AA 242A Midterm Exam

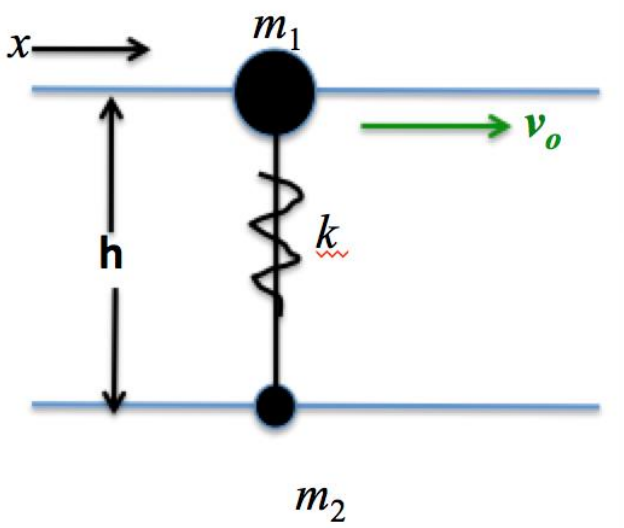
October 27, 2020

12:30 – 2:00 PM

- This exam is open NOTES/open TEXTBOOK.
- There are three problems, each with multiple parts, and 100 possible points; the value of each problem is indicated. You are to work all of the problems individually.
- The completed exam is to be submitted by 2:00 PM.
- Good luck!

"You can use logic to justify almost anything." – Capt. Janeway

Problem 1 (30 points). Janeway and Torres are fixing the warp core. They are sliding without friction on parallel fixed horizontal wires, separated by a distance h. Approximate Janeway as a particle of mass $m_1 = 2m$, and Torres as a particle of mass $m_2 = m$. A spring of stiffness k and unstretched length h connects Janeway and Torres. Define their positions on the wires as x_1 and x_2, respectively. Assume Janeway has an initial velocity v_o and that Torres is initially motionless; ignore gravity.

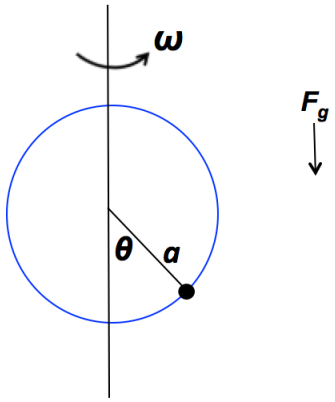


a. (15 points) What is and is not conserved in this system? Justify your answer.

b. (15 points) Find the maximum velocity of Torres.

Problem 2 (35 points)

Tuvok has developed a new *effective* weapon to battle Species 8472. His weapon can be approximated as a point particle of mass m that slides without friction under the influence of gravity on a rotating wire hoop of radius a. The hoop rotates with a constant angular velocity ω about its vertical axis.



a. (4 points) Write down the constraint equation(s).
b. (6 points) Write down the generalized coordinate(s). How many degrees of freedom are there?
c. (3 points) Write an expression for the angular velocity of the particle with respect to the inertial frame.
d. (10 points) Write down the Lagrangian of the system.
e. (10 points) Find the EOM(s).
f. (2 points) In two sentences or less, explain what happens to your EOM if ω is 0, and therefore what the term with ω represents.

Problem 3 (35 points)

(Same setup as for Problem 2)

g. (4 points) Draw a FBD in the inertial frame.
h. (25 points) Find the EOM(s) using Force Balance. (Use your result from part c.)
i. (6 points) Draw a FBD in the rotating frame.