Machine Learning 1

Lecture 5.3 - Supervised Learning
Classification - Decision Regions

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(Bishop 1.5, 4.1)

Slide credits: Rianne van den Berg

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Classification through decision regions

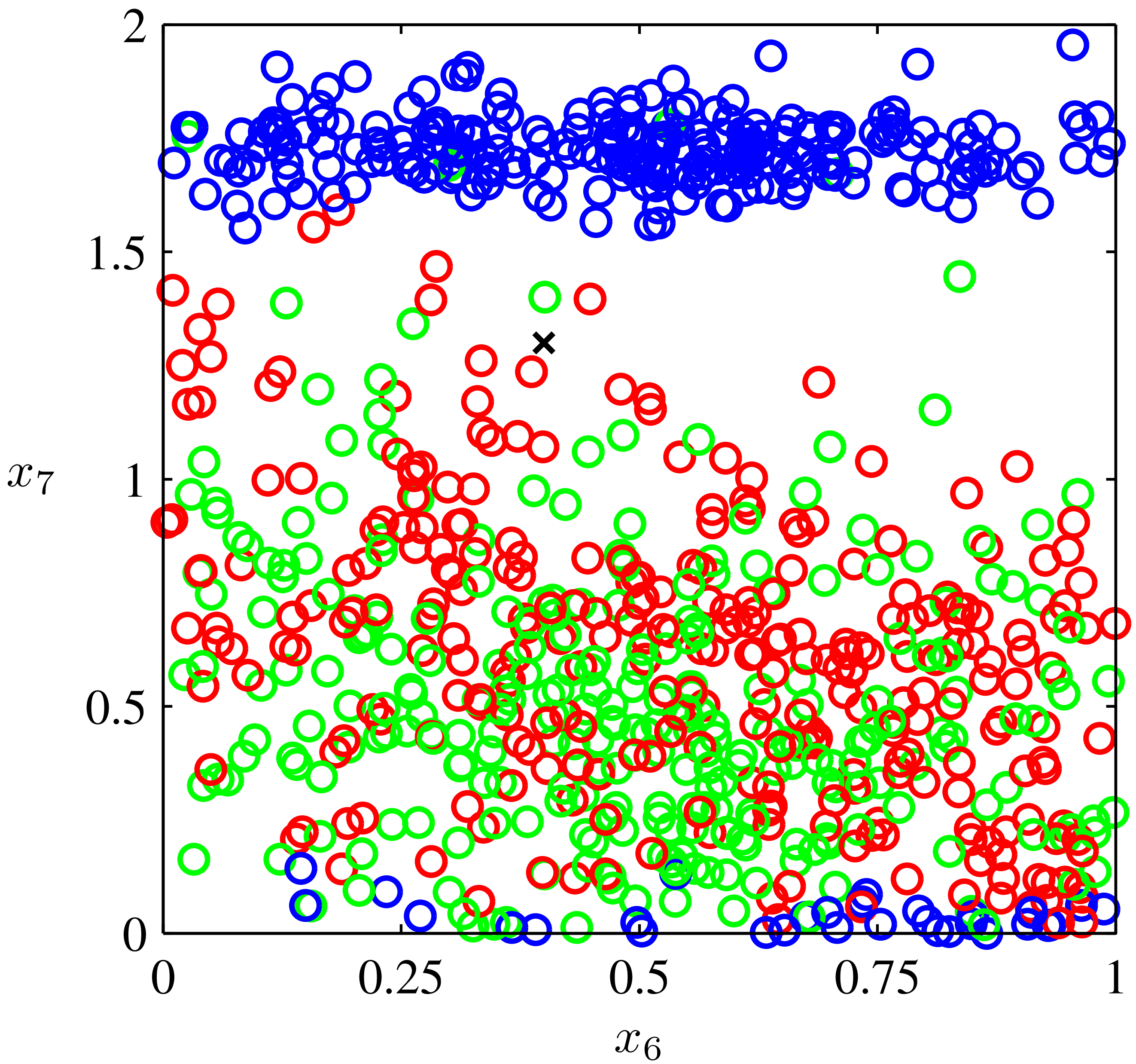
- Input: $\mathbf{x} = (x_1, ..., x_D)^T$
- Target: $t \in \{C_1, C_2, ..., C_K\}$
    - 2-class targets: $t = C_1$, $t = C_2$ $\Longleftrightarrow$ $t = 0$, $t = 1$
    - Multi-class targets e.g. $K = 5$, $t = C_3$ $\Longleftrightarrow$ $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

