AA 242A Homework 4

Assigned: **Thursday, October 20th, 2022**

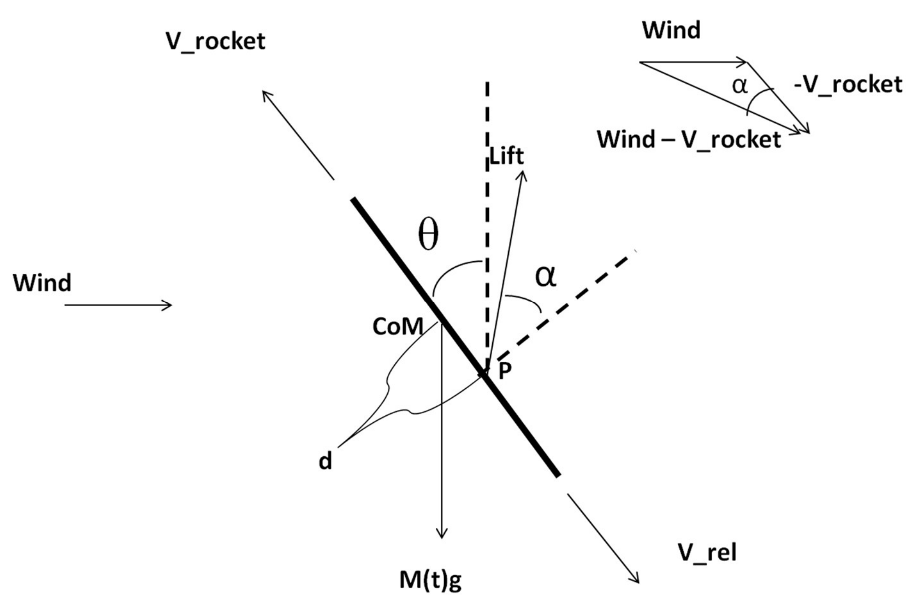
Due: **Thursday, October 27th, 2022**

1. [*Classical Mechanics,* Problem 1-13] Rockets are propelled by the momentum reaction of the exhaust gases expelled from the tail. Since these gases arise from the reaction of the fuels carried in the rocket, the mass of the rocket is not constant, but decreases as fuel is expended. Show that the equation of motion for a rocket projected vertically upward in a uniform gravitational field, neglecting friction, is

$$m\frac{\mathrm{dv}}{\mathrm{dt}} = -v'\frac{\mathrm{dm}}{\mathrm{dt}} - \mathrm{mg}$$

where m is the mass of the rocket and v' is the velocity of the escaping gases relative to the rocket. Integrate this equation to obtain v as a function of m, assuming a constant time rate of loss of mass. Show, for a rocket starting initially from rest, with v' equal to 2.1 km/s and a mass loss per second equal to 1/60th of the initial mass, that in order to reach the escape velocity the ratio of the weight of the fuel to the weight of the empty rocket must be almost 300!

2. You are the launch director for a sounding rocket. On the day of the launch, you find that there is a strong surface wind blowing along the x-direction. The surface wind creates a lift on the rocket, which acts at the "center of pressure" P and causes a deviation from the nominal trajectory. This phenomenon is called "rocket weather-cocking". We will now construct a simple model for this.
   a. Assume that the rocket, which is a slender body, may be approximated as a straight rod with uniform mass.
   b. Assume that the mass ejected from the rocket is along the body of the rocket at all times – this means that the thrust always points along the rocket.
   c. Assume there is no drag.
   d. Assume air density remains constant.
   e. Assume that the center of mass and center of pressure remain at the same location throughout (not true in the real situation).
   f. Treat this problem only as a 2-dimensional problem. A free body diagram is given below:



Here α is the "angle of attack" of the rocket – the angle that the relative wind makes with the rocket. Note that mass is a function of time. The mass loss rate is constant. You launch from $(x,y)=(0,0)$. As launch director, you determine that the launch is still good as long as the rocket is above 2km in altitude, and between 1-2km in the $-x$ direction after 30 seconds of flight.

The magnitude of lift is given by the following equation:

$$L = 0.09621 * \left(\frac{\alpha}{cos(\alpha)}\right)\rho_{air}\left((w - v_x)^2 + v_y^2\right)$$

The equation for the attitude angle θ is

$$\frac{d^2\theta}{dt^2} = \frac{1}{I}(TorqueaboutCoM)$$

Here w is the wind velocity, v_x and v_y are components of the rocket's velocity and I is the moment of inertia of the rocket about the CoM.

