

# STA 521 - Final Project Part I

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*December 7th, 2019*

## 1. Introduction: Summary of problem and objectives

Our team of esteemed statisticians was recently hired by a prestigious Art historian for a consulting project. We were asked to help build a predictive model in exchange for an A on our STA 521 Final Exam. After much discussion, our team accepted the historian's offer.

We were given the task of predicting paintings' selling prices at auctions in 18th century Paris. To accomplish this, we used a dataset containing information about each painting's buyer, seller, painter, and characteristics of the painting. These variables were all possible predictor variables in modeling the response variable, the selling price of a painting.

There were two primary objectives in our analysis:

- 1) To determine which variables (or interactions) drove the price of a painting
- 2) To determine which paintings were overpriced or and which were underpriced.

After arriving at a final model, we are able to answer these primary questions. Any variables that appear in the model will be important in driving painting prices, and observing residuals will enable us to determine if a painting was over or underpriced.

We had 1,500 observations to train the model on, along with 750 observations held out as a testing set. There were a total of 59 variables in the dataset, both categorical and continuous.

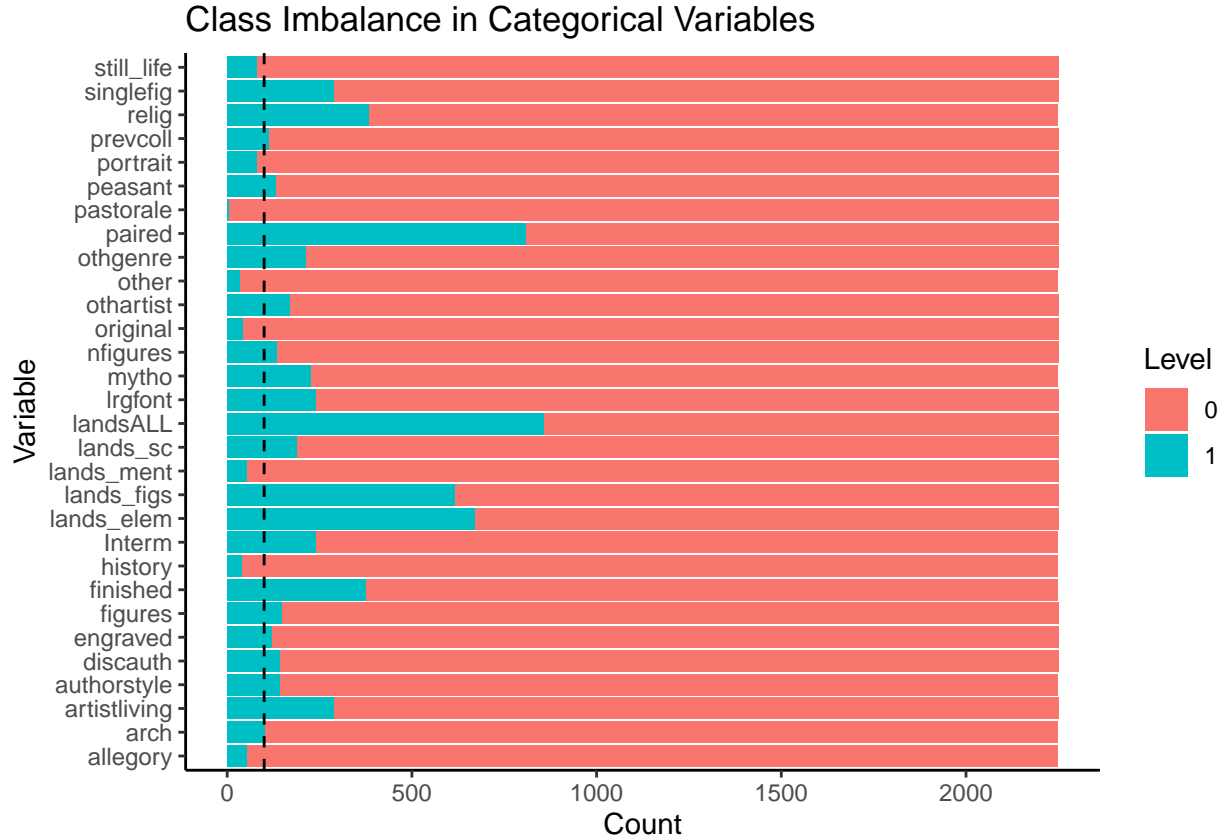
## 2. Exploratory Data Analysis:

### Categorical Variables

We began our data cleaning process by reading the codebook for a better understanding of what each variable in the data represented. Several predictors in the dataset were redundant and therefore removed to avoid high correlation among the predictors. Examples of this include the variable 'sale', which is a combination of 'dealer' and 'year'. Additionally, there were other predictors that we deemed would not be useful for prediction, such as 'count' which was 1 for every observation, or 'subject' which was a short description of the content in the painting. After simplifying the data by eliminating unnecessary predictors, we recoded each categorical variable to be a factor.

We created a visualization of the binary categorical variables to observe the balance between classes below.

Plot 1

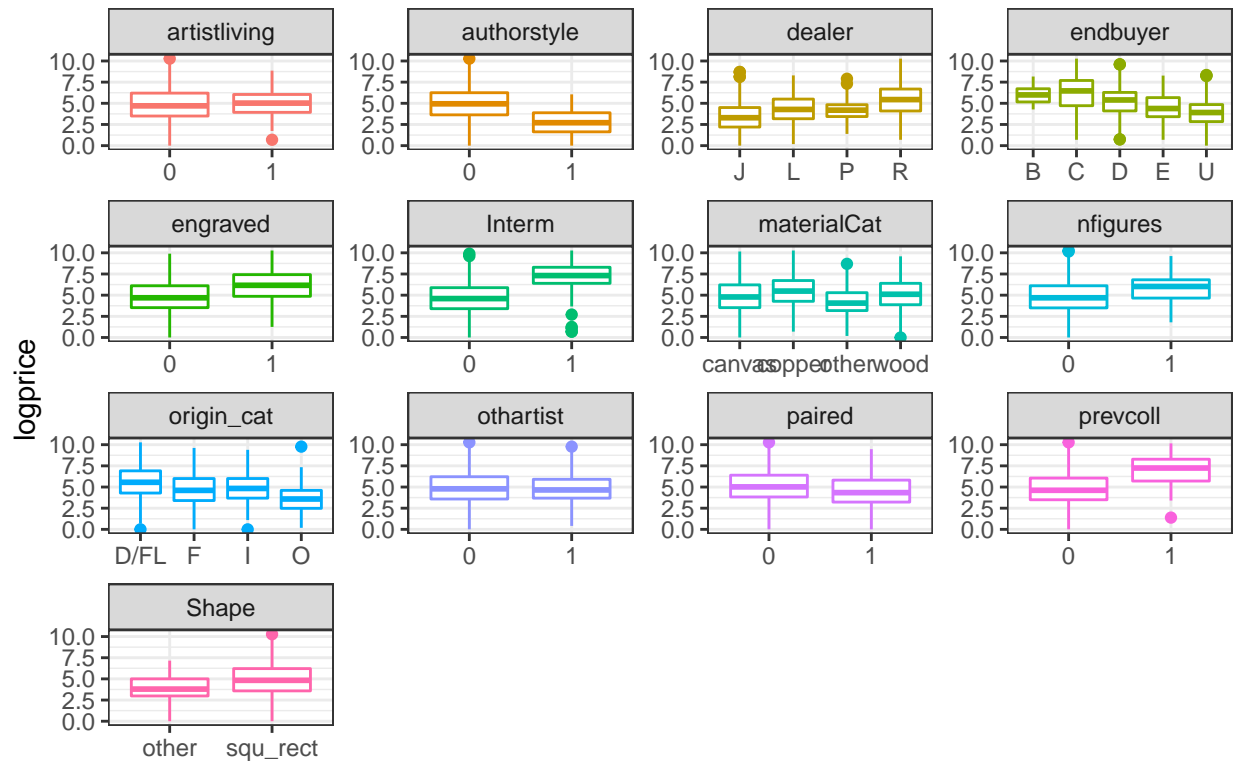


Imbalanced classes can lead to unstable estimates if the underrepresented class does not have a sufficient amount of data. This was our motivation to remove any variable that had less than an arbitrary 100 observations in a class, which is denoted by the dotted black line in our visualization above.

