Control of Aerial Manipulator

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***Abstract*—This paper presents two controllers for quadtoror equipped with a two-DOF robot manipulator that physically interacts with the environment. Cartesian impedance and Adaptive sliding node controller are implemented and compared for simple hovering task in presence of some bounded disturbance. A full dynamic model is developed and two above controllers are implemented with same initial conditions and desired position.**

I. SUMMARY

A. Cartesian Impedance Control

As a part of ARCAS[3] project, Lippiello and Ruggiero[1] proposed a nonlinear model-based Cartesian impedance controller in which the nonlinear dynamics of system is taken into account. Their Cartesian impedance model is a dynamic relationship between the end-effector motion in Cartesian coordinates and the generalized external forces and moments acting on the system. Impedance control is often used in robot manipulation task. Cassical impedance control requires the measurement of external force, which typically acts on the manipulator end-effector. So, Lippiello and Ruggiero[1] modified the classical impedance control as suggested by Ott [4] since it's unfeasible to measure such external force in aerial robotics.

1) Control Problem: UAV equipped with a robotic arm can be efficient solution for dexterous manipulation. It can also be used for rescue operations, construction in inaccessible and inspection of power lines. However, the presence of robot arm changes the dynamic behavior of the system and creates coupling effects. The center of mass is no longer constant and moment of inertia changes significantly. So, in this paper [1], the focus in on hovering control in presence of some bounded disturbances.

2) Control Law:

$$\dot{x} = J_a \dot{q} \tag{1}$$

$$\ddot{x} = J_a \ddot{q} + \dot{J}_a \dot{q} \tag{2}$$

$$\tilde{x} = x_d - x \tag{3}$$

$$\tau = J_a^T f \tag{4}$$

From (1),(2),(3),(3),(4) the joint space dynamics can be transformed into manipulator end-effector Cartesian coordinates (x). Here, J_a is the Jacobian of the system. τ is the generalized torque related to four motor thrust and joint actuator.

$$M_x \ddot{x} + C_x(x, \dot{x})\dot{x} + G_x(x) = f + f_{ext} \tag{5}$$

where, $M_x = J_a^{-T} M_q J_a^{-1}$, $C_x = J_a^{-T}(C_q - M_q J_a^{-1} \dot{J}_a) J_a^{-1}$, $G_x = J_a^{-T} G_q$,$f = J_a^{-T}\tau$, $f_ext = J_a^{-T}\tau_{ext}$

The matrices in (5) have following properties:

- **Property 1.** The inertia matrices M and M_x are symmetric and positive definite.
- **Property 2.** $(M - 2C)$ and $(M_x - 2C_x)$ are skew symmetric.

Following control law is defined where K_P and K_D are $n_q \times n_q$ symmetric and positive definite matrices.

$$\tau = G + J_a^T (M_x \ddot{x_d} + C_x \dot{x_d} + K_D \dot{\tilde{x}} + K_P \dot{x}) \tag{6}$$

Closed loop dynamics of the system is:

$$M_x \ddot{\tilde{x}} + (C_x + K_D)\dot{\tilde{x}} + K_P \tilde{x} = f_{ext} \tag{7}$$

3) Stability Analysis: Positive definite candidate Lyapunov function is defined as:

$$V(x,t) = \frac{1}{2}\dot{\tilde{x}}^T M_x \dot{\tilde{x}} + \frac{1}{2}\tilde{x}^T K_P \tilde{x} \tag{8}$$

Taking time derivative of $V(x,t)$:

$$\dot{V} = \frac{1}{2}\dot{\tilde{x}}^T \dot{M}_x \tilde{x} + \dot{\tilde{x}}^T M_x \ddot{\tilde{x}} + \tilde{x}^T K_P \dot{\tilde{x}} \tag{9}$$

substituting equation 7 into 9, we get:

$$\dot{V} = -\dot{\tilde{x}}^T K_D \dot{\tilde{x}} + \dot{\tilde{x}}^T f_{ext} \tag{10}$$

If $f_{ext} = 0$ for free motion case, the stability analysis of the system is not obvious since the matrix M_x is time varying, so we cannot use invariance principle. Hence, only stability can be insured. Also, it is mentioned in the paper that by considering Cartesian configuration space in the which the Jacobian is invertible, it is possible to show the asymptotic stability of the system. It also states that in case of constraint motion , $f_{ext} \neq 0$, only stability can be ensured only in the unconstrained Cartesian directions.

