

# Information Retrieval 1

## Term-based Retrieval

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# Document representation and matching

Evaluation

Document  
representation  
& matching

Conversational  
search

Learning to rank

IR—user  
interaction

Recommender  
systems

# Outline

- 1 Vector space model
- 2 Language modeling in IR
- 3 BM25

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# Documents as vectors

vocabulary of all documents

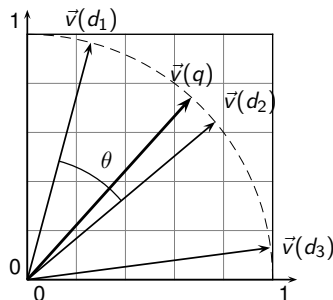
a document = a vector.  
eg. doc1 = (1, 1, 1, 0, 1, 1, 1, ...)

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
Anthony	1	1	0	0	0	1	
Brutus	1	1	0	1	0	0	
Caesar	1	1	0	1	1	1	
Calpurnia	0	1	0	0	0	0	
Cleopatra	1	0	0	0	0	0	
mercy	1	0	1	1	1	1	
worser	1	0	1	1	1	0	
...							

1: this word occurs  
0: this word does not occur

roduction to Information Retrieval"

# Match using cosine similarity



similarity between a word in doc and a word in query

$$\text{sim}(d, q) = \cos(\vec{v}(d), \vec{v}(q)) = \frac{\vec{v}(d) \cdot \vec{v}(q)}{\|\vec{v}(d)\| \cdot \|\vec{v}(q)\|}$$

$v$ 的绝对值=size of vocab

$$= \frac{\sum_{i=1}^{|V|} d_i \cdot q_i}{\sqrt{\sum_{i=1}^{|V|} d_i^2} \cdot \sqrt{\sum_{i=1}^{|V|} q_i^2}}$$

Manning et al., "Introduction to Information Retrieval"

# Term frequency

word anthony has  
occured 157 times in  
doc1

and  
Cleopatra

		Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
Anthony	157	73	0	0	0	1	
Brutus	4	157	0	2	0	0	
Caesar	232	227	0	2	1	0	
Calpurnia	0	10	0	0	0	0	
Cleopatra	57	0	0	0	0	0	
mercy	2	0	3	8	5	8	
worser	2	0	1	1	1	5	

...

Manning et al., "Introduction to Information Retrieval"

# Term frequency

we can use either raw  
tf, or log tf.  
depends on the  
project

Raw term frequency	$tf(t, d)$	
Log term frequency	$\begin{cases} 1 + \log tf(t, d) \\ 0 \end{cases}$	$\begin{matrix} \text{if } tf(t, d) > 0 \\ \text{otherwise} \end{matrix}$



# Inverse document frequency

分子分母互换位置

$$idf(t) = \log \frac{N}{df(t)}$$

total nb of doc

the nb of doc that contain term (or word)  $t$

- $df(t)$  – document frequency of term  $t$
- $N$  – total number of documents in a collection

# Inverse document frequency

the word with low tf has high idf. this is the effect of inverted df

10000 docs contain fly

Term	$df(t)$	$idf(t)$
calpurnia	1	6
animal	100	4
sunday	1000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

for  $N = 1,000,000$  and  $\log_{10}$

Manning et al., "Introduction to Information Retrieval"

# TF-IDF

$$\text{TF-IDF}(t, d) = tf(t, d) \cdot idf(t)$$

- Term frequency

- $tf(t, d)$

- $\begin{cases} 1 + \log tf(t, d) & \text{if } tf(t, d) > 0 \\ 0 & \text{otherwise} \end{cases}$

- Inverse document frequency

- $\log \frac{N}{df(t)}$

- $\max\{0, \log \frac{N - df(t)}{df(t)}\}$

these two are standard tf and standard idf

# Vector space model summary

- Documents and queries as vectors
- Match using cosine similarity
- Weights can be
  - ① binary
  - ② term frequency
  - ③ TF-IDF

# Outline

- 1 Vector space model
- 2 Language modeling in IR
  - Method
  - Smoothing
- 3 BM25

# Outline

- 2 Language modeling in IR
  - Method
  - Smoothing

# Language model

A statistical language model is a probability distribution over sequences of words.