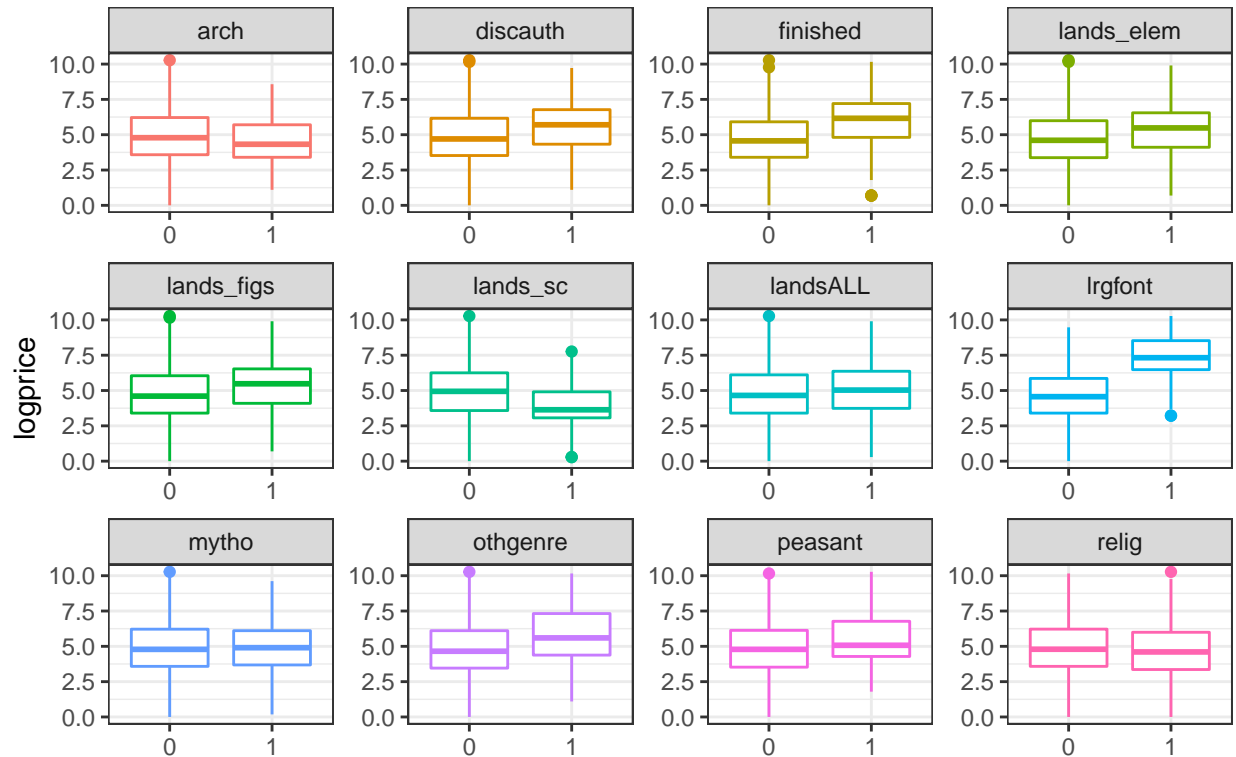
To identify important categorical variables, we created a boxplot for each variable that compared the distribution of logPrice over every level of the factor. The results are shown below.

Plot 2

Boxplots of Log Price for Categorical Variables



Boxplots of Log Price for Categorical Variables (continued)



