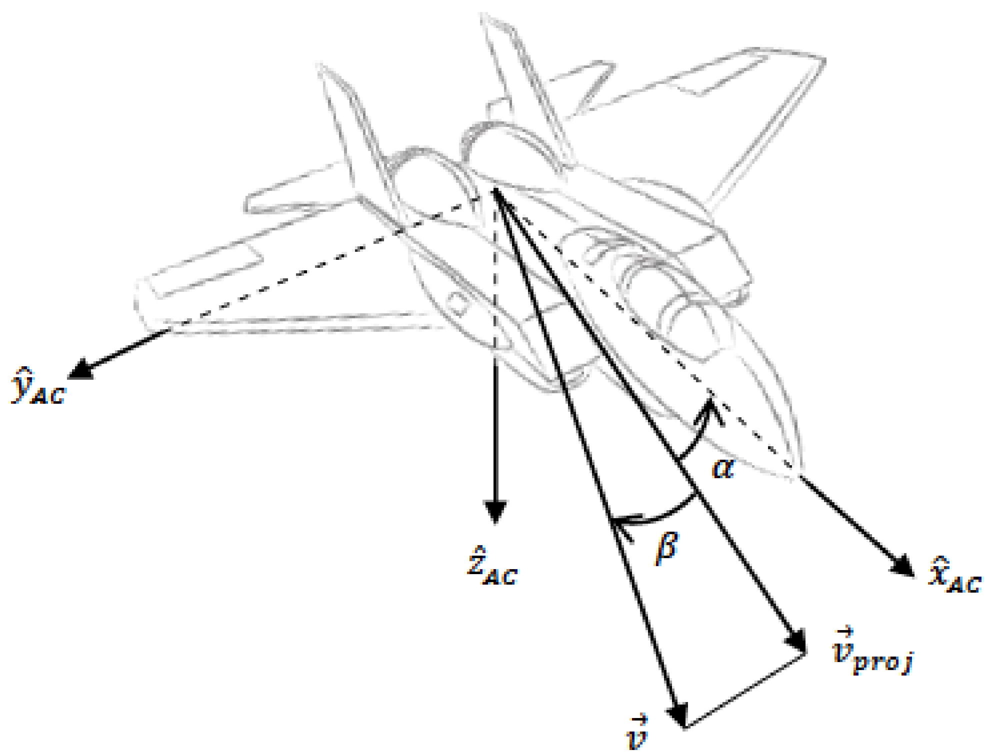
AA 242A Homework 7

Assigned: 11/17/22

Due: 12/01/22

1. In aircraft flight dynamics, control and stability is typically analyzed using a standard set of four frames. For short duration simulations, one typically makes a flat Earth approximation and uses an inertial frame with downward-pointing *z*-axis (towards the ground). This is called the Earth frame. Three moving frames are then defined relative to the Earth frame. The first is the aircraft body-fixed frame, defined by the 3-2-1 Euler angles ψ (yaw), θ (pitch), and φ (roll). The aircraft body-fixed frame uses the aircraft center of gravity (CG) as its origin, and defines the +*x*-axis as through the aircraft nose, the +*y*-axis as through the right wing of the plane, and the +*z*-axis pointing downwards according to the right-hand rule. A schematic of the aircraft frame can be seen below.



The aircraft velocity vector, $\vec{v}$, is also shown in the diagram. This is the velocity of the aircraft with respect to the inertial Earth frame, which means, if we assume a stagnant atmosphere, $\vec{v}$ is equivalent to the velocity of the aircraft with respect to the atmosphere (an important assumption for aerodynamics analysis). Define the projection of the velocity vector on the aircraft frame $\hat{x}_{AC} - \hat{z}_{AC}$ plane as $\vec{v}_{proj}$. The stability frame is the moving frame with origin at the aircraft CG that uses this projection for the direction of its *x*-axis instead of the aircraft nose. The angle from the stability frame *x*-axis to the aircraft frame *x*-axis is defined as the aircraft angle of attack, α, defined positive counterclockwise when looking along the aircraft *y*-axis towards the origin. The wind frame is the moving frame with origin at the aircraft CG that uses the aircraft velocity vector as the direction of its *x*-axis, which is defined by the sideslip angle β between the stability frame *x*-axis and the wind frame *x*-axis. The angle β is positive clockwise when looking along the stability *z*-axis away from the origin. Another way to visualize this is that β is positive when the wind is "in the pilot's right ear."

