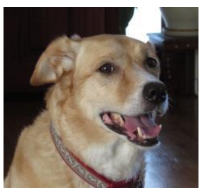
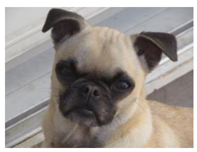
AA 242A Final Exam

December 10, 2021

- You may reference two pages of equations (front and back) in addition to the conservation, Lagrange and Euler sheets given in class.
- There are 100 possible points, split up into five problems, each with multiple parts. The value of each problem is indicated. You are to work all of the problems individually.
- Good luck!

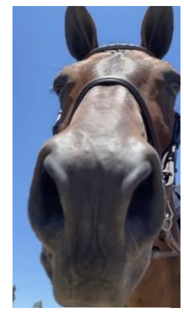

Sundance


Cookie

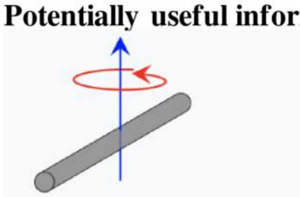

Tigress


Muffin & Coconut

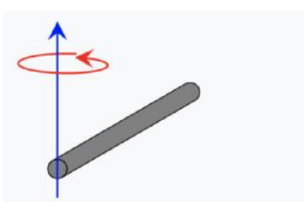

Biscuit


Cintas

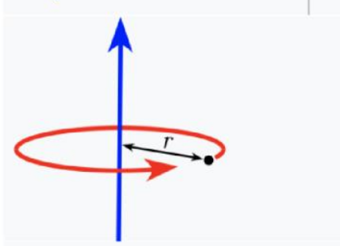
Potentially useful information:



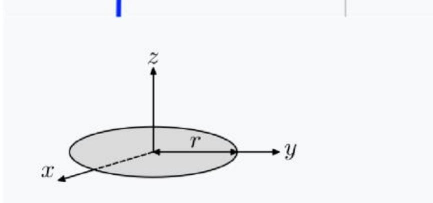
$$I_{\text{center}} = \frac{1}{12} mL^2$$



$$I_{\text{end}} = \frac{1}{3} mL^2$$

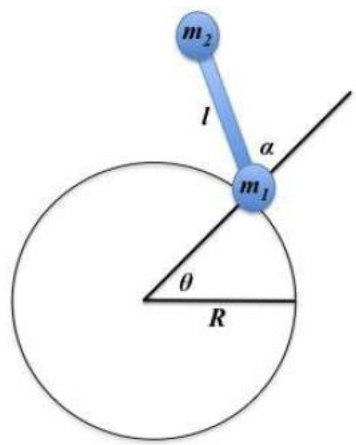


$$I = Mr^2$$



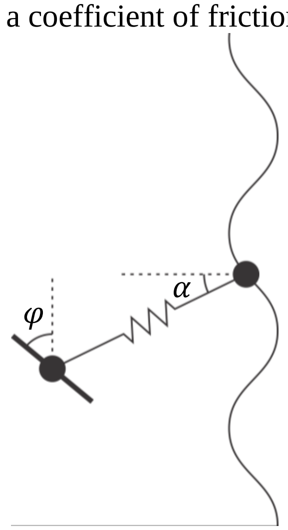
$$I_z = \frac{1}{2} mr^2$$
$$I_x = I_y = \frac{1}{4} mr^2$$

Problem 1 (20 points). Sundance (point mass m_1) and Cookie (point mass m_2) are playing tug-of-war with a rope, which can be approximated as a massless rod of length l. Sundance is pulling Cookie in a horizontal, circular motion. Approximate this motion as Sundance being fixed on a frictionless circular track of radius R with angles θ and α in a horizontal plane.



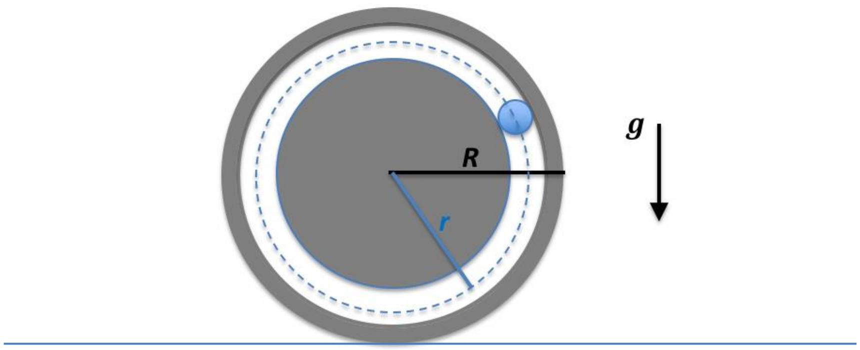
a. (3 points). What is and is not conserved and why?
b. (2 points). Write expressions for the constraint(s).
c. (15 points). Find the EOMs. Let m_1=3m and m_2=m for simplicity.