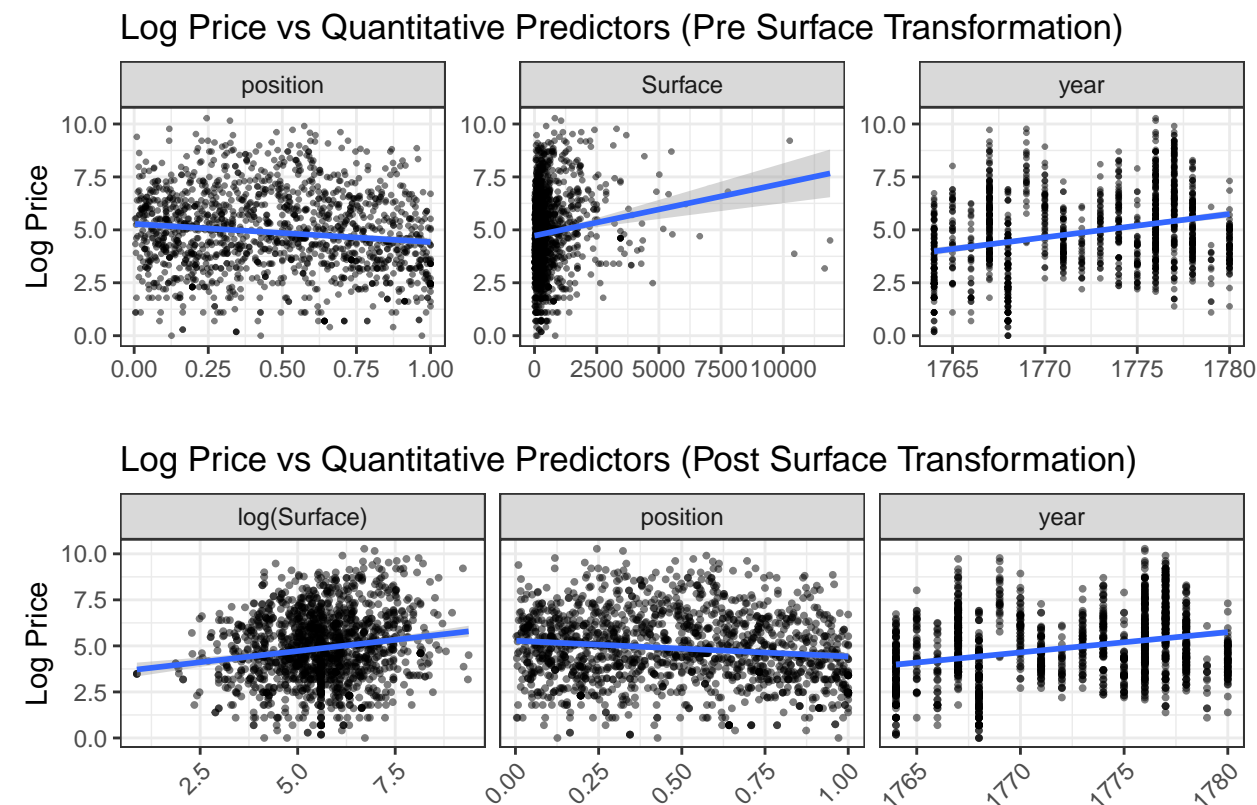
The boxplots above help us identify which variables could be important in predicting a painting's price. They also help us in our variable selection process by displaying variables that have similar prices in all of their categories. After inspecting the boxplots, we determined that 'mytho', 'landsALL', 'relig', and 'othartist' were not useful for prediction. Variables that may be important include, but are not limited to, 'lrgfont', 'Interm', 'authorstyle', and 'prevcoll'.

## Quantitative Variables

There are also quantitative variables in our data that could be used for prediction. Like the categorical variables, many of these predictors were redundant. For example, we were given the surface area of a painting. Additionally, we were given a column for surface area if the painting was round and a column if the painting was rectangular. We also were given the height, the width, and the diameter of the painting. We determined that all this information could be condensed to a single column, 'Surface'.

There was missing data in our 'Surface' column that we had to address. Surface area intuitively seems like it could drive the price of a painting, so we had to develop a strategy for handling the missing observations. With the help of the plot below, we determined that imputing the median surface area size of the dataset would be a good estimation for missing values. Since the distribution of 'Surface' is so skewed, we wanted an imputation strategy that would be robust to outliers. Thus, we opted for the median over the mean.

Plot 3



We created scatterplots to observe the relationship between our three quantitative predictor variables and the log price of a painting. The distribution of 'Surface' was skewed right and a log transformation was necessary. We plot the relationship of logprice and the transformed Surface column in the lower graph.

