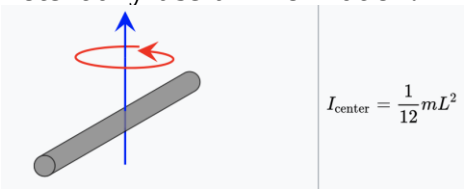
AA 242A Final Exam

December 13, 2019

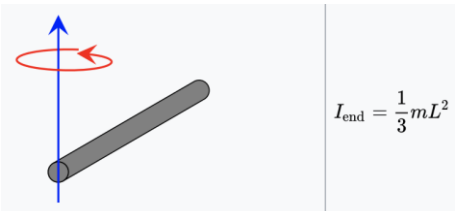
12:15 – 3:15 PM

- You may reference two pages of equations (front and back) in addition to the conservation, Lagrange and Euler sheets given in class.
- There are 100 possible points, split up into five problems, each with multiple parts. The value of each problem is indicated. You are to work all of the problems individually.
- Good luck!

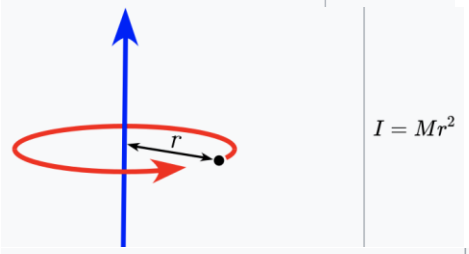
Potentially useful information:



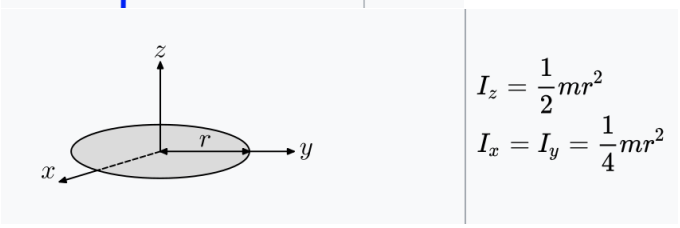
$$I_{\text{center}} = \frac{1}{12}mL^2$$



$$I_{\text{end}} = \frac{1}{3}mL^2$$



$$I = Mr^2$$



$$I_z = \frac{1}{2}mr^2$$

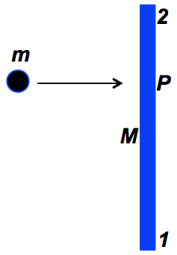
$$I_x = I_y = \frac{1}{4}mr^2$$

"Higher, further, faster…I'm kind of done with you telling me what I can't do"

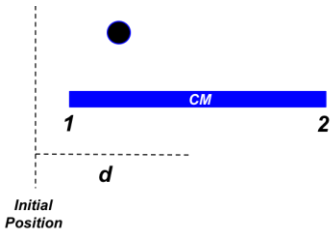
Problem 1 (20 points). Captain Marvel uses a Kree pistol to fire a projectile, which can be approximated as a point mass with mass m, upward from the surface of the earth at a latitude θ. Its initial velocity as seen by Captain Marvel is vertically upwards with speed v_o. The angular velocity of the earth's rotation is ω. Assume uniform gravity g.

a. (6 points). Neglecting the rotation of the earth, write expressions for the velocity and position of the projectile in terms of the initial speed v_o, g, and r_{Earth} (Earth radius).
b. (7 points). Now considering the rotation of the Earth using a rotating surface-fixed frame (south, east up), derive the total effective force on the projectile including gravity, centrifugal, and Coriolis terms in terms of $\vec{v}(t)$ and $\vec{r}(t)$.
c. (7 points). Use the trajectories derived in part a) as a first approximation to solve for an expression for the Coriolis force in terms of v_o. Will the projectile curve north/east/south/westward on the way up?

