

Control of Aerial Manipulator

MAE 5463 Nonlinear Systems

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Introduction

- Aerial Manipulator: Rotorcraft Equipped with a robotic arm
- Current research in Aerial Manipulator:
 - ▶ Installing a gripper at the bottom of UAV to hold a payload
 - ▶ Suspending payload with cables
 - ▶ Autonomous navigation
- Use:
 - ▶ Inspection and maintenance of power lines
 - ▶ Rescue operations
 - ▶ Construction in inaccessible sites

Dynamics of Combined System

- Based on Euler-Lagrangian formulation

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + \tau_{ext} \quad (1)$$

- Where, q is the generalized joints vector of $6 + n$ components. In this case, the arm has 2 DOF and $n = 2$.
- M is the inertia matrix, C is the centripetal/Coriolis matrix and G is the gravity.

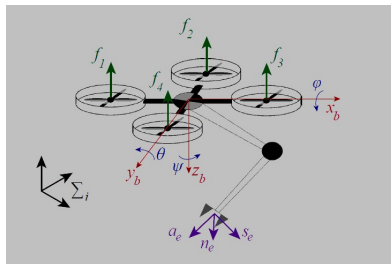


Figure 1: UAV/Arm system with corresponding reference frame [1].

Control Problem

- Complicated coupled dynamics
- Displacement of center of mass
- Variation of mass distribution, moment of inertia

Cartesian Impedance Control

Control Law:

$$\dot{x} = J_a \dot{q} \quad (2)$$

$$\ddot{x} = J_a \ddot{q} + \dot{J}_a \dot{q} \quad (3)$$

$$\tilde{x} = x_d - x \quad (4)$$

$$\tau = J_a^T f \quad (5)$$

From (1),(2),(3),(3),(4) & (5),the joint space dynamics can be transformed into manipulator end-effector Cartesian coordinates(x). Here, J_a is the Jacobian of the system. τ is the generalized torque related to four motor thrust and joint actuator.

$$M_x \ddot{x} + C_x(x, \dot{x}) \dot{x} + G_x(x) = f + f_{ext} \quad (6)$$

where, $M_x = J_a^{-T} M_q J_a^{-1}$, $C_x = J_a^{-T} (C_q - M_q J_a^{-1} \dot{J}_a) J_a^{-1}$,
 $G_x = J_a^{-T} G_q$, $f = J_a^{-T} \tau$, $f_{ext} = J_a^{-T} \tau_{ext}$

Cartesian Impedance Control

- Following control law is defined where K_P and K_D are $n_q \times n_q$ symmetric and positive definite matrices.

$$\tau = G + J_a^T (M_x \ddot{x}_d + C_x \dot{x}_d + K_D \dot{\tilde{x}} + K_P \tilde{x}) \quad (7)$$

- Closed loop dynamics of the system is:

$$M_x \ddot{\tilde{x}} + (C_x + K_D) \dot{\tilde{x}} + K_P \tilde{x} = f_{ext} \quad (8)$$

Cartesian Impedance Control

Stability Analysis:

- Positive definite candidate Lyapunov function

$$V(x, t) = \frac{1}{2} \dot{\tilde{x}}^T M_x \dot{\tilde{x}} + \frac{1}{2} \tilde{x}^T K_P \tilde{x} \quad (9)$$

Time derivative of V:

$$\dot{V} = -\dot{\tilde{x}}^T K_D \dot{\tilde{x}} + \dot{\tilde{x}}^T f_{ext} \quad (10)$$

- If $f_{ext} = 0$ for free motion case, Using Theorem 8.5, from Khalil, we can conclude that $\dot{\tilde{x}} \rightarrow 0$ as $t \rightarrow \infty$
- In case of constraint motion, $f_{ext} \neq 0$, only stability can be ensured.

Cartesian Impedance Control

Simulation Results:

- ASCTEC PELLICAN, $m_b = 2$ kg, $I_b = \text{diag}(1.24, 1.24, 2.48)$
- 2-DOF robotic arm, $l_1 = 15$ cm $l_2 = 5$ cm with 3 revolute joints,
- $m_1 = 0.05$ kg, $I_1 = 0.0019$ m²kg , $m_2 = 0.05$ kg, $I_2 = 0.0011$ m²kg
- Focus on hovering control
 - ▶ initial conditions, $q = [0, 0, 0, 0, 0, 0, 0, 0, \frac{\pi}{2}]^T$
 - ▶ desired position, $q_d = [0, 0, 6, 0, 0, 0, 0, 0, 0.5, 0.5]$
- A force of 1 N is modelled through of sine wave of $\pi/4$ rad/s which acts along the x_b axis and another force of 0.5 N magnitude acts along the x-direction of end-effector of 20 s to model the windy situation.