### 3. Development and Assessment of Initial Model:

```
paint_train_trans <- paint_train%>% select(-c(othartist,mytho,relig, landsALL))%>%
  mutate(Surface = log(Surface))

model0 <- lm(logprice~(.), data=paint_train_trans)

n = nrow(paint_train_trans)

#use bic to select the variables
step(model0,k=log(n),trace = F)
```

```
##
## Call:
## lm(formula = logprice ~ dealer + year + origin_cat + artistliving +
##     authorstyle + endbuyer + Interm + Surface + engraved + prevcoll +
##     paired + finished + lrgfont + lands_sc + discauth, data = paint_train_trans)
##
## Coefficients:
## (Intercept)      dealerL      dealerP      dealerR      year
## -211.1924      1.2877      0.2917      1.7553      0.1206
## origin_catF origin_catI origin_cat0 artistliving1 authorstyle1
## -0.7389      -0.6956      -0.8543      0.4179      -1.0523
## endbuyerC endbuyerD endbuyerE endbuyerU Interm1
## -0.2727      -0.4361      -0.8255      -1.0593      0.7800
## Surface engraved1 prevcoll1 paired1 finished1
## 0.3367      0.6554      0.8523      -0.2376      0.7782
## lrgfont1 lands_sc1 discauth1
## 0.8594      -0.3878      0.5074
```

```
#model without interaction
```

```
model1 <- lm(formula = logprice ~ (dealer + year + origin_cat + artistliving +authorstyle +
      endbuyer + Interm + Surface + engraved + prevcoll +
      paired + finished + lrgfont + lands_sc + discauth),
      data = paint_train_trans)
```

```
#prepare for BIC
```

```
# model2 <- lm(formula = logprice ~ (dealer + year + origin_cat + artistliving + authorstyle + endbuyer +
#     paired + finished + lrgfont + lands_sc + discauth)^2, data = #paint_train_trans)
```

```
# use this step to have a look : step(model2, k = 2, trace = F)
```

```
# the following model is modified based on BIC selection, I deleted some interactions, feel free to che
```

```
finalmodel <- lm(formula = logprice ~ dealer + year + origin_cat + artistliving +
      authorstyle + endbuyer + Interm + Surface + engraved + prevcoll +
      paired + finished + lrgfont + lands_sc + discauth + dealer:year +
      dealer:Interm + dealer:discauth + year:artistliving + year:endbuyer + year:Surface + year:discauth +
      endbuyer:discauth + Interm:Surface + Interm:prevcoll + Interm:lrgfont +
      Interm:discauth + Surface:discauth + prevcoll:finished +
      paired:finished + paired:lrgfont + paired:lands_sc + paired:discauth +
```

```
lands_sc:discauth, data = paint_train_trans)
```

```
summary(finalmodel)
```

```
##
## Call:
## lm(formula = logprice ~ dealer + year + origin_cat + artistliving +
##     authorstyle + endbuyer + Interm + Surface + engraved + prevcoll +
##     paired + finished + lrgfont + lands_sc + discauth + dealer:year +
##     dealer:Interm + dealer:discauth + year:artistliving + year:endbuyer +
##     year:Surface + year:discauth + artistliving:authorstyle +
##     artistliving:lands_sc + artistliving:discauth + authorstyle:Surface +
##     endbuyer:finished + endbuyer:discauth + Interm:Surface +
##     Interm:prevcoll + Interm:lrgfont + Interm:discauth + Surface:discauth +
##     prevcoll:finished + paired:finished + paired:lrgfont + paired:lands_sc +
##     paired:discauth + lands_sc:discauth, data = paint_train_trans)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.6808 -0.6900  0.0038  0.7026  3.8641
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -2.582e+02  2.000e+02  -1.291  0.196841