Problem 2 (20 points). Biscuit has a lot of energy and needs to be lunged. Model the system as two point masses (you and Biscuit) on ice skates, each with point mass m, connected by a spring with stiffness k. You are constrained to move according to $x = \sin(ay)$ as you play tug of war with her, and she is constrained as well (see diagram). The spring has a rest length d, and each point mass has a coefficient of friction μ_k.



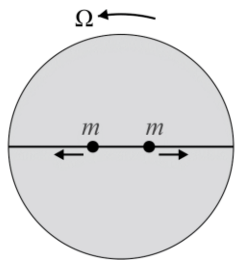
a. (2 points). What set of coordinates would you use to describe the system?
b. (4 points). Write the two equations of constraint.
c. (4 points). Draw a FBD for both masses.
d. (6 points). Write down the Lagrangian and the specific Lagrangian equation(s) you would use (no need to solve, you can show the general form with indexing).
e. (4 points). Write an expression for the generalized force for each coordinate.

Problem 3 (20 points). Tigress is playing with a ball constrained within a disk that rolls on a smooth, horizontal surface. Model her toy as a solid disk of external radius R with a circular guide of radius r for a ball of point mass m that can move freely inside of it. Assume the disk (without the ball) has a moment of inertia I and mass M and is rolling without slip in a vertical plane.



a. (10 points). Solve for the EOMs using Lagrange. *Hint: find the position, velocity, and acceleration of the ball with respect to an inertial-fixed point.*
b. (2 points). Draw a FBD for both the disk and the ball.
c. (2 points). Find an expression for the normal force of the disk on the ball using Force Balance.
d. (2 points). Find an expression for the static friction on the disk using Force Balance.
e. (4 points). Solve for the EOMs using Force Balance. You are allowed to hold normal force and static friction as variables (i.e. F_N, F_f).

Problem 4 (20 points). Coconut and Muffin are playing in their disk, which has mass *M* and radius *r*, with a rail that runs along the diameter. Approximate Coconut and Muffin as two point particles, each of mass *m*. Initially, the disk is spinning at Ω and Coconut and Muffin are symmetrically fixed on the rail at r/4 from the center. But Coconut and Muffin want to slow the spin of the disk. Being smart guinea pigs, they devise a plan to slow the spin by simultaneously running along the rail to the outer edge of the cylinder, where they stick. All motion is in a horizontal plane.



a. (6 points). What is and is not conserved in this system? Justify your reason for making any statement.
b. (5 points). What is the angular momentum of the system about the CM immediately after Coconut and Muffin start running towards the edge? What is the kinetic energy at this time?
c. (4 points). Assume that the spin rate of the system when the guinea pigs reach the edge is Ω/2. Using parts a) and b), find an expression for the masses of the guinea pigs in order to achieve this spin condition.
d. (5 points). In the rotating frame, what fictitious forces act on one of the pigs after she begins to move outward but before she reaches the edge (use either mass)? Draw a diagram showing the directions of each fictitious force.

Problem 5 (20 points). Cintas and his rider are heading for a jump. At the last moment, Cintas decides to stop, and the rider goes flying off like a missile.

a. (5 points). Use a 1-3-1 Euler angle sequence with the angles ϕ, θ, and ψ about the inertial *x*, intermediate *z*, and body *x* axes, respectively. What is the rotation matrix describing this sequence of rotations from the inertial to body frames? You can express this as a product of separate rotation matrices – no need to multiply them together.
b. (7 points). If the rider can be represented by a cylinder with mass *M*, radius *R* and height *H*, derive her moment of inertia tensor about her center of mass in a principal axis system (do not just write it down). *Hint: use cylindrical coordinates.*
c. (8 points). The rider is spinning about her axis of symmetry (initially horizontal) at a rate Ω. In order to avoid hitting a fence, she applies a constant moment *M* about her transverse axis. What is the rate of change of her angular velocity (body-frame components)?

BONUS (+3): In 1-2 sentences, state one concept that you studied that wasn't asked on the exam and describe a dynamics problem where it's applicable.