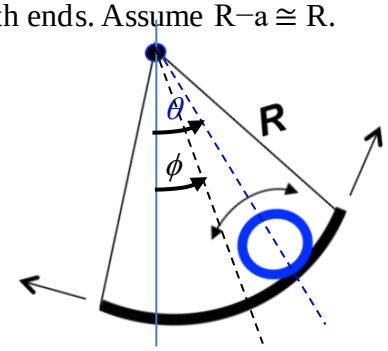
Problem 2 (20 points). One of the Kree lies **horizontally on a frictionless surface**. Approximate him as a thin uniform rod of mass M and length L. A point mass m moving with speed $\vec{v} = v\hat{x}$ makes an **elastic** collision with him at a point P. Ignore gravity.



a. (3 points). What is and is not conserved?
b. (10 points). At what point should the mass hit the Kree so that immediately after the collision, he has an instantaneous axis of pure rotation about his toes (point 1)?
c. (7 points). Now assume that the mass makes an elastic collision with him at a distance 3L/4 from point 1 where the mass has $\vec{v}' = v'\hat{x}$ after collision. At the instant the Kree first becomes aligned in the $\hat{x}$ direction, what is the distance (d) between the Kree's initial CM position and his final CM position? Assume $m = M$ and express your answer in terms of L.

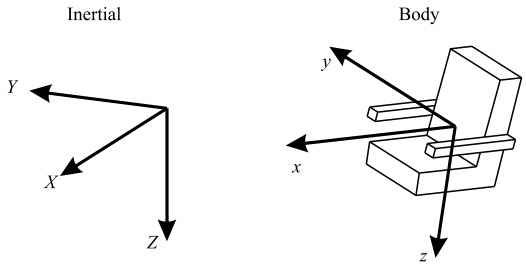


Problem 3 (20 points). Captain Marvel is swinging a hoop as she runs on the LA subway. Approximate the hoop as having a mass m and radius a that rolls without slipping on a swing under the influence of **gravity**. Assume the rolling of the hoop is characterized by an angular velocity ω, which is positive for counter-clockwise rotation. The angular displacement of the swing and hoop from equilibrium are ϕ and θ, respectively. The swing, also of mass m, is made from an arc section of radius R and is suspended from a frictionless pivot by massless ropes at both ends. Assume $R - a \cong R$.



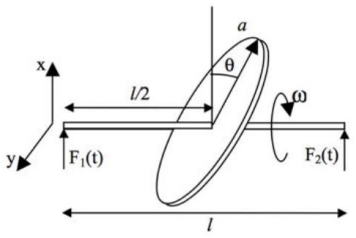
a. (5 points). Calculate the equation(s) of constraint assuming the hoop remains on the swing.
b. (15 points). Find the equations of motion. You may choose your favorite (but appropriate!) method.

Problem 4 (20 points). Captain Marvel sits in a rotating chair within her ship.



a. (8 points). The chair has a mechanism to rotate using an unconventional 3-1-2 Euler angle sequence with the angles ψ, θ, and ϕ about the inertial Z, intermediate x', and body y axes, respectively. What is the rotation matrix R describing the passive rotation of a vector from inertial to body coordinates?
b. (4 points). Gravity points down along the positive Z axis in the inertial frame. What is its representation in body coordinates, as a function of the Euler angles?
c. (8 points). At one point, the actuators internal to the chair are moving her chair with the Euler angles in degrees described by $\psi = 3t - 30$, $\theta = 2t - 15$, and $\phi = t - 15$. What is the angular velocity vector expressed in the inertial frame at $t = 30$?

Problem 5 (20 points). Captain Marvel is lifting weights. Model one of the weights as a homogeneous disk of mass m and radius a that is mounted in the middle of a massless rod of length l. The plane of the disk is tilted at an angle θ away from the normal to the rod. At $t = 0$, it begins to rotate around the axis of the rod with a **constant** angular velocity ω. The rod is supported at two ends with forces $F_1(t)$ and $F_2(t)$ such that it rotates without wobble. Ignore gravity.



a. (5 points). Calculate the principal moments of inertia of the disk.
b. (3 points). Is the disk a spherical, symmetric or asymmetric object? Define each one.
c. (3 points). Write down the components of the angular velocity in the body frame.
d. (3 points). Write down the body-coordinate-system components of the external torques needed to keep the disk spinning without wobble.
e. (6 points). Determine the components of the forces in the inertial frame that are needed to keep the disk spinning without wobble. Leave your expression in terms of (F_1 - F_2); do not try to resolve absolute F_1 and F_2 separately.

Choose one bonus!

BONUS (+2): What is one of the biggest challenges in space travel?

OR

BONUS (+2): What is one of the biggest challenges in air travel?