II. Critique

A. Methodology

The Cartesian impedance model is a model based controller which needs the exact values of all parameters in the dynamics of the UAV and manipulator. Moreover, in this paper[1], some of the impedance control parameters cannot be specified arbitrarily and they are dependent on the parameters of the system dynamics. Also, their controller is focused in hovering task. For tracking problem, the authors have not given any results. Very few reports exists on handling the aerial vehicle and the robotic arms as an integrated system. This report[1] was one of the early papers to develop full dynamic model of a multi-DOF robot arm using Euler-Lagrangian formalism.

B. Literature Review

The motivation behind introducing this paper was to develop full dynamic model of the UAV plus robotic arm as a single rigid body. Since, the coupling effects and change of center of mass and inertia are significant, the dynamic model given seems to be the best way to go forward for this kind of system. Previous reports have controlled the UAV and arm separately using different controller for UAV and robot arm.

C. Dynamic Model/Mathematics

This paper gave a complete formulation for complete dynamics of the UAV plus a robot as a single system. In doing so they have made many assumptions which might not always hold true. The Jacobain J_a might not be always invertibe. Also, the computation of C matrix becomes very complicated. Their stability analysis is not detailed. In order to prove global asymptotic stability, we need to find a different Lyapunov function for this specific controller mentioned in this paper. Furthermore, they went on to assume that just the controller used in ground manipulator can be used to prove globally asymptotic stability in UAV plus manipulator situation, which might not be the case always.

Details in derivation are lacking. While deriving the Jacobain, the author did not mentioned any formula for the Jacobain. They also did make gave any clear distinction about the manipulator velocity and angular Jacobian. I had to find that using different paper [2].

D. Simulation Results

Based on the results provided in the paper, the simulation results shows agreement with the theoretical analysis. But they are lacking details on that too. Complete results of torque and generalized force should have been provided in the paper to check the range of torque involved. Any experimental data is lacking so it hard to analyze the practical aspect of the controller given. Their controller is used just for hovering and perturbations. So, the results are not comprehensive. It would have been a better idea if they have provided plot for tracking error along with hovering.

E. Suggestions

I would like to give following suggestions to the paper I used to simulation:

- Use of better notation while deriving the complete dynamics.
- Providing better equations for system Jacobian and manipulator Jacobain.
- Use of better label in the figure while marking inertial and body frame of the UAV.
- Improving the Stability analysis
- Include the experimental data.

F. Sliding Mode Control

I used another controller from [2], to compare with the controller mentioned above.

Control Law:

$$e = q - q_d \tag{11}$$

$$s = \dot{e} - \Lambda e \tag{12}$$

$$q_r = \dot{q} - s = \dot{q_d} - \Lambda e \tag{13}$$

To design the controller, we define error e and sliding surface s. Now the control law becomes:

$$\tau = \hat{M}\ddot{q_r} + \hat{C}\dot{q_r} + \hat{\Delta} - As - Ksgn(s) \tag{14}$$

here, $\hat{M}$, $\hat{C}$, $\hat{G}$ represents estimation of each matrix. A and K are positive gain matrices.

$$M\dot{s} + Cs + \Lambda s = -\Delta + \hat{\Delta} - Ksgn(s) \tag{15}$$

where,

$$\Delta = \tilde{M}\ddot{q_r} - \tilde{C}\dot{q_r} - \tilde{G} \tag{16}$$

Stability Analysis: Let a Lyapunov candidate function be

$$V = \frac{1}{2}s^T M s + \frac{1}{2}\tilde{\Delta}^T\tilde{\Delta} > 0 \tag{17}$$