Cartesian Impedance Control

Simulation Results: $K_P = 80I_8$, $K_D = 16I_8$

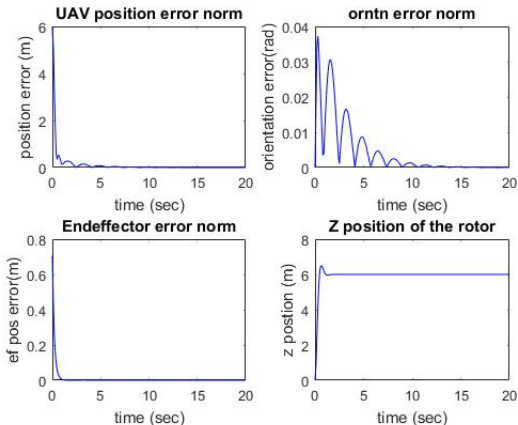


Figure 2: Time histories simulation for Rigid Case.

Cartesian Impedance Control

Simulation Results: $K_P = 5I_8$, $K_D = 2I_8$

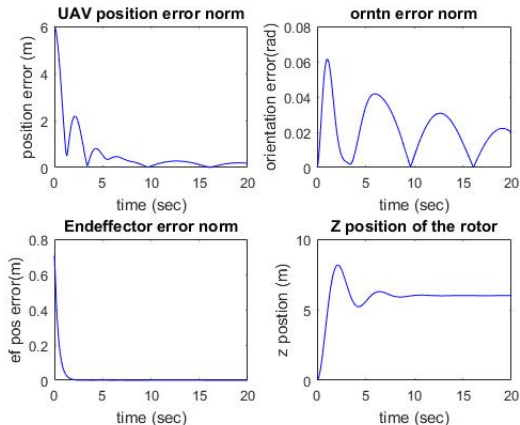


Figure 3: Time histories simulation for Compliant Case.

Cartesian Impedance Control

Simulation Results: $K_P = \text{diag}(10I_6, 100I_2)$, $K_D = \text{diag}(3I_6, 50I_2)$

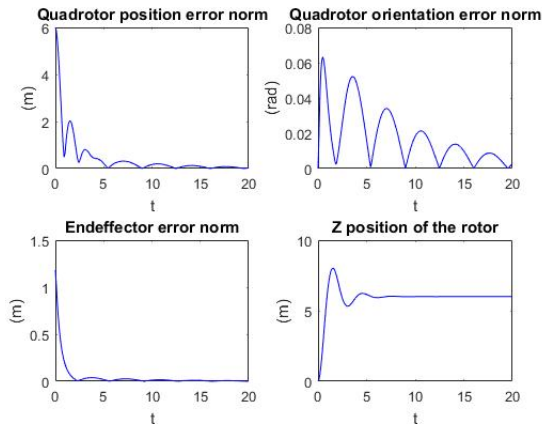


Figure 4: Time histories simulation for Compliant Rigid Case.

Adaptive Sliding Mode Control

Control Law:

$$e = q - q_d \quad (11)$$

$$s = \dot{e} - \Lambda e \quad (12)$$

$$q_r = \dot{q} - s = \dot{q}_d - \Lambda e \quad (13)$$

To design the controller, we define error e and sliding surface s . Now the control law becomes:

$$\tau = \hat{M}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{\Delta} - As - K\text{sgn}(s) \quad (14)$$

here, \hat{M} , \hat{C} , \hat{G} represents estimation of each matrix. A and K are positive gain matrices.

$$M\dot{s} + Cs + \Lambda s = -\Delta + \hat{\Delta} - K\text{sgn}(s) \quad (15)$$

where,

$$\Delta = \tilde{M}\ddot{q}_r - \tilde{C}\dot{q}_r - \tilde{G} \quad (16)$$

Adaptive Sliding Mode Control

Stability Analysis: Let a Lyapunov candidate function be

$$V = \frac{1}{2}s^T Ms + \frac{1}{2}\tilde{\Delta}^T \tilde{\Delta} > 0 \quad (17)$$

Its time derivative is :

$$\dot{V} = \frac{1}{2}s^T(-\Lambda s + \tilde{\Delta} - K \operatorname{sgn}(s) + \tau_{ext}) + \tilde{\Delta}^T \dot{\tilde{\Delta}} = -s^T \Lambda s - S^T(K \operatorname{sgn}(s) - \tau_{ext}) \leq 0 \quad (18)$$

Adaptation law:

$$\dot{\tilde{\Delta}} - \dot{\Delta} = -s \quad (19)$$

with the assumption that Δ changes very slow compared with the adaptation rate, we get

$$\dot{\tilde{\Delta}} = -s \quad (20)$$

Free motion: If we assume that $\tau_{ext} = 0$, from (19) and (20), we can conclude that $s \rightarrow 0$ as $t \rightarrow \infty$, so $e \rightarrow 0$ exponentially.