- Given a sequence of length  $m$
- A language model assigns probability  $P(w_1, \dots, w_m)$  to this sequence
- Unigram language model

$$P(w_1, \dots, w_m) = P(w_1) \dots P(w_m)$$

- Bi-gram language model

$$P(w_1, \dots, w_m) = P(w_1)P(w_2 \mid w_1)P(w_3 \mid w_2) \dots P(w_m \mid w_{m-1})$$

[https://en.wikipedia.org/wiki/Language\\_model](https://en.wikipedia.org/wiki/Language_model)

# Unigram language model example

Model $M_1$		Model $M_2$	
the	0.2	the	0.15
a	0.1	a	0.12
frog	0.01	frog	0.0002
toad	0.01	toad	0.0001
said	0.03	said	0.03
likes	0.02	likes	0.04
that	0.04	that	0.04
dog	0.005	dog	0.01
cat	0.003	cat	0.015
monkey	0.001	monkey	0.002
...	...	...	...

Manning et al., "Introduction to Information Retrieval"



# Documents as distributions

- Unigram language model

$$P(t \mid M_d) = \frac{tf(t, d)}{dl(d)}$$

- A document is a multinomial distribution over words
- If some vocabulary terms do not appear in document  $d$ , then  $P(t \mid M_d) = 0$
- This is addressed by smoothing

# Match using query likelihood model (QLM)

- Likelihood of a document given a query

$$P(d | q) = \frac{P(q | d)P(d)}{P(q)}$$

- The prior distribution over queries  $P(q)$  does not affect matching for a particular query

$$P(d | q) \stackrel{\text{rank}}{=} P(q | d)P(d)$$

- Usually, the prior distribution over documents  $P(d)$  is assumed to be uniform

$$P(d | q) \stackrel{\text{rank}}{=} P(q | d) = P(q | M_d)$$

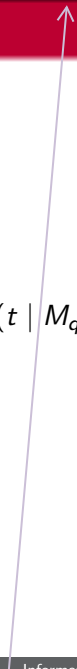
- “Bag of words” assumption: terms are independent

$$P(q | M_d) = \prod_{t \in q} P(t | M_d) = \prod_{t \in q} \frac{tf(t, d)}{dl(d)}$$

how many times the word  $t$  occurs  
in doc  $d$

len of a doc

# Match using KL-divergence

$$KL(M_d \| M_q) = \sum_{t \in V} P(t | M_q) \log \frac{P(t | M_q)}{P(t | M_d)}$$


# Outline

- 2 Language modeling in IR
  - Method
  - Smoothing

# Jelinek-Mercer smoothing

the probability estimate for a word present in the document

smoothing weight  
 $= 1 - \lambda$   $\lambda = 0.1$

$$P_s(t | M_d) = \lambda P(t | M_d) + (1 - \lambda) P(t | M_c)$$

$$= \lambda \frac{tf(t, d)}{dl(d)} + (1 - \lambda) \frac{cf(t)}{cl}$$

how many times term  $t$  occurs in all documents

vocabulary size

- $cf(t)$  – collection frequency of term  $t$
- $cl$  – collection length

# Dirichlet smoothing

- A unigram language model can be seen as a multinomial distribution over words  $\mathcal{L}_d(n_1, \dots, n_k \mid p_1, \dots, p_k)$ 
  - $n_i = tf(t_i, d)$
  - $p_i = P(t_i \mid M_d)$
- The conjugate prior for multinomial is the Dirichlet distribution  $P_{prior}(p_1, \dots, p_k; \alpha_1^{pr}, \dots, \alpha_k^{pr})$ 
  - $\alpha_i^{pr} = \mu P(t_i \mid M_c)$
  - $\mu$  is a smoothing parameter ( $\lambda = \frac{dl}{dl + \mu}$ )
- The posterior is the Dirichlet distribution with parameters  $\alpha_i^{po} = n_i + \alpha_i^{pr} = tf(t_i, d) + \mu P(t_i \mid M_c)$
- Dirichlet smoothing

$$P_s(t \mid M_d) = \frac{tf(t_i, d) + \mu P(t_i \mid M_c)}{dl(d) + \mu}$$

# Language modeling for IR summary

- Documents and queries as distributions
- Match using QLM or KL-divergence
- Smoothing
  - Jelinek-Mercer smoothing
  - Dirichlet smoothing