It time derivative is :

$$\dot{V} = \frac{1}{2}s^T \dot{M} s + s^T M\dot{s} + \tilde{\Delta}^T\dot{\tilde{\Delta}} \tag{18}$$

$$\dot{V} = \frac{1}{2}s^T(-\Lambda s + \tilde{\Delta} - Ksgn(s) + \tau_{ext}) + \tilde{\Delta}^T\dot{\tilde{\Delta}} \tag{19}$$

$$\dot{V} = -s^T\Lambda s - S^T(Ksgn(s) - \tau_{ext}) \leq 0$$

Adaptation law:

$$\dot{\hat{\Delta}} - \dot{\Delta} = -s \tag{20}$$

with the assumption that Δ changes very slow compared with the adaptation rate, we get

$$\dot{\hat{\Delta}} = -s \tag{21}$$

Free motion:If we assume that $\tau_{ext} = 0$, from (19) and (20) , we can conclude that $s \to 0$ as t $\to$ 0, so $e \to 0$ exponentially.If $\tau_{ext} \neq 0$, we can conclude that s is bounded, so e is bounded.

III. Simulation Experiment

- ASCTEC PELLICAN, with a mass m_b = 2 kg, inertia matrixI_b= $diag(1.24, 1.24, 2.48)$
- 2-DOF robotic arm, l_1 =15 cm $l_2$5 cm with 3 revolute joints, mass of link m_1 = 0.05 kg,m_2 = 0.05 kg, inertia if link I_1 = 0.0019 m^2kg I_1 = 0.0011 m^2kg
- initial conditions, $q = \left[0, 0, 0, 0, 0, 0, 0, 0, \frac{\pi}{2}\right]^T$
- desired position, $q_d = [0, 0, 6, 0, 0, 0, 0, 0, 0.5, 0.5]$

A force of 1 N is modelled through of sine wave of $\pi/4$ rad/s which acts along the x_b axis and another force of 0.5 N magnitude acts along the x-direction of end-effector of 20 s to model the windy situation.There cases are simulated as done in the paper and compared with another adaptive sliding mode controller.

A. *Cartesian Impedance Controller*

Case I (Rigid Behavior): $K_P = 80I_8$, $K_D = 16I_8$

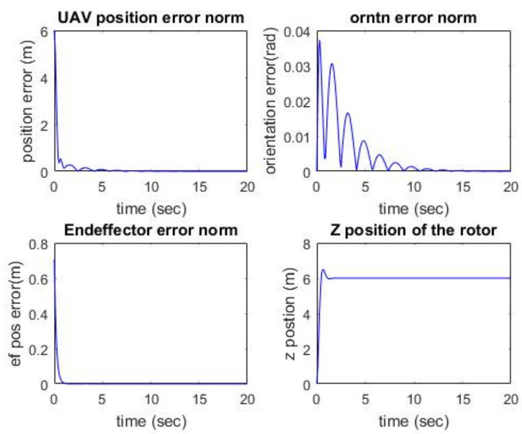

Fig. 1. Time histories simulation for Rigid Case.

Case II (Compliant): $K_P = 5I_8,$, $K_D = 2I_8$

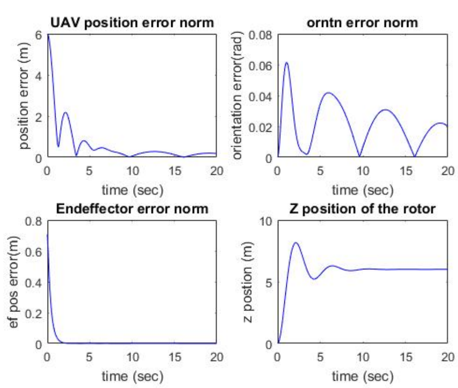

Fig. 2. Time histories simulation for Compliant Case.

Rigid UAV and Compliant manipulator:$K_P = diag(10I_6, 100I_2)$, $K_D = diag(3I_6, 50I_2)$

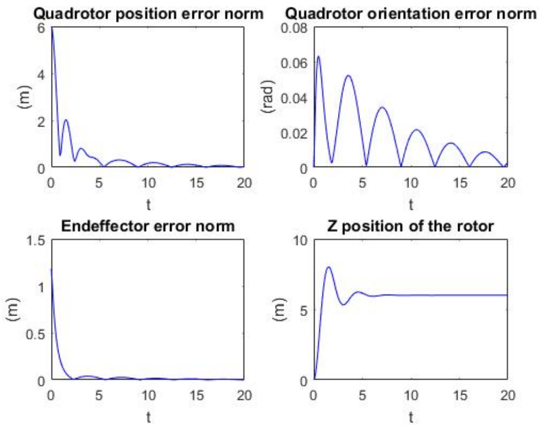

Fig. 3. Time histories simulation for Compliant Rigid Case.

