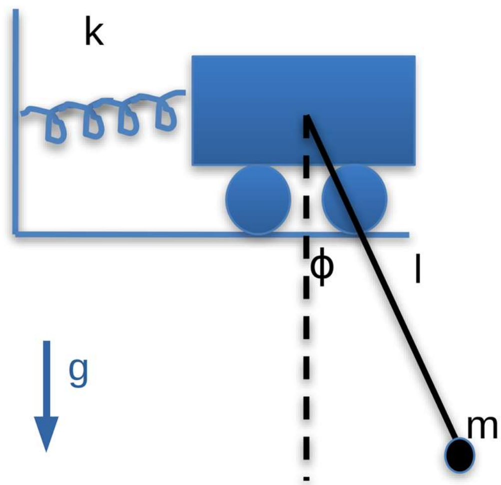
AA 242A Homework 5

Assigned: **Thursday, October 27th, 2022**

Due: **Thursday, November 3rd, 2022**

1. A pendulum of mass *m* is attached to a cart of mass *M* on a spring. Take the rest length of the spring to be zero.



a. Find EOMs by Lagrange method and discuss the physical significance any of them might have

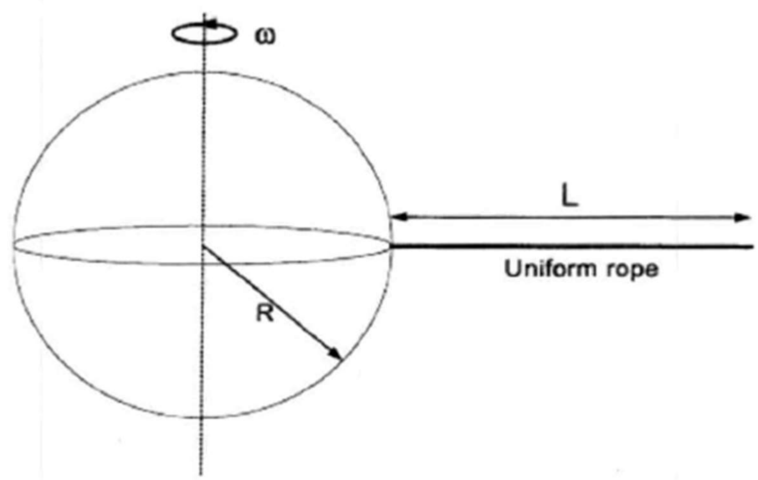
b. Find the frequency of the oscillating pendulum for small motions about the vertical when the cart is fixed.

c. Use Matlab to find the frequency of the oscillating pendulum when the cart is fixed. DO NOT simplify the EOM by making the small angle assumption. Plot ϕ vs. time, and find the frequency of oscillation for the following initial conditions: $\phi'(0) = 0$, $\phi(0) = 5^\circ, 10^\circ, 15^\circ, 30^\circ, 60^\circ$. Compare the frequencies of oscillation found to the result of part b (you can do this graphically). For what $\phi(0)$ is the small angle assumption accurate? Use numerical values: $m = 1kg$, $l = 2$ meters, $k = 5\ N/m$, $M = 2\ kg$.

d. Use MATLAB to simulate the full system when the cart is NOT fixed. Use initial cart conditions: $x(0) = 0$, $x'(0) = 0.5m/s$. Plot ϕ vs. time for the ϕ initial conditions from part c. Compare these plots to those of part (c). Use same numerical values as part (c).

2. As early as 1895, it was proposed that we could build something called a "space elevator" to get objects into orbit about the Earth. This is a free hanging rope in stationary orbit

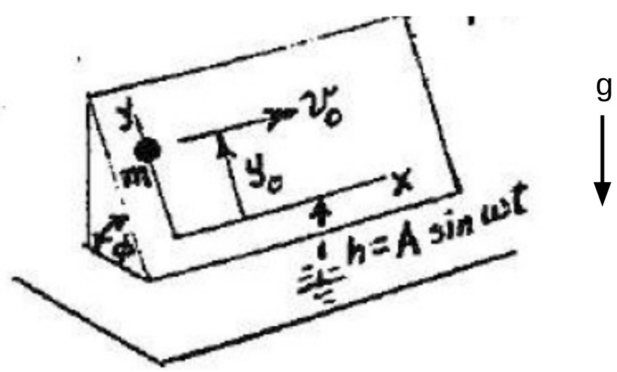
around the earth above the equator. You could send an elevator up this rope to launch objects into space at less cost than required for shuttle flights. Imagine such a rope just reached the Earth's surface. Assume the rope has length L, mass m, and that the Earth has radius R and mass M and rotates at angular velocity ω.



a. Find an expression for the tension in the rope as a function of height, y, off the Earth's surface.

b. What length, L, allows the rope to hang freely (i.e. without being attached to the Earth's surface)?

3. A particle of mass m is constrained to move without friction in the xy plane. The x axis remains horizontal and the y axis is inclined at a constant angle ϕ with the horizontal. In addition, the xy plane translates vertically such that the height of the x axis above a given reference level is $h = A\sin(\omega t)$.

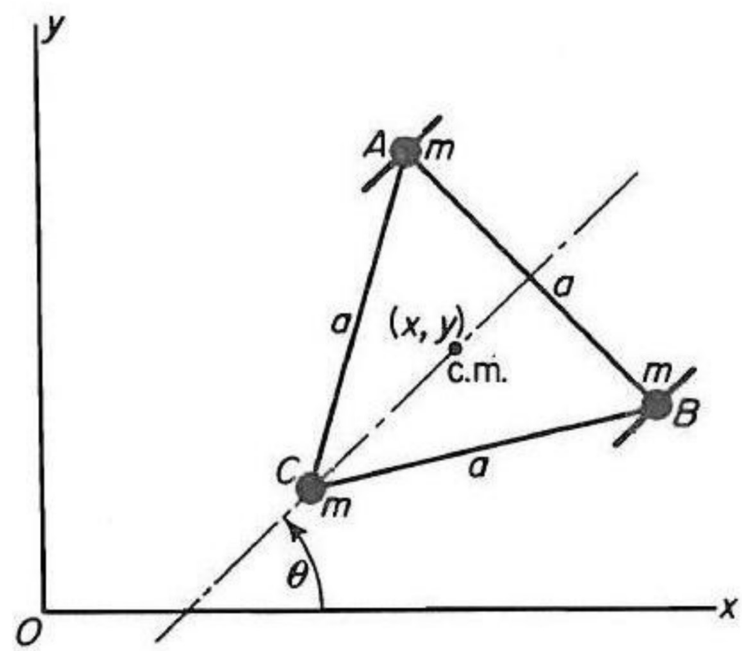


If the particle has the initial conditions, $y(0) = y_0$, $\dot{y}(0) = 0$, $x(0) = x_0$, $\dot{x}(0) = v_0$, solve for its position (x,y) as a function of time using the following:

a. Force balance.

b. Lagrange's equation with Lagrange multipliers.

4. Three particles, each of mass m, lie in a horizontal plane at the corners of an equilateral triangle. Massless rods rigidly connect the particles and the whole system can roll without slipping on a horizontal floor. The wheels at A and B revolve about a common axle AB and permit the motion of these two particles in a direction perpendicular to AB. A castered wheel at C allows its motion in any direction consistent with the other constraints.



Using coordinates (x,y) to specify the CM position, and coordinate θ to specify the orientation of the plane of the wheel at A and B with respect to the horizontal x-axis, obtain the equations of motion using the method of Lagrange multipliers.