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# BM25

$$BM25 = \sum_{t \in q} \log \left[ \frac{N}{df(t)} \right] \cdot \frac{(k_1 + 1) \cdot tf(t, d)}{k_1 \cdot \left[ (1 - b) + b \cdot \frac{dl(d)}{dl_{avg}} \right] + tf(t, d)}$$

- $k_1$ ,  $b$  – parameters
- $dl(d)$  – length of document  $d$
- $dl_{avg}$  – average document length

$k_1$  controls  $tf$   
 $b$  controls doc len

if 10 doc in total.  
then  $dl_{avg} = (dl(doc1) + \dots + dl(doc10)) / 10$

# BM25

BM25 is a  
weighting scheme  
in inverted index

IDF = inverse doc  
frequency

$$BM25 = \sum_{t \in q} \log \left[ \frac{N}{df(t)} \right] \cdot \frac{(k_1 + 1) \cdot tf(t, d)}{k_1 \cdot \left[ (1 - b) + b \cdot \frac{dl(d)}{dl_{avg}} \right] + tf(t, d)}$$

BM25 = sum of IDF

- What if  $k_1 \in \{0, \infty\}$ ?
- What of  $b \in \{0, 1\}$ ?
- What if  $tf(t, d)$  is small/large?  $k_1 \in [1.2, 2]$ ,  $b = 0.75$

# BM25 for long queries

$$BM25 = \sum_{t \in q} \log \left[ \frac{N}{df(t)} \right] \cdot \frac{(k_1 + 1) \cdot tf(t, d)}{k_1 \cdot \left[ (1 - b) + b \cdot \frac{dl(d)}{dl_{ave}} \right] + tf(t, d)} \cdot \frac{(k_3 + 1)tf(t, q)}{k_3 + tf(t, q)}$$

# Experimental comparison

Collection	Method	Parameter	MAP	R-Prec.	Prec@10
Trec8 T	Okapi BM25	Okapi	0.2292	0.2820	0.4380
	JM	$\lambda = 0.7$	0.2310 (p=0.8181)	0.2889 (p=0.3495)	0.4220 (p=0.3824)
	Dir	$\mu = 2,000$	<b>0.2470</b> (p=0.0757)	0.2911 (p=0.3739)	<b>0.4560</b> (p=0.3710)
	Dis	$\delta = 0.7$	0.2384 (p=0.0686)	0.2935 (p=0.0776)	0.4440 (p=0.6727)
	Two-Stage	auto	0.2406 (p=0.0650)	<b>0.2953</b> (p=0.0369)	0.4260 (p=0.4282)

Figure: TREC-8 Newswire, ad-hoc track, queries 401–450, title-only

G. Bennett, "A Comparative Study of Probabilistic and Language Models for Information Retrieval"

# Experimental comparison

Collection	Method	Parameter	MAP	R-Prec.	Prec@10
TREC-2001 T	Okapi BM25	Okapi	0.1522	0.2056	0.2918
	JM	$\lambda = 0.7$	0.1113 (p=0.0003)	0.1505 (p=0.0037)	0.2122 (p=0.0003)
	Dir	$\mu = 2,000$	<b>0.1774</b> (p=0.0307)	<b>0.2238</b> (p=0.3236)	<b>0.3184</b> (p=0.3165)
	Dis	$\delta = 0.7$	0.1370 (p=0.0511)	0.1906 (p=0.053)	0.2653 (p=0.1348)
	Two-Stage	auto	0.1441 (p=0.2963)	0.1934 (p=0.3992)	0.2898 (p=0.8962)

Figure: TREC-2001 Web data, ad-hoc track, queries 501–550, title-only

G. Bennett, "A Comparative Study of Probabilistic and Language Models for Information Retrieval"

# Content-based retrieval summary

- Vector space model
  - Documents and queries as vectors
  - Match using cosine similarity
- Language modeling in IR
  - Documents and queries as distributions
  - Match using QLM or KL-divergence
- BM25

# Materials

Query likelihood



- Manning et al., Chapters 6, 9, 11, 12
  - Croft et al., Chapter 7
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