a. Draw a complete diagram or sequence of diagrams indicating how the stability and wind frames relate to the aircraft frame. Determine the direction cosine matrices that relate the aircraft frame to each the stability frame and the wind frame. If you knew the individual rotation matrices $\overleftrightarrow{A}_{\Psi}, \overleftrightarrow{A}_{\Theta}, \overleftrightarrow{A}_{\Phi}$ corresponding to yaw, pitch and roll (you do not need to write these out), how could you resolve a vector in the wind frame into the Earth frame, and vice versa?

b. The drag *D*, side force *S*, and lift *L* are the negative *x*-, positive *y*-, and negative *z*-components of the resultant force on the aircraft resolved in the wind frame. Find expressions for the resultant force resolved in the aircraft frame in terms of *D, S, L*, α and β.

c. Express the angular velocity and angular acceleration of the wind frame relative to the aircraft frame in terms of α and β and their derivatives. Resolve these vectors in the aircraft frame.

2. Find the inertia tensor, expressed in the principal axis frame, of a homogeneous ellipsoid of density ρ about its center of mass. The ellipsoid has semi-major axes a, b, and c in the x, y, and z directions, respectively. Express your answer in terms of the ellipsoid mass, m. Then find the inertia tensor resolved in a frame rotated by θ about the principal y-axis. What happens when $a=c$? What does this imply about the rotated frame? (Hint! Try using symmetry arguments and change of variables techniques to simplify the integration)

3. Greenwood 7-13

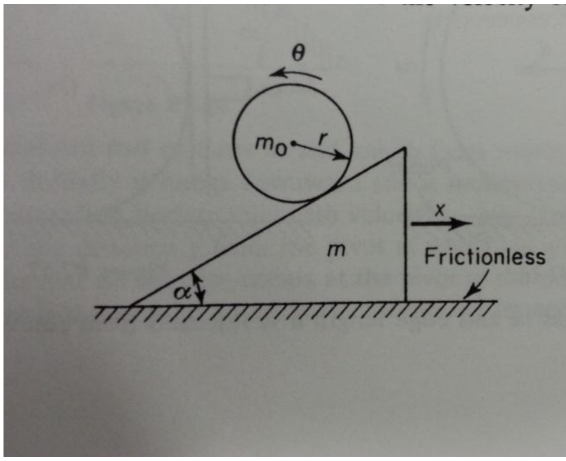


Figure P7-13

A solid homogeneous sphere of mass m_0 and radius r rolls without slipping on a triangular block of mass m which can slide on a frictionless floor. Assuming that the system is initially motionless, solve for the velocity of the block as a function of time.

4. Greenwood 7-22

A thin uniform rod of mass m and length l can rotate freely about a fixed pivot at one end. Initially it hangs downward and is motionless. Then a particle of mass m_0 which is traveling horizontally with velocity v_0 hits the rod and sticks to it.

a.) Find the distance y from the pivot at which the particle must strike the rod in order that no impulse occurs at the pivot at the instant of impact. *Hint: what is conserved under this condition?*

b.) What is the angular rotation rate of the rod immediately after impact?

5. We will continue to define the more complex classical dynamics problem that you chose and began last week. For this week, once again draw the picture, including reference frames and coordinate systems, and provide a concise description of what you are trying to solve. State what quantities are conserved and not conserved, and justify your answers. Attempt a solution using conservation equations – if the method does not work, explain why (i.e. not enough quantities conserved, some constraint that requires another method, etc.).