Solutions

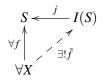
Basic Category Theory

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Introduction

Excise 0.10

Proof. It is symmetric to the Example 0.5, just need to inverse all the arrows. Here are the details:



For any X a topological space, and any continuous map f, we gain an unique continuous map \bar{f} make the above diagram commutes. Which is the universal property of I(S).

Actually, we can say I(S) is the coarsest topology make all topological spaces and all maps from them to I(S) continuous.

Excise 0.11

Proof. See page 112 of the textbook, it is a simple case of the equalizer.

Excise 0.12

Proof. Notice: $f^{-1}(U_Y) \cap g^{-1}(U_Y)$ is still open.

Excise 0.13

normal check

Excise 0.14

Proof. Refer to the definition of product and coproduct.

Chapter 1

Excise 1.2.23

Categories functors and natural transformations

Excise 1.1.12 *Proof.* 1) the chain complex; 2) the modules complex; 3) the algebras complex \Box **Excise 1.1.13** *Proof.* Like what we do in the group theory. **Excise 1.1.14** *Proof.* $(f_A, f_B) \circ (g_A, g_B) = (f_A \circ g_A, f_B \circ g_B)$ **Excise 1.1.15** *Proof.* Iff they are homotopy equivalent. **Excise 1.2.20** *Proof.* 1) the irreducible representations of the algebra; 2) from simplex to the chain complex; 3) from affine varieties to the spectrum to the coordinate ring **Excise 1.2.21** *Proof.* Notice: $1_{F(A)} = F(1_A) = F(f \circ g) = F(f) \circ F(g)$. **Excise 1.2.22** Proof. Just by definition.

Proof. (a) Just sent $x \in G$ to $(x^{op})^{-1} \in G^{op}$, and check it induced a group homomorphism.

(b) This question take me long time, because I want to construct a simple finite monoid to meet all the requirement. By the proof of the last question, we know if we want to construct such a monoid, we have to break its symmetry, at least I don't want it commutative. (Notice: $x^{op}y^{op} = (yx)^{op}$)

My idea is to use the projections of one finite set. We want this set simple as well as possible. let's start with a set $S = \{1,2,3\}$ (Actually, you can check that the set had one or two elements can not do this job)