Machine Learning 1

Lecture 5.3 - Supervised Learning
Bayesian Linear Regression - Approximating the Model Evidence

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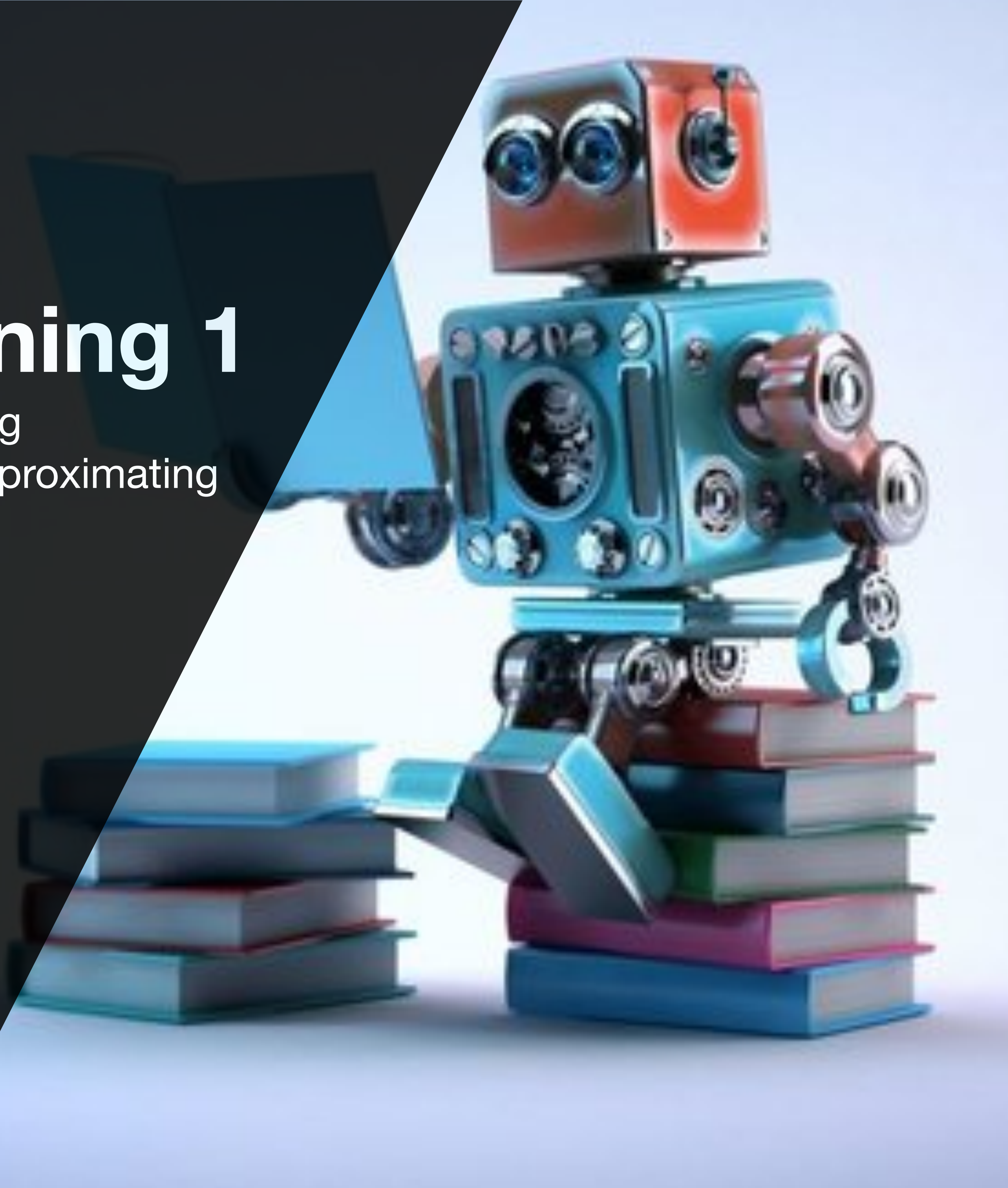
(Bishop 3.5.0)



Slide credits: Rianne van den Berg

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Model Evidence for Linear Basis Models

- Full Bayesian treatment to model evidence:

$$p(\mathbf{t}|\mathbf{X}, \mathcal{M}_i) = \iiint \underline{p(\mathbf{t}|\mathbf{X}, \mathbf{w}, \beta, \mathcal{M}_i)p(\mathbf{w}|\mathbf{t}, \mathbf{X}, \alpha, \mathcal{M}_i)}p(\alpha, \beta|\mathbf{t}, \mathbf{X}, \mathcal{M}_i)\mathrm{d}\mathbf{w}\mathrm{d}\alpha\mathrm{d}\beta$$

$$= \iint p(\underline{t}|X, \alpha, \beta, \mathcal{M}_i)\, \underbrace{p(\alpha, \beta|\underline{t}, X, \mathcal{M}_i)}_{\text{assume sharply peaked at } \alpha^*, \beta^*}\, d\alpha\, d\beta$$

- Empirical Bayes / evidence approximation:

done iterative way (Bishop 3.5.2)

$$p(\mathbf{t}|\mathbf{X}, \mathcal{M}_i) \approx p(\underline{t}|X, \alpha^*, \beta^*, \mathcal{M}_i)$$

$$\alpha^*, \beta^* = \underset{\alpha,\beta}{\arg\max}\; p(\alpha, \beta|\underline{t}, X, \mathcal{M}_i)$$

$$\approx \underset{\alpha,\beta}{\arg\max}\; p(\underline{t}|X, \alpha, \beta, \mathcal{M}_i)\, \cancel{p(\alpha, \beta, \mathcal{M}_i)}$$

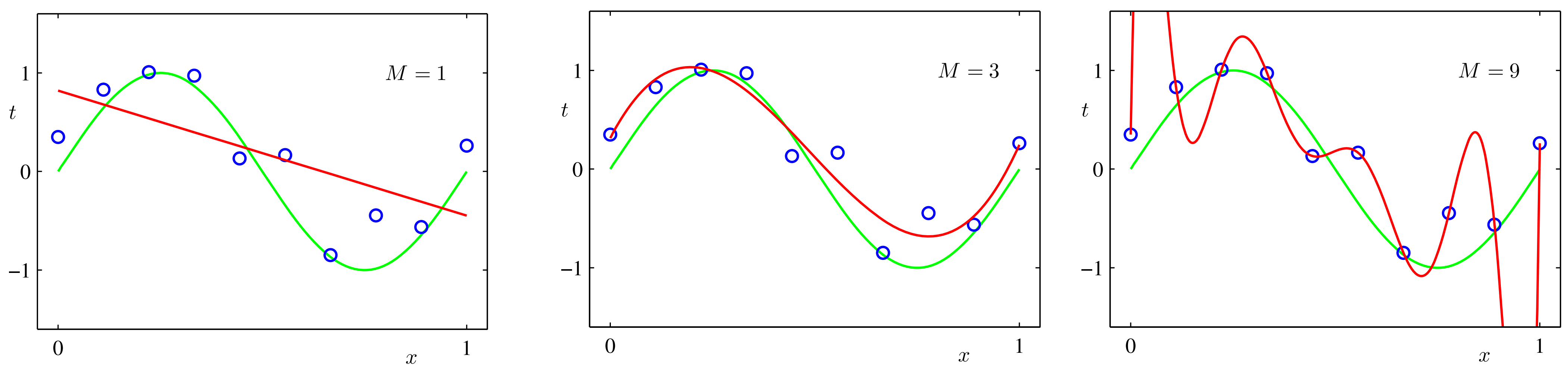
model selection

- Predictive distribution:

$$p(t'|\mathbf{x}', \mathbf{t}, \mathbf{X}, \mathcal{M}_i) \approx p(t'|\underline{x}', \underline{t}, X, \alpha^*, \beta^*, \mathcal{M}_i)$$

Polynomial Regression: Choosing M

Model complexity: trade-off between model fit and model complexity



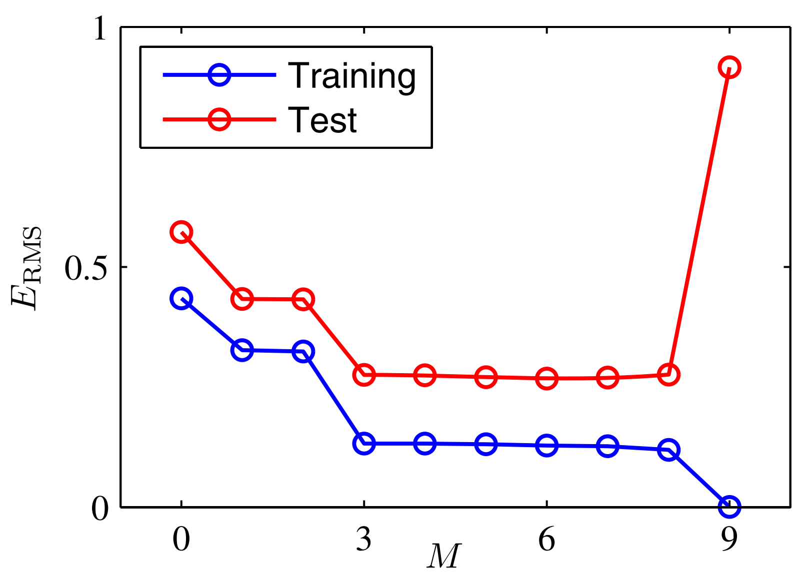


Figure: E_{rmse} (Bishop 1.5)

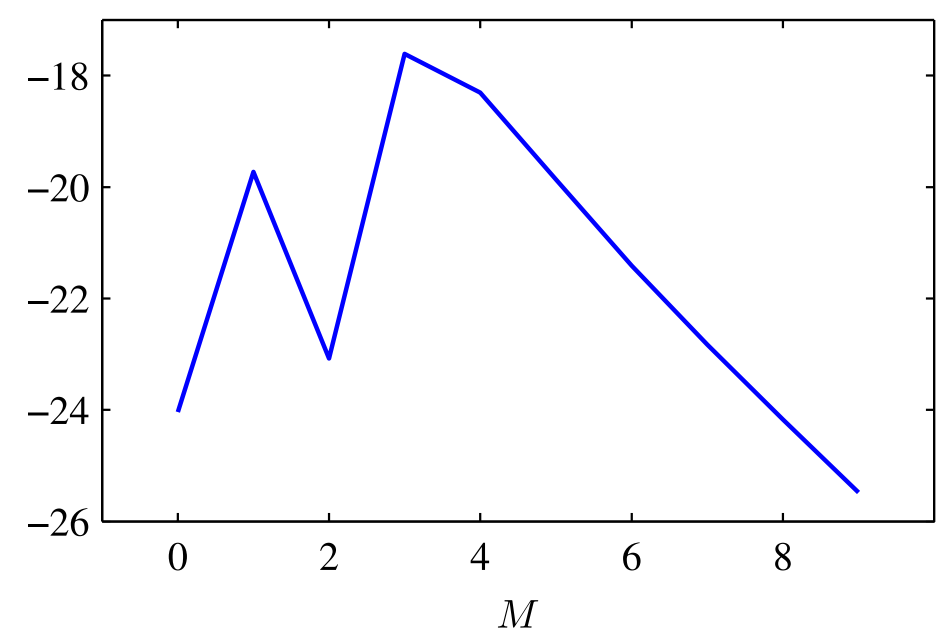


Figure: log model evidence (Bishop 3.14)