If $\tau_{ext} \neq 0$, we can conclude that s is bounded, so $\Rightarrow e$ is bounded.

Adaptive Sliding Mode Control

Simulation Results:

- Design Parameters: $K = 8I_{8 \times 8}$, $\Lambda = 2I_{8 \times 8}$, $A = 10I_{8 \times 8}$

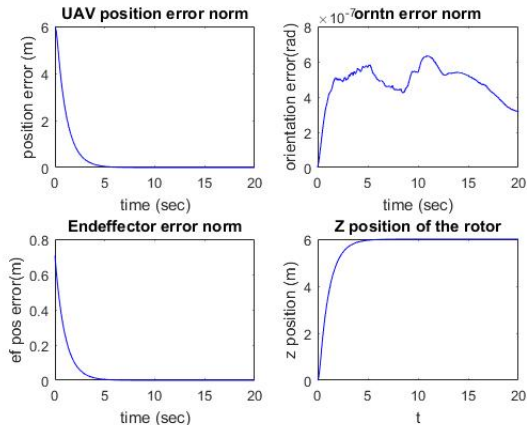


Figure 5: Time histories simulation for adaptive sliding mode controller.

Simulation Results: Position History

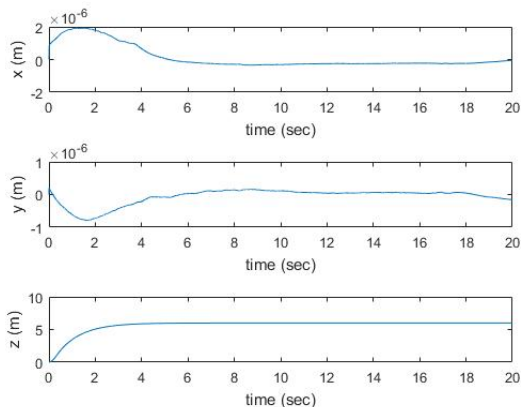


Figure 6: Position history of the quadrotor.

Simulation Results: Attitude History

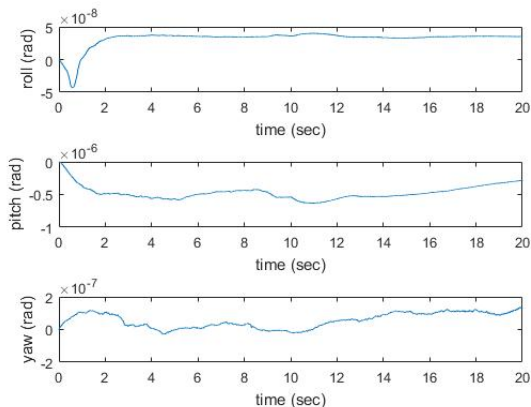
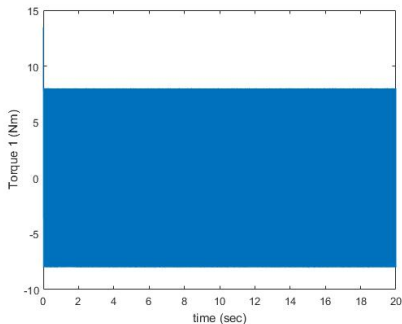


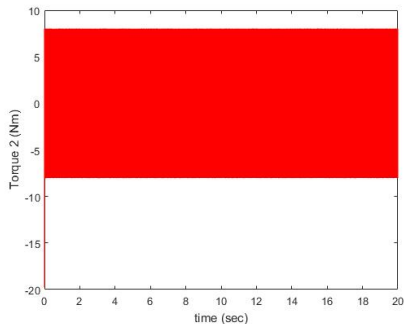
Figure 7: Attitude history of the quadrotor.

Adaptive Sliding Mode Control

Simulation Results: Torque Plots



(a) Torque1 History.



(b) Torque2 History

Figure 8: Torque History for both links

Conclusion and Future Work

- Coupled dynamics of aerial manipulator makes the control problem challenging
- Computation of C matrix
- Can implement a hierarchical control separately for both arm and the rotor.
- Future work on other applicable control strategies and platform for testing

References

- [1] V. Lippiello and F. Ruggiero, *Cartesian impedance control of a UAV with a robotic arm*. In 10th International IFAC Symposium on Robot Control, Dubrovnik, Croatia, Sep 2012
- [2] Kim S. et al. *Aerial Manipulation Using a Quadrotor with a Two DOF Robotic Arm*, 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2013) November 3-7, 2013, Tokyo, Japan.

Questions?