Solutions

Basic Category Theory

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Introduction

Excise 0.10

Proof. It is symmetric to the Example 0.5, just need to inverse all the arrows. Here are the details:

$$\begin{tikzcd} S & I(S) \arrow[l, "j"'] \\ \forall X \arrow[u, "\forall f"] \arrow[ur, dashed, "\exists!\bar{f}"'] & \end{tikzcd}$$

For any X a topological space, and any continuous map f, we gain an unique continuous map $\bar{f}$ make the above diagram commutes. Which is the universal property of $I(S)$.
Actually, we can say $I(S)$ is the coarsest topology make all topological spaces and all maps from them to $I(S)$ continuous . □

Excise 0.11

Proof. See page 112 of the textbook, it is a simple case of the equalizer. □

Excise 0.12

Proof. Notice: $f^{-1}(U_Y) \cap g^{-1}(U_Y)$ is still open. □

Excise 0.13
normal check

Excise 0.14

Proof. Refer to the definition of product and coproduct. □

Chapter 1

Categories functors and natural transformations

Excise 1.1.12

Proof. 1) the chain complex;
2) the modules complex;
3) the algebras complex □

Excise 1.1.13

Proof. Like what we do in the group theory. □

Excise 1.1.14

Proof. $(f_A, f_B) \circ (g_A, g_B) = (f_A \circ g_A, f_B \circ g_B)$ □

Excise 1.1.15

Proof. Iff they are homotopy equivalent. □

Excise 1.2.20

Proof. 1) the irreducible representations of the algebra;
2) from simplex to the chain complex;
3) from affine varieties to the spectrum to the coordinate ring □

Excise 1.2.21

Proof. Notice: $1_{F(A)} = F(1_A) = F(f \circ g) = F(f) \circ F(g)$. □

Excise 1.2.22

Proof. Just by definition. □

Excise 1.2.23

Proof. (a) Just sent $x \in G$ to $(x^{op})^{-1} \in G^{op}$, and check it induced a group homomorphism.
(b) This question take me long time, because I want to construct a simple finite monoid to meet all the requirement. By the proof of the last question, we know if we want to construct such a monoid, we have to break its symmetry, at least I don't want it commutative. (Notice: $x^{op}y^{op} = (yx)^{op}$)
My idea is to use the projections of one finite set. We want this set simple as well as possible. let's start with a set $S = \{1,2,3\}$ (Actually, you can check that the set had one or two elements can not do this job)

$\square$