Machine Learning 1

Lecture 11.3 - Kernel Methods
Support Vector Machines - Maximum Margin Classifier

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(Bishop 7.1.0)

Slide credits: Rianne van den Berg

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Support vector machines

- Kernel method with sparse solutions:
  - prediction for new inputs depend only on kernel function evaluated at a **subset** of the training points
- Applications:
  - Classification
  - Regression
  - novelty detection/anomaly detection

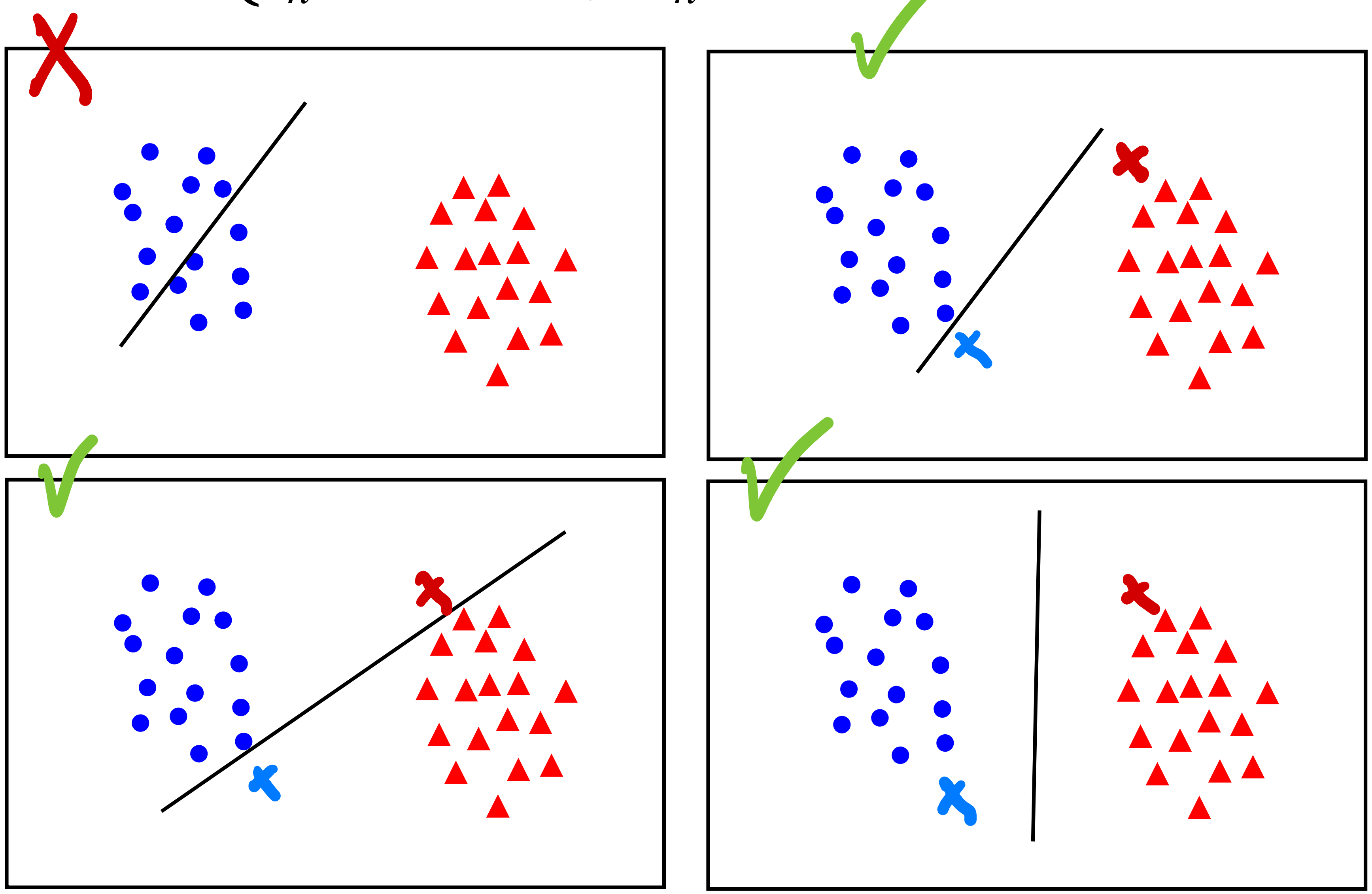
Primal viewpoint $y(\underline{x}, \underline{w}) = \underline{w}^T \underline{\phi}(\underline{x})$