B. *Adaptive Sliding Mode Controller:*

Design Parameters: $K = 8I_{8\times8}$, $\Lambda = 2I_{8\times8}$, $A = 10I_{8\times8}$

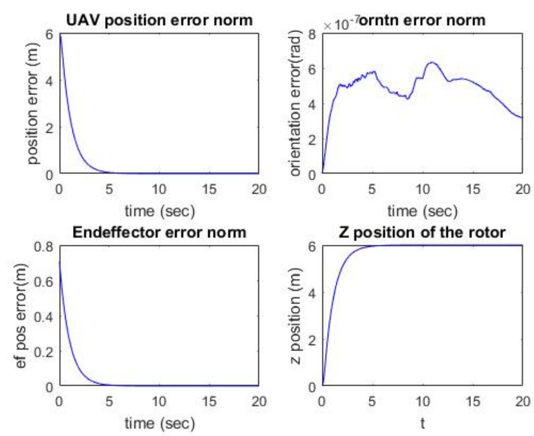

Fig. 4. Time histories simulation for adaptive sliding mode controller.

Position History

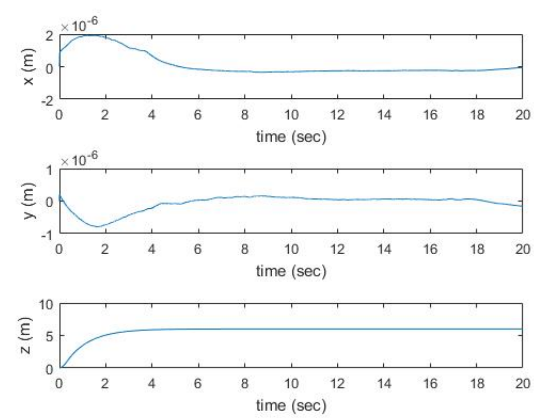

Fig. 5. Position history of the quadrotor.

Attitude History

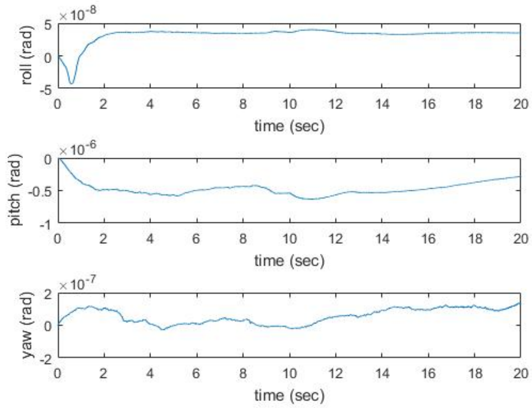

Fig. 6. Attitude history of the quadrotor.

Torque Plots

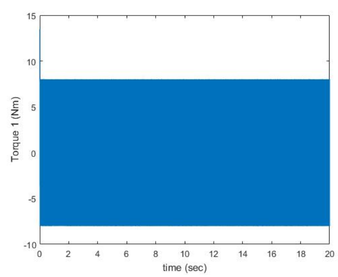

Fig. 7. Torque History for link 1

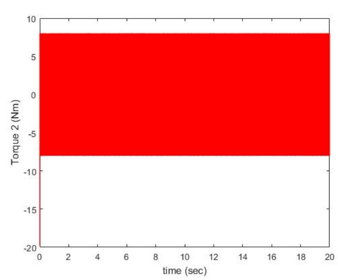

Fig. 8. Torque History for link 2

TABLE I
COMPARISON OF RESPONSE FOR Z-POSITION

Z position	**Rigid**	**Compliant**	**CompRigid**	**AdpSMC**
overshoot	8%	36.05%	33.66	0
rise time (sec)	0.8	1.8	1.5	4.4
settling time (sec)	1.8	7	5	5

