^{\top}\widetilde{\nabla}\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\right]\\ &=\eta_{\mathrm{LPG}}(m)\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))^{\top}\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\\ &\le\mathbb{E}\left[\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m+1))-\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\right]*(-1)\\ &\quad+\frac{\beta}{2}\eta^2_{\mathrm{LPG}}(m)\nu_2^{\mathrm{OM}},\ \forall m\in [\![0:M]\!]\,.\end{aligned}$$

Hence,

$$\begin{aligned}&\mathbb{E}\left[\min_{m\in[\![0:M]\!]}\eta_{\mathrm{LPG}}(m)||\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))||_2^2\right]\\ &\le\frac{1}{M+1}\mathbb{E}\left[\sum_{m=0}^{M}\eta_{\mathrm{LPG}}(m)||\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))||_2^2\right]\\ &\le\frac{1}{M+1}\sum_{m=0}^{M}\mathbb{E}\left[\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m+1))-\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))\right]*(-1)\\ &\quad+\frac{\beta}{2}\frac{1}{M+1}\sum_{m=0}^{M}\eta^2_{\mathrm{LPG}}(m)\nu_2^{\mathrm{OM}},\\ &\le\frac{1}{M+1}\mathbb{E}\left[\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(0))-\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}^{\star})\right]+\frac{\beta\nu_2^{\mathrm{OM}}}{2(M+1)}\sum_{m=0}^{M}\eta^2_{\mathrm{LPG}}(m).\end{aligned}$$

Note that

$$\sum_{m=0}^{M} \eta_{\mathtt{LPG}}^2(m) = \sum_{m=0}^{M} \left(\frac{\eta_{\mathtt{LPG}}(0)}{m+1}\right)^2 \leq \frac{\pi^2}{6}\eta_{\mathtt{LPG}}^2(0), \tag{24}$$

we have

$$\begin{aligned}
&\mathbb{E}\left[\min_{m\in[\![0:M]\!]} \eta_{\mathtt{LPG}}(m)||\nabla\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(m))||_2^2\right] \\
&\leq \frac{1}{M+1}\mathbb{E}\left[\bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}(0)) - \bar{\mathsf{G}}(\mathbf{S}_0;\mathbf{z}^\star) + \frac{\pi^2}{12}\nu_2^{\mathtt{OM}}\right].
\end{aligned} \tag{25}$$

$\square$

As M tends to infinity, the convergence of the proposed SGD-based Algorithm is assured by Theorem 1.

REFERENCES

[1] S. Bubeck *et al.*, "Convex optimization: Algorithms and complexity," *Foundations and Trends® in Machine Learning*, vol. 8, no. 3-4, pp. 231–357, 2015.