Dual viewpoint $y(\underline{x}, \underline{a}) = \sum_{n}^{N} a_n k(\underline{x}, \underline{x}_n)$

- Convex optimization problem, any local solution is at global optimum!
- No good probabilistic interpretation
- Today: SVM for binary classification -> maximum margin classifier!

Linearly separable dataset

- Linear classifier: $y(\mathbf{x_n}) = \mathbf{w}^t \mathbf{x}_n + b$
- Classification: $\begin{cases} t_n = +1 & \text{if } \; y(\mathbf{x}_n) \geq 0 \\ t_n = -1 & \text{if } \; y(\mathbf{x}_n) < 0 \end{cases}$



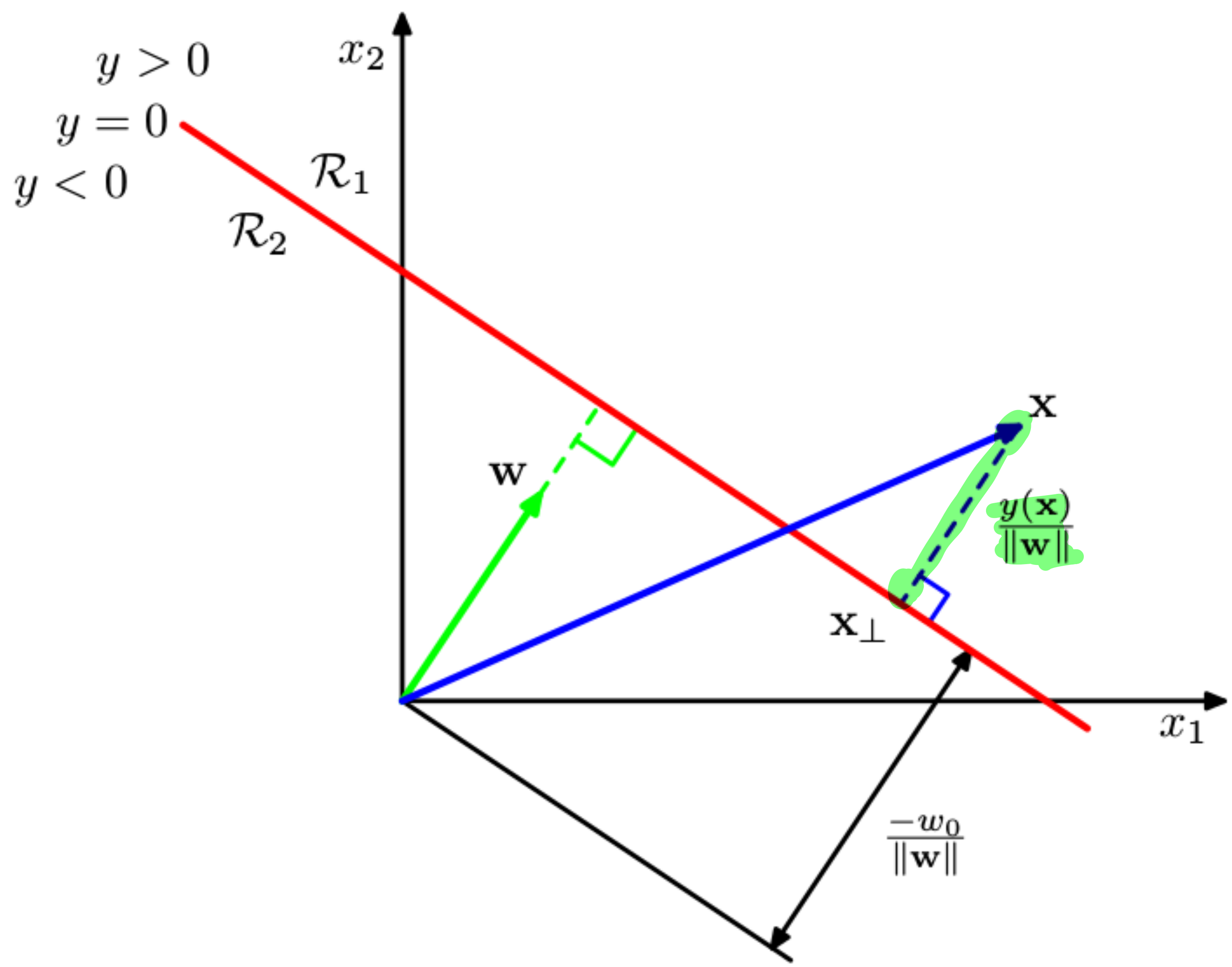
- Maximum Margin: most stable under perturbations of the input

