<u>**AA 242A Homework 1**</u>

Assigned: **Monday, September 29th, 2025**

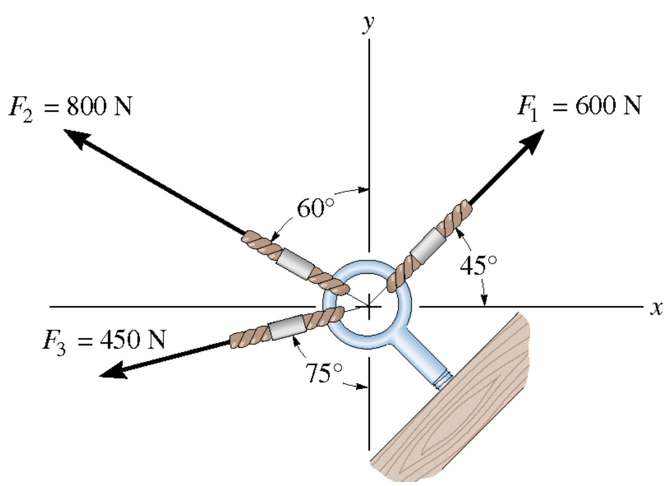
Due: **Monday, October 6th, 2025 at 11:59pm**

Submission on [Gradescope]

1. Consider the vector $A = A_x i + A_y j + A_z k$. Find the components A_1, A_2, A_3 of the vector $\boldsymbol{A}$ in a skewed coordinate system whose axes have directions specified by the following unit vector triad:

$$e_1 = i; \qquad e_2 = (i + j)/\sqrt{2}; \qquad e_3 = (i + k)/\sqrt{2}.$$

2. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



3. An airplane of mass m starts at a speed v_0 in level flight (lift = weight so no vertical motion). Its engine exerts a constant thrust T. The drag experienced by the aircraft is $F_D = bv^2$. You may find it useful to assume that initially, $T > F_D$.

a. Find the speed of the airplane as a function of time.

b. Find the eventual steady-state speed of the airplane.

c. If this were a rocket plane (like the Bell X-1) and you had to consider the mass loss of fuel, how would you change your formulation of the problem? (You don't have to actually solve this problem unless you really want to. Just describe your approach.)

4. MATLAB primer: This homework problem is intended so that you can begin to familiarize yourself with MATLAB. **Please submit your code for ALL MATLAB problems in this class.** Use MATLAB's *publish* feature on newer versions to export to PDF. Answers should be clearly indicated; this problem should be solved in MATLAB (even if it is easy for you to do by hand).

a. Find the unit vector in the direction of (1, -2, 4).

b. Find the angle (in radians) between the vectors (1, -2, 3) and (3, 1, 4).

c. Generate a plot of the level curves for the function

$$f(x,y) = (x^3 - 3x)(2y^3 - y)$$

for $-2 \leq x \leq 2$ and $-1 \leq y \leq 1$. On your plot of the level curves, draw vectors in the direction of the gradient of f.

Hint: look into MATLAB's *contourf* and *quiver* functions for plotting.

5. Matlab ODE45/ODE113 Primer:

In this class, we will be deriving the equations of motion of many systems. Very often, these equations are second order, coupled and nonlinear Ordinary Differential Equations (ODEs) which are difficult (if not impossible) to solve analytically. Matlab's ODE45 and ODE 113 are very useful packages that numerically integrate ODEs. This problem is intended for you to familiarize yourself with these packages which will be useful for future problem sets.

For this problem, your choice of ODE45 vs ODE113 does not matter. ODE45 is more general-purpose, whereas ODE113 is preferable for orbital mechanics problems where the solutions are smooth but high precision is required (i.e. for a 2-body problem, error must not cause the solution to spiral inward or outward). For more information, see the MATLAB documentation:

https://blogs.mathworks.com/loren/2015/09/23/ode-solver-selection-in-matlab/

Using ODE45 or ODE113, numerically integrate the following set of equations:

$$\ddot{x}_1 = -ax_1 + b(x_2 - x_1)$$
$$\ddot{x}_2 = -c(x_2 - x_1)$$

where the dots indicate time derivatives (this is typically the set of equations one arrives at for coupled oscillating systems, like a set of masses and springs). Solve these equations from t=0 to t=100 for the following initial conditions:

$$x_1(0) = 1, \dot{x}_1(0) = 0$$
$$x_2(0) = 1, \dot{x}_2(0) = 0$$

Solve the equations for 2 cases and plot x_1, x_2 and x_1+x_2 vs. time:

a. a=4, b=1, c=7.5

b. a=1, b=1, c=1.5

Notice anything interesting in the second case? Try changing initial conditions for this case. What happens?

Please submit MATLAB code for all coding problems.

6. This problem is the beginning of a 5-week "thought experiment" where you will identify and solve a "Classical Dynamics" problem of your choosing. For this week, describe in words and draw a picture of a dynamics problem that you recognize right now in your immediate vicinity (e.g. your desk, the book on your desk, a scene outside the nearest window). Be creative!