## dealerL         1.664e+02  4.668e+01   3.565  0.000376 ***
## dealerP         1.469e+02  9.328e+01   1.575  0.115474
## dealerR         4.195e+01  4.186e+01   1.002  0.316433
## year           1.469e-01  1.128e-01   1.302  0.193073
## origin_catF     -6.406e-01  8.031e-02  -7.977  3.04e-15 ***
## origin_catI     -6.804e-01  1.033e-01  -6.585  6.35e-11 ***
## origin_cat0     -1.102e+00  1.194e-01  -9.229  < 2e-16 ***
## artistliving1    2.884e+01  3.079e+01   0.937  0.349078
## authorstyle1     -6.263e-01  6.066e-01  -1.032  0.302042
## endbuyerC       -1.183e+01  1.902e+02  -0.062  0.950403
## endbuyerD        8.703e+01  1.900e+02   0.458  0.646994
## endbuyerE       -1.251e+02  1.915e+02  -0.653  0.513612
## endbuyerU        2.517e+01  1.895e+02   0.133  0.894366
## Interm1         -1.386e+00  6.132e-01  -2.260  0.023940 *
## Surface         -6.975e+00  9.093e+00  -0.767  0.443166
## engraved1       7.204e-01  1.390e-01   5.183  2.50e-07 ***
## prevcoll1       1.161e+00  1.680e-01   6.912  7.17e-12 ***
## paired1         -2.262e-01  7.455e-02  -3.034  0.002456 **
## finished1       1.218e+00  8.993e-01   1.355  0.175748
## lrgfont1        1.162e+00  1.487e-01   7.814  1.06e-14 ***
## lands_sc1       -2.942e-01  1.659e-01  -1.774  0.076346 .
## discauth1       4.129e+02  8.853e+01   4.664  3.39e-06 ***
## dealerL:year     -9.300e-02  2.632e-02  -3.534  0.000422 ***
## dealerP:year     -8.245e-02  5.249e-02  -1.571  0.116484
## dealerR:year     -2.258e-02  2.362e-02  -0.956  0.339168
## dealerL:Interm1  1.433e+00  8.099e-01   1.770  0.076950 .
## dealerP:Interm1  1.196e-01  1.240e+00   0.096  0.923161
## dealerR:Interm1  1.400e+00  4.796e-01   2.920  0.003560 **
## dealerL:discauth1 4.696e-01  8.637e-01   0.544  0.586739
```

```
## dealerP:discauth1      -1.764e+00  8.930e-01  -1.975  0.048406 *
## dealerR:discauth1      -2.850e+00  4.660e-01  -6.116  1.24e-09 ***
## year:artistliving1     -1.610e-02  1.737e-02  -0.927  0.354060
## year:endbuyerC         6.652e-03  1.073e-01   0.062  0.950588
## year:endbuyerD        -4.920e-02  1.072e-01  -0.459  0.646404
## year:endbuyerE         7.031e-02  1.081e-01   0.651  0.515371
## year:endbuyerU        -1.465e-02  1.069e-01  -0.137  0.891088
## year:Surface           4.120e-03  5.131e-03   0.803  0.422161
## year:discauth1        -2.284e-01  4.978e-02  -4.589  4.85e-06 ***
## artistliving1:authorstyle1 9.199e-01  8.214e-01   1.120  0.262908
## artistliving1:lands_sc1 7.111e-01  3.142e-01   2.263  0.023776 *
## artistliving1:discauth1 1.341e+00  5.597e-01   2.396  0.016713 *
## authorstyle1:Surface   -5.179e-02  1.034e-01  -0.501  0.616566
## endbuyerC:finished1    -2.961e-01  9.122e-01  -0.325  0.745555
## endbuyerD:finished1    -5.606e-01  9.092e-01  -0.617  0.537612
## endbuyerE:finished1    -2.918e-01  9.394e-01  -0.311  0.756153
## endbuyerU:finished1    -9.915e-01  9.157e-01  -1.083  0.279090
## endbuyerC:discauth1    -4.677e+00  1.661e+00  -2.815  0.004939 **
## endbuyerD:discauth1    -3.847e+00  1.588e+00  -2.423  0.015533 *
## endbuyerE:discauth1    -3.589e+00  1.631e+00  -2.201  0.027901 *
## endbuyerU:discauth1    -4.199e+00  1.632e+00  -2.573  0.010185 *
## Interm1:Surface        1.652e-01  8.579e-02   1.925  0.054414 .
## Interm1:prevcoll1      -5.061e-01  3.358e-01  -1.507  0.132002
## Interm1:lrdfont1       -5.261e-01  2.627e-01  -2.003  0.045381 *
## Interm1:discauth1      1.644e+00  5.482e-01   2.998  0.002760 **
## Surface:discauth1      -3.646e-01  1.224e-01  -2.978  0.002947 **
## prevcoll1:finished1    -1.030e+00  3.118e-01  -3.304  0.000978 ***
## paired1:finished1      5.796e-01  1.961e-01   2.956  0.003164 **
## paired1:lrdfont1       -6.353e-01  2.352e-01  -2.701  0.006996 **
## paired1:lands_sc1      -3.026e-01  2.233e-01  -1.355  0.175507
## paired1:discauth1      -5.982e-01  3.482e-01  -1.718  0.086051 .
## lands_sc1:discauth1    -2.058e+00  8.764e-01  -2.349  0.018980 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.117 on 1438 degrees of freedom
## Multiple R-squared:  0.6746, Adjusted R-squared:  0.6608
## F-statistic: 48.87 on 61 and 1438 DF, p-value: < 2.2e-16
```