Linearly Separable Dataset

- If $\mathbf{x}'$ lies on decision boundary: $y(\mathbf{x}') = \mathbf{w}^T\mathbf{x}' + b = 0$

- Recall: distance from $\mathbf{x}$ to decision boundary is

$$r = \frac{|y(\mathbf{x}_n)|}{||\mathbf{w}||} = \frac{t_n y(\mathbf{x}_n)}{||\mathbf{w}||}$$



- For correct classification:

$$y(\mathbf{x}_n) \geq 0 \text{ if } t_n = +1$$

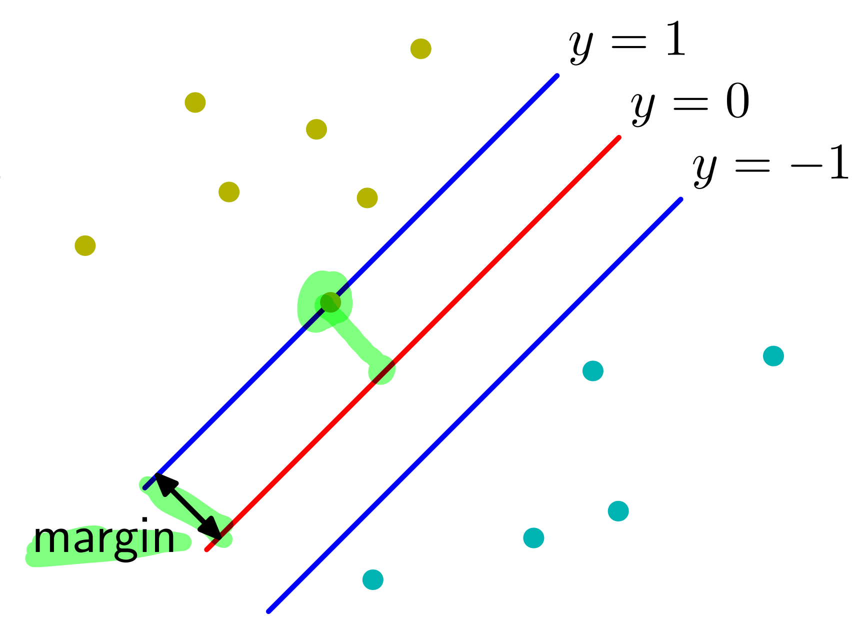
$$y(\mathbf{x}_n) < 0 \text{ if } t_n = -1$$

- So for all $n = 1,\ldots,N$

$$t_n y(\mathbf{x}_n) \geq 0$$

Maximum Margin Classifier

- Margin: perpendicular distance from decision boundary to closest point $\mathbf{x}_n$

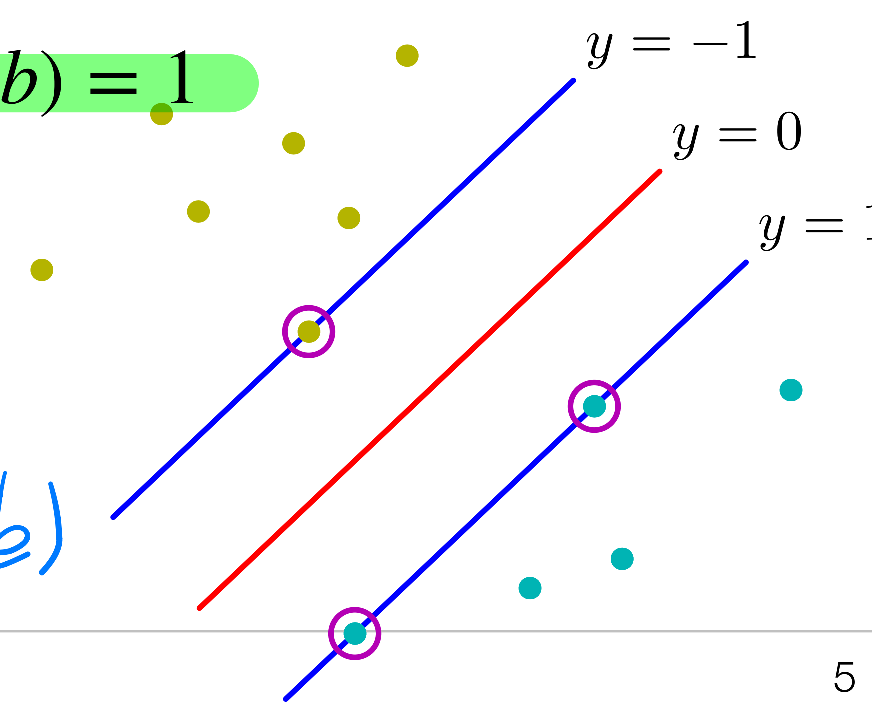


- For all data points distance to decision boundary is

$$r = \frac{t_n y(\mathbf{x}_n)}{||\mathbf{w}||} = \frac{t_n(\mathbf{w}^T\mathbf{x}_n + b)}{||\mathbf{w}||}$$

- Margin: $\min_n \frac{t_n(\mathbf{w}^T\mathbf{x}_n + b)}{||\mathbf{w}||} = \min_n \frac{t_n(\kappa\mathbf{w}^T\mathbf{x}_n + \kappa b)}{||\kappa\mathbf{w}||}$

- For point closest to decision boundary $t_n(\mathbf{w}^T\mathbf{x}_n + b) = 1$



- For all data points: $t_n(\mathbf{w}^T\mathbf{x}_n + b) \geq 1$

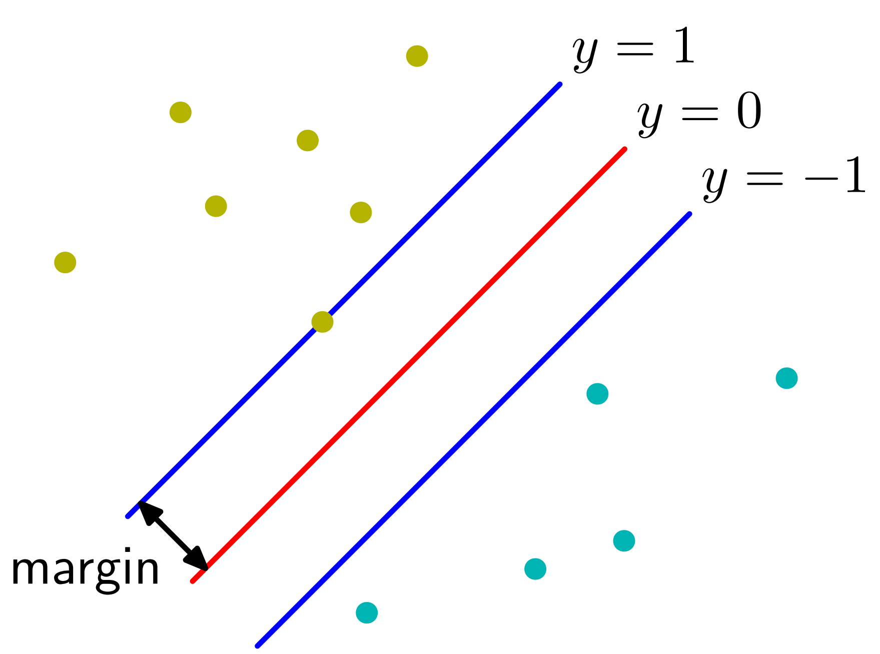
- Maximum margin classifier: $\max_{\underline{w},\,\underline{b}} \text{margin}(\underline{w}, \underline{b})$

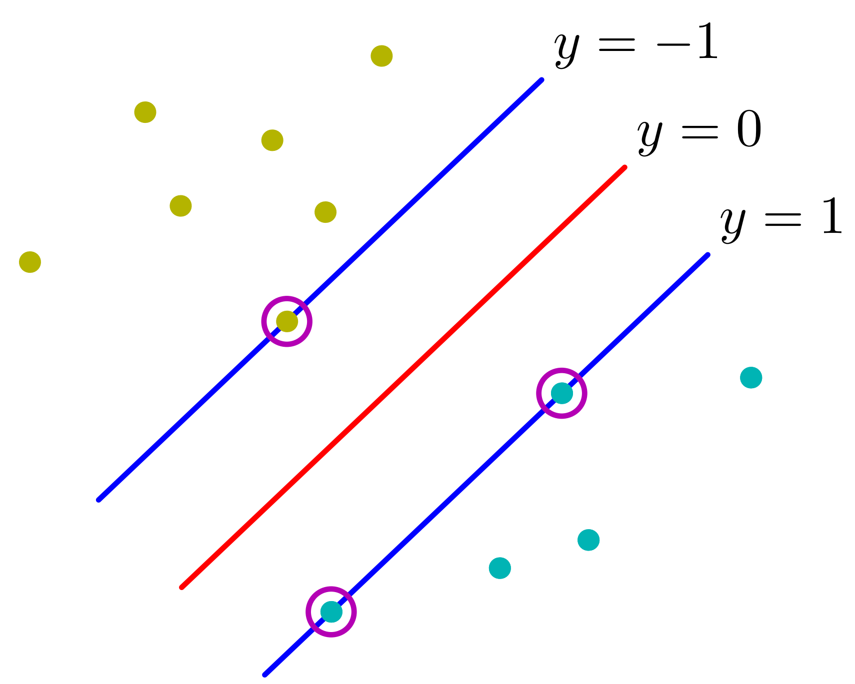
Maximum Margin Classifier

- For all data points distance to decision boundary is

$$r = \frac{t_n y(\mathbf{x}_n)}{||\mathbf{w}||} = \frac{t_n(\mathbf{w}^T \mathbf{x}_n + b)}{||\mathbf{w}||}$$

- For point closest to decision boundary $t_n(\mathbf{w}^T \mathbf{x}_n + b) = 1$
- For all data points: $\boxed{t_n(\mathbf{w}^T \mathbf{x}_n + b) \geq 1}$
- Size of the margin: $\dfrac{1}{||\mathbf{w}||}$ ← maximize





Maximum Margin Classifier

- Size of the margin: $\dfrac{1}{||\mathbf{w}||}$
- For all data points: $t_n(\mathbf{w}^T\mathbf{x}_n + b) \geq 1$
- **Maximizing the margin**:

$$\arg\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 \textit{ subject to } N \textit{ constraints } t_n(\mathbf{w}^T\mathbf{x}_n + b) \geq 1$$

- Quadratic programming problem!