Setup the equations of motion of the rocket, then using ode45, ode113 (or your own ODE solver) find *x(t)*, *y(t)* and *θ(t)*. Use the following parameters:

a. Length of the rocket = 6m

b. Initial mass = 600kg

c. Density of air= 1.2041 kg/m^3

d. *d* (distance between CoM and P) = 0.35m

e. *I* (moment of inertia) = 1800 kg m^2

f. Mass exhaust velocity (with respect to rocket), v_{rel} = 2500m/s

h. Constant mass loss rate $\frac{dm}{dt} = -c$ = -4kg/s

Start with initial conditions $[x, y, \theta, v_x, v_y, \omega] = [0{,}0{,}1°, 0{,}0{,}0]$. That is the rocket starts with 0 velocity, but inclined 1 degree to the west. Simulate this for two wind speeds-

1. *W*= 5m/s
2. *W*=15 m/s

On the same graph, plot the trajectory for both cases. In which case would you give the go-ahead to launch the sounding rocket?

Bonus (2 points):

Do you know or can you think of any ways in which weather-cocking can be reduced for a rocket?

3.

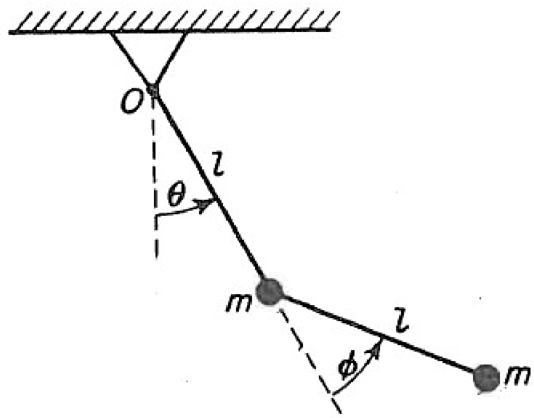


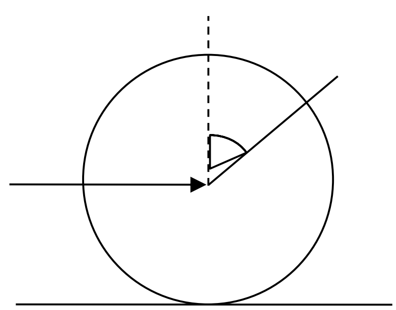
Fig. P6-4

A double pendulum consists of two massless rods of length l and two particles of mass m which can move in a given vertical plane, as shown.

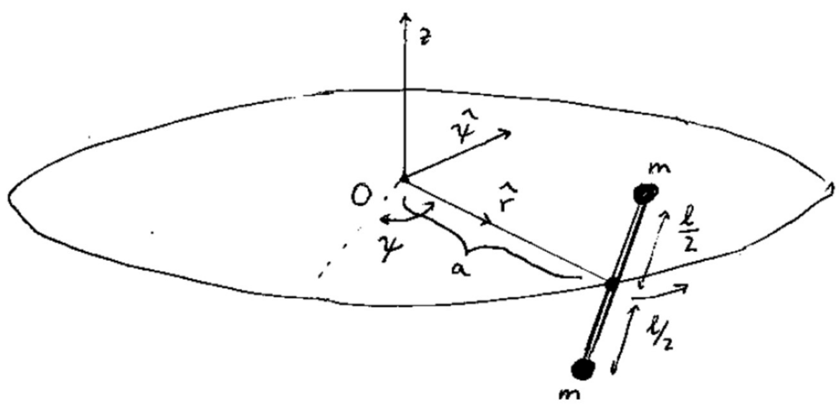
a. Assuming frictionless joints, and using θ and φ as coordinates, obtain the differential equations of motion.

b. What are the linearized equations for small θ and φ?

4. [Goldstein, *Classical Mechanics,* Problem 1-11] Consider a uniform thin disk that rolls without slipping on a horizontal plane. A horizontal force is applied to the center of the disk and in a direction parallel to the plane of the disk and parallel to the ground.
   a. Derive Lagrange's equations and find the generalized force.
   b. Discuss the motion if the force is not applied parallel to the plane of the disk (i.e. now with a component out of/into the page).



5. [Goldstein, *Classical Mechanics,* Problem 1-14] Two points, each of mass , are joined by a rigid weightless rod of length , the center of which is constrained to move on a circle of radius . Express the kinetic energy of the system in generalized coordinates.



(<u>Hint</u>: In addition to the cylindrical coordinate system shown, define an *x-y-z* coordinate system that has a fixed orientation in space but *translates with the rod center of mass*. For each particle, use a spherical coordinate system with angles defined relative to this frame)

6. This is the final week for your "simple" Classical Dynamics problem that you chose and began on the first homework. Once again draw a picture and provide a short description of what you are trying to solve. For this week, attempt a solution using Lagrange's approach. Define your generalized coordinates and write the Lagrangian. Use the Lagrangian to solve your EOM. Compare this approach to force balance and discuss. How does your answer this week compare to your answer using force balance? If you are still unable to obtain a solution this week, discuss how you might simplify the problem.