## finalmodel deleted some interactions

```
finalmodel <- lm(formula = logprice ~ dealer + year + origin_cat + artistliving +
  authorstyle + endbuyer + Interm + Surface + engraved + prevcoll +
  paired + finished + lrdfont + lands_sc + discauth + dealer:year +
  dealer:Interm + dealer:discauth + year:artistliving + year:endbuyer + year:Surface + year:discauth +
  Interm:discauth + Surface:discauth + prevcoll:finished +
  paired:finished + paired:lrdfont + paired:discauth, data = paint_train_trans)

summary(finalmodel)
```

```
##
```

```
## Call:
## lm(formula = logprice ~ dealer + year + origin_cat + artistliving +
##     authorstyle + endbuyer + Interm + Surface + engraved + prevcoll +
##     paired + finished + lrgfont + lands_sc + discauth + dealer:year +
##     dealer:Interm + dealer:discauth + year:artistliving + year:endbuyer +
##     year:Surface + year:discauth + artistliving:authorstyle +
##     artistliving:discauth + authorstyle:Surface + endbuyer:finished +
##     endbuyer:discauth + Interm:Surface + Interm:prevcoll + Interm:lrgfont +
##     Interm:discauth + Surface:discauth + prevcoll:finished +
##     paired:finished + paired:lrgfont + paired:discauth, data = paint_train_trans)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.6957 -0.6962 -0.0090  0.7223  3.8998
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -2.603e+02  2.004e+02  -1.299  0.194261
## dealerL         1.727e+02  4.673e+01   3.696  0.000227 ***
## dealerP         1.435e+02  9.352e+01   1.534  0.125269
## dealerR         4.539e+01  4.195e+01   1.082  0.279366
## year            1.481e-01  1.131e-01   1.309  0.190582
## origin_catF     -6.516e-01  8.024e-02  -8.120  9.92e-16 ***
## origin_catI     -7.001e-01  1.034e-01  -6.773  1.84e-11 ***
## origin_cat0     -1.108e+00  1.196e-01  -9.262  < 2e-16 ***
## artistliving1    2.767e+01  3.069e+01   0.902  0.367446
## authorstyle1    -6.403e-01  6.079e-01  -1.053  0.292378
## endbuyerC       -6.758e+00  1.906e+02  -0.035  0.971715
## endbuyerD        9.524e+01  1.903e+02   0.500  0.616829
## endbuyerE       -1.156e+02  1.918e+02  -0.603  0.546832
## endbuyerU        3.089e+01  1.898e+02   0.163  0.870737
## Interm1         -1.366e+00  6.141e-01  -2.225  0.026239 *
## Surface         -8.435e+00  9.101e+00  -0.927  0.354144
## engraved1        7.211e-01  1.392e-01   5.180  2.53e-07 ***
## prevcoll1        1.206e+00  1.673e-01   7.207  9.20e-13 ***
## paired1         -2.583e-01  7.177e-02  -3.600  0.000329 ***
## finished1        1.253e+00  9.016e-01   1.389  0.164984
## lrgfont1         1.146e+00  1.490e-01   7.695  2.62e-14 ***
## lands_sc1       -3.676e-01  1.128e-01  -3.258  0.001149 **
## discauth1        4.179e+02  8.874e+01   4.710  2.72e-06 ***
## dealerL:year     -9.657e-02  2.635e-02  -3.666  0.000256 ***
## dealerP:year     -8.049e-02  5.262e-02  -1.530  0.126339
## dealerR:year     -2.452e-02  2.367e-02  -1.036  0.300305
## dealerL:Interm1  1.461e+00  8.111e-01   1.802  0.071830 .