TABLE II
COMPARISON OF RMS ERROR FOR TWO CONTROLLER

error	position (m)	orientation (rad)	end eff pos (m)
Rigid	0.6265	0.0088	0.0628
Compliant	1.2006	0.0258	0.0811
CompRigid	1.0148	0.023	0.0879
AdpSMC	1.0769	4.90E-07	0.1128

		x	y	z	roll	pitch	yaw	x1	y1
Rigid	average	2.27E-05	0.000857	0.074617	-9.5E-05	-6E-06	-3.9E-05	0.00633	-0.01245
	max error	0.0004618	0.275189	6	0.037219	0.000638	0.000446	0.5	0.258405
	min error	-0.0003305	-0.31501	-0.48734	-0.03057	-0.00065	-0.0018	-0.00012	-1.0708
	rms	1.23E-04	0.0789	0.6265	0.0088	8.74E-05	2.68E-04	0.0444	0.0444
compliant	average	0.0001205	0.003905	0.134819	-0.00041	-4.7E-05	-0.00011	0.011395	-0.02044
	max error	0.0007372	0.370054	6	0.06127	0.001432	0.003117	0.5	0.344783
	min error	-0.00082	-0.48702	-2.16279	-0.04177	-0.00209	-0.00323	-0.00073	-1.0708
	rms	4.10E-04	0.2311	1.1781	0.0258	4.86E-04	0.0012	0.0573	0.0573
gid Compli	average	3.328E-06	0.004496	0.104867	-0.00049	-5.8E-05	-0.00011	0.018652	-0.02905
	max error	0.0012899	0.481499	6	0.063231	0.000527	0.002825	0.5	0.038599
	min error	-0.0019837	-0.47836	-2.01989	-0.05204	-0.0017	-0.00328	0.004976	-1.0708
	rms	5.89E-04	0.2117	0.9925	0.023	3.24E-04	0.001	0.023	0.0622
ADSMC	average	1.35E-07	-6.33E-08	-0.3452	3.19E-08	-4.72E-07	5.55E-08	-0.0253	0.0253
	max error	1.92E-06	2.02E-07	6	4.02E-08	0	1.38E-07	5.00E-01	0.5
	min error	-3.47E-07	-7.94E-07	-2.76E-06	-4.34E-08	-6.33E-07	-2.88E-08	-8.04E-08	-7.87E-08
	rms	7.13E-07	2.49E-07	1.0769	3.45E-08	4.84E-07	7.20E-08	0.0798	0.0798

Fig. 9. Error comparison for both of the controllers

The simulation results of Cartesian impedance controller matches with paper. However, their desired trajectory is not given, so I used above stated desired conditions for my simulation purposes. Comparing my plot with the one from paper, it shows good agreement.

We can observe from the figure 1-3, that error is bounded. The orientation error still won't go to zero because of the external disturbance in the system. Z position converges from 0 to 6 in less than 5 seconds in all the three cases. Also, in adaptive controller the error remains bounded as we can observe from figure 4, 5 & 6.

While simulating the Cartesian impedance controller, three cases were done in order to find the rigid behavior of the manipulator and compliant behavior of the UAV. One of the major challenges while implementing the controller was tuning the gain matrices. Comparing Cartesian controller with Adaptive Sliding mode controller, it can be clearly seen that just from the z-position history it is better in terms of performance. There is no overshoot with the Adaptive controller. However, in all three case for the Impedance controller, there was significant overshoot. Adaptive Controller has better response but higher error in the end effector position. From figure 9, we can observe that error is significantly less in adaptive controller expect the end effector position.

IV. CONCLUSION

Coupled dynamics of aerial manipulator makes the control problem challenging. This paper contributed towards dynamic modeling of aerial manipulator. Cartesian impedance control is used widely in robotics. However, it's relevance in aerial robotics is still not being properly analyzed. Comparing all the results we can see that Cartesian impedance controller is better in task space but in other cases like the position and orientation of the UAV, adaptive controller is better. So, the solution might be combination of two using a Adaptive Cartesian Impedance Controller.

V. References

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