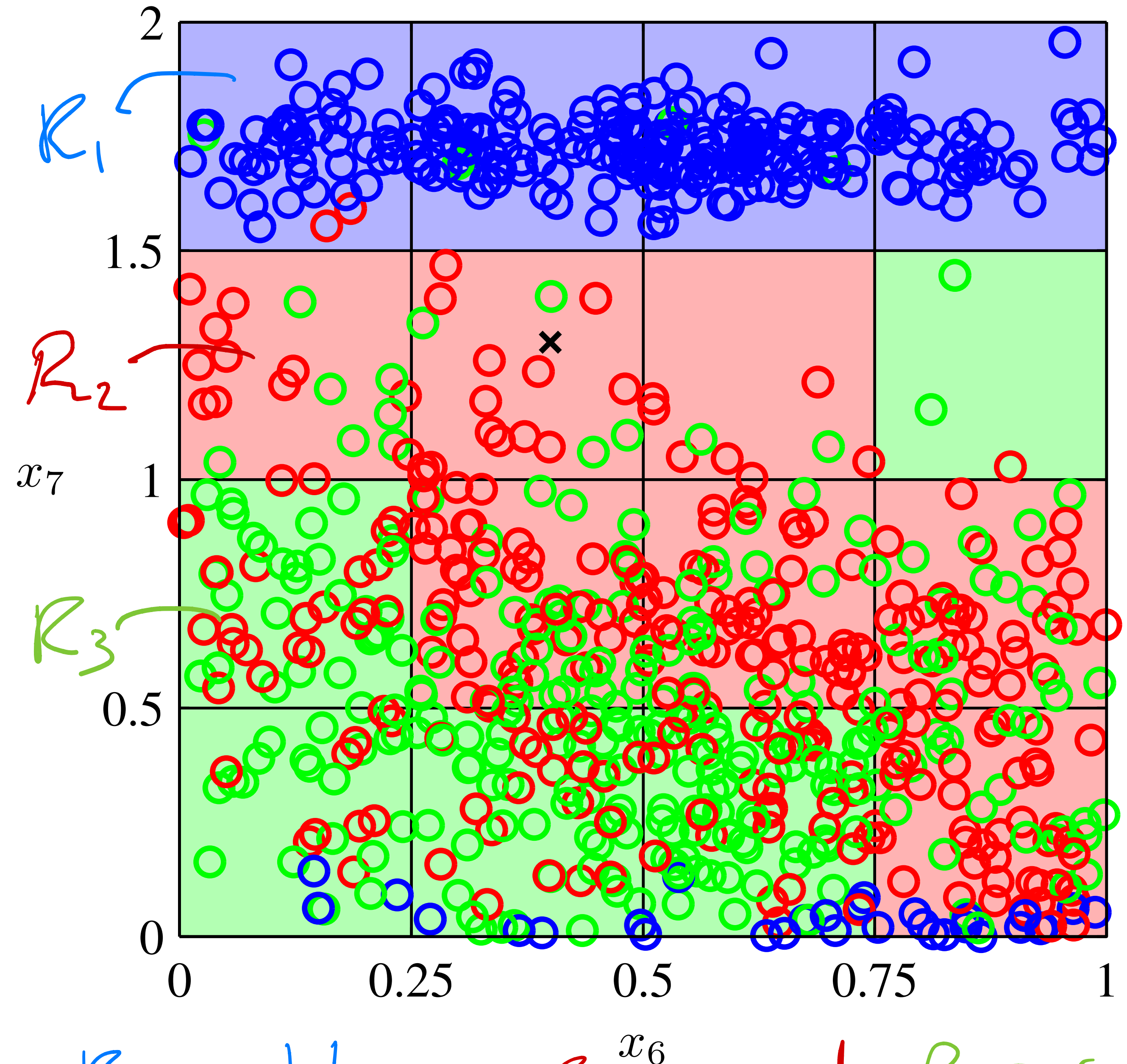
one-hot encoding of class C_3

Strategy:

- Divide input space $\mathbb{R}^D$ into K *decision regions*. $\mathcal{R}_k$
- Assign each decision region to a class $\mathcal{R}_k \rightarrow C_k$
- Boundaries of decision regions are called *decision boundaries/surfaces.*

Classification through Decision Regions





$R_1 \to$ blue $R_2 \to$ red $R_3 \to$ green

Figures: 3 class problem with decision boundaries. (Bishop 1.19 & 1.20)

$K=3$

Linear Classification

- Linear Classification: consider only *linear* decision boundaries
- For *D* - dimensional input space: $\underline{x} \in \mathbb{R}^D$
  decision surface is a $D-1$ dimensional hyperplane
- Datasets whose classes can be separated *exactly* by linear decision surfaces are called linearly separable

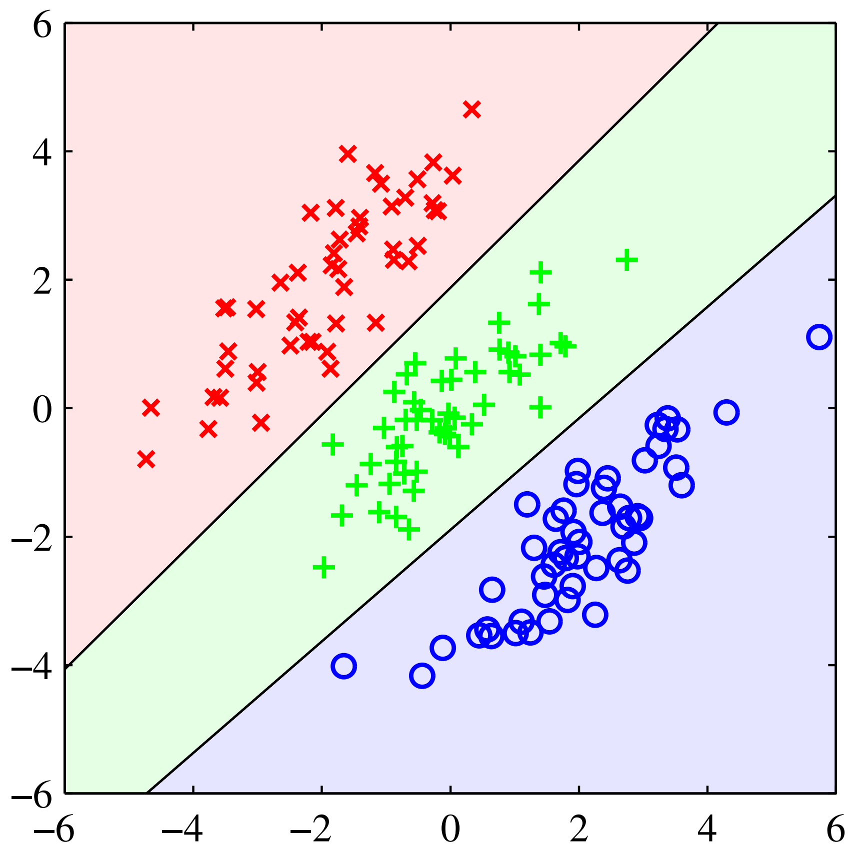


Figure: Linearly separable dataset (Bishop 4.5)

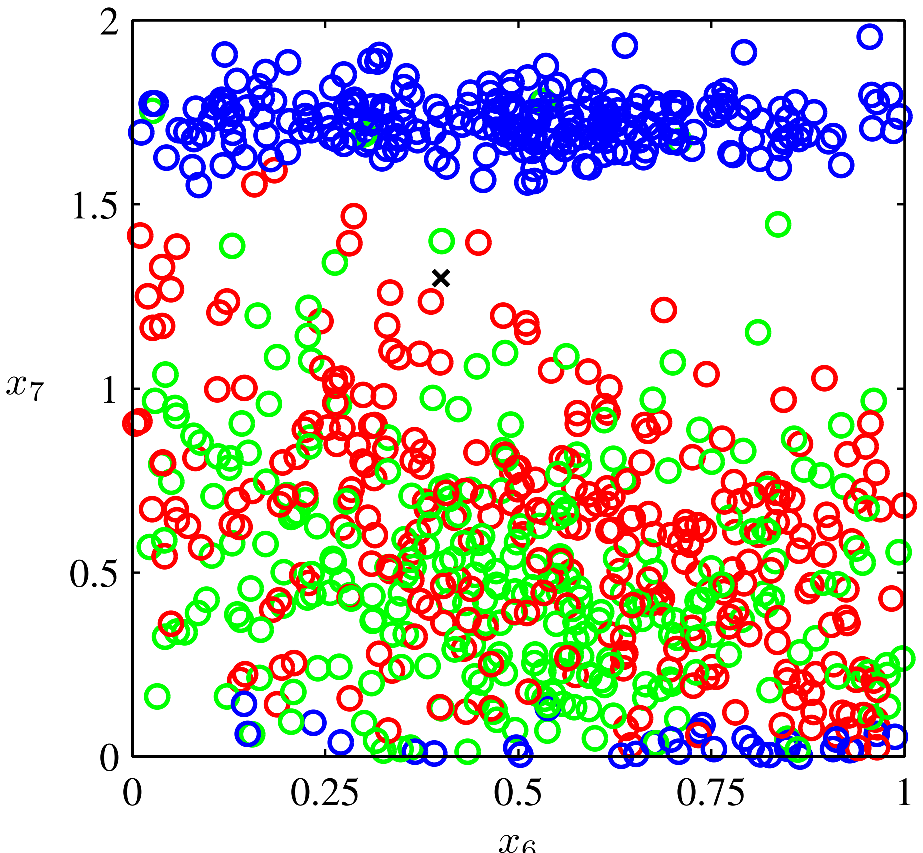


Figure: Not linearly separable dataset (Bishop 1.19)

Multiple Classes ($K > 2$)

$t = C_1$ & C_2 ??

- K=2 classes:
  - 1 classifier determines $t = C_1$, $t = C_2$
- Multiple classes: $K > 2$
  - K-1 classifiers: $t = C_k$, $t \neq C_k$
- One-versus-the-rest

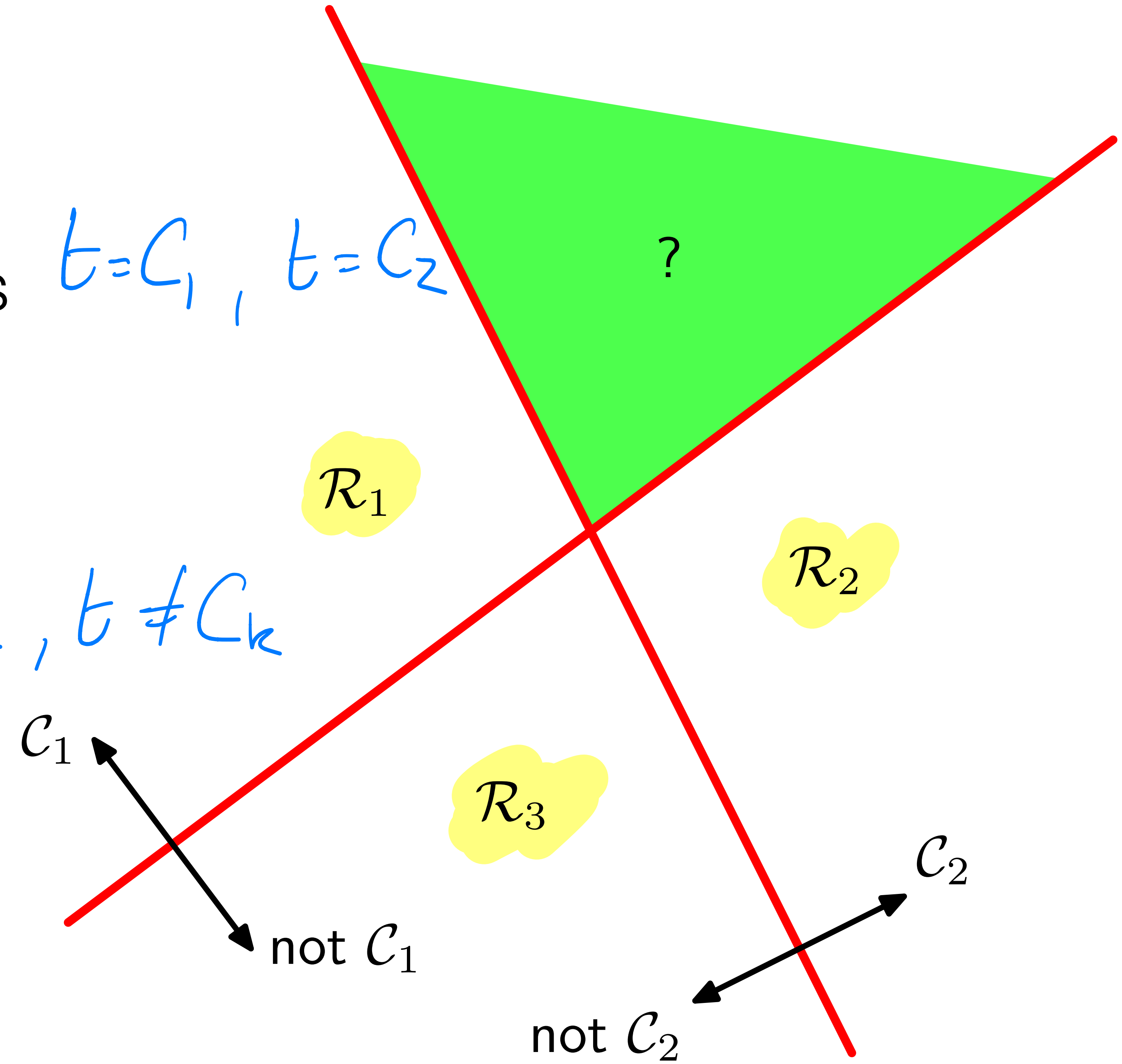
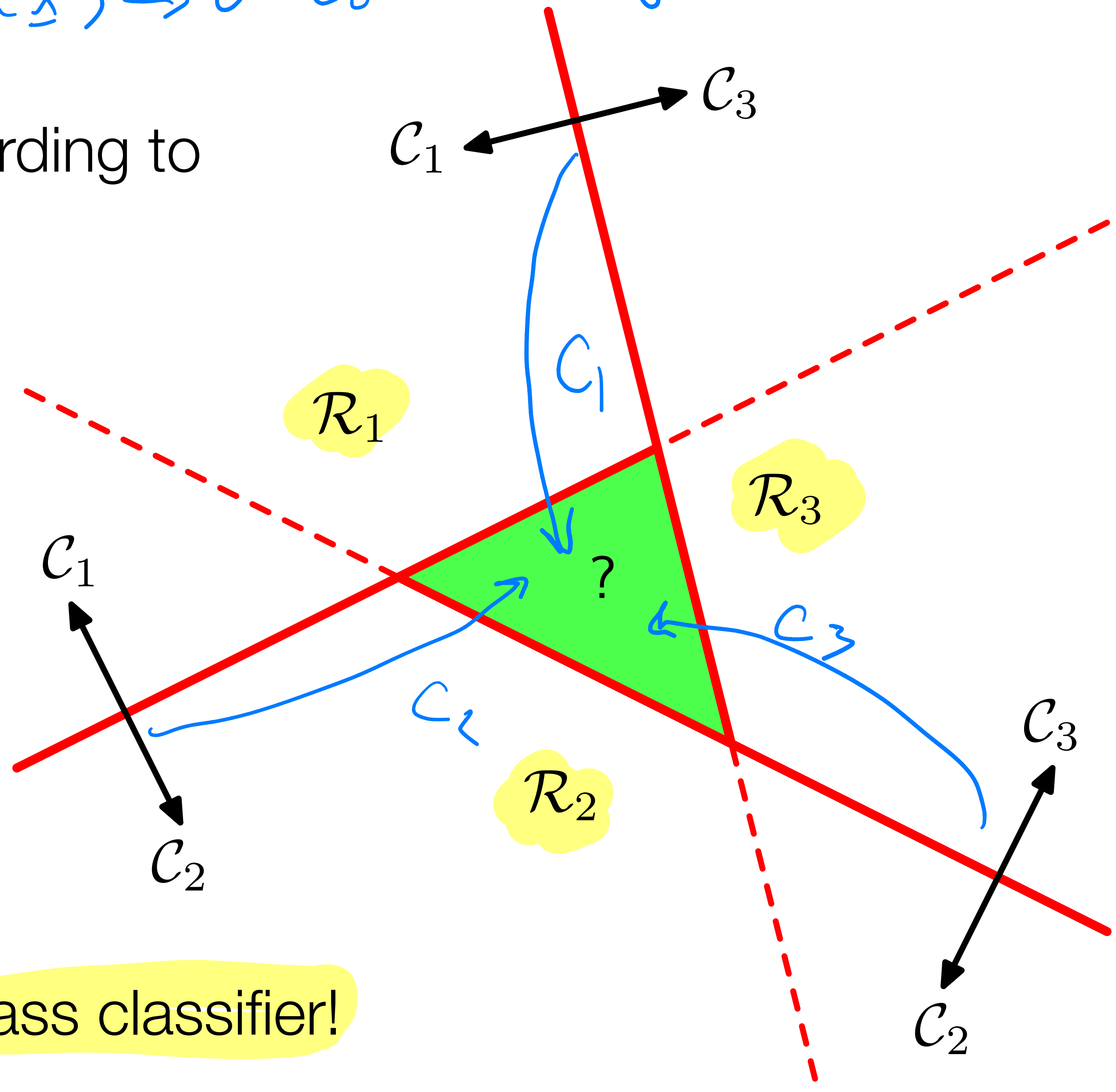


Figure: one-versus-the-rest classifiers (Bishop 4.2)

Multiple Classes ($K > 2$)

- K(K-1)/2 classifiers: $\phi_{ij}(\underline{x}) \rightarrow t = C_i$ or $t = C_j$
- Points are classified according to majority vote of classifiers
- one-versus-one



- **Solution**: Make one K-class classifier! (See later)

Figure: one-versus-one classifiers (Bishop 4.2)