## dealerP:Interm1  3.329e-02  1.243e+00   0.027  0.978635
## dealerR:Interm1  1.424e+00  4.791e-01   2.971  0.003017 **
## dealerL:discauth1 4.535e-01  8.658e-01   0.524  0.600524
## dealerP:discauth1 -1.587e+00  8.906e-01  -1.782  0.075003 .
## dealerR:discauth1 -2.809e+00  4.661e-01  -6.028  2.11e-09 ***
## year:artistliving1 -1.539e-02  1.731e-02  -0.890  0.373837
## year:endbuyerC    3.786e-03  1.075e-01   0.035  0.971915
## year:endbuyerD   -5.382e-02  1.074e-01  -0.501  0.616297
## year:endbuyerE    6.494e-02  1.082e-01   0.600  0.548562
## year:endbuyerU   -1.786e-02  1.071e-01  -0.167  0.867568
```



```

## year:Surface          4.946e-03  5.135e-03   0.963  0.335651
## year:discauth1       -2.326e-01  4.988e-02  -4.663  3.40e-06 ***
## artistliving1:authorstyle1  8.557e-01  8.231e-01   1.040  0.298719
## artistliving1:discauth1    6.666e-01  4.633e-01   1.439  0.150369
## authorstyle1:Surface    -4.916e-02  1.036e-01  -0.474  0.635341
## endbuyerC:finished1    -3.112e-01  9.145e-01  -0.340  0.733669
## endbuyerD:finished1    -5.894e-01  9.115e-01  -0.647  0.517967
## endbuyerE:finished1    -3.346e-01  9.414e-01  -0.355  0.722336
## endbuyerU:finished1    -1.064e+00  9.178e-01  -1.159  0.246547
## endbuyerC:discauth1    -2.507e+00  1.442e+00  -1.739  0.082317 .
## endbuyerD:discauth1    -1.764e+00  1.376e+00  -1.282  0.200078
## endbuyerE:discauth1    -1.418e+00  1.399e+00  -1.014  0.310741
## endbuyerU:discauth1    -2.003e+00  1.394e+00  -1.436  0.151193
## Interml:Surface        1.612e-01  8.599e-02   1.874  0.061090 .
## Interml:prevcoll1     -5.465e-01  3.363e-01  -1.625  0.104399
## Interml:lrgfont1      -5.416e-01  2.632e-01  -2.058  0.039778 *
## Interml:discauth1      1.589e+00  5.492e-01   2.893  0.003871 **
## Surface:discauth1     -3.279e-01  1.210e-01  -2.710  0.006807 **
## prevcoll1:finished1   -1.029e+00  3.118e-01  -3.299  0.000992 ***
## paired1:finished1      5.911e-01  1.960e-01   3.015  0.002612 **
## paired1:lrgfont1     -5.903e-01  2.351e-01  -2.510  0.012171 *
## paired1:discauth1     -7.151e-01  3.425e-01  -2.088  0.036964 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.12 on 1441 degrees of freedom
## Multiple R-squared:  0.6722, Adjusted R-squared:  0.659
## F-statistic: 50.94 on 58 and 1441 DF, p-value: < 2.2e-16

```

I deleted the interactions `lands_sc:discauth`, `paired:lands_sc` and `artistliving:lands_sc`. First of all, for the interactions related to `lands_sc`, we think the `lands_sc`—if described as a plain landscape, is a variable describing a single dimension of painting content and we don't think it will interact with the variables describing the dealers' behaviour, the pairing of a painting, or the living info of the artists. Though the Adjusted R-squared decreased a little, from 0.66 to 0.659, we think it worth to make the model more reasonable.

#### 4. Summary and Conclusions: