# **Language Models: Auto-Complete**

In this assignment, you will build an auto-complete system. Auto-complete system is something you may see every day

- When you google something, you often have suggestions to help you complete your search.
- When you are writing an email, you get suggestions telling you possible endings to your sentence.

By the end of this assignment, you will develop a prototype of such a system.



stanford is be

stanford is better than harvard stanford is best known for is stanford better than ivy league is stanford better than berkeley

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A key building block for an auto-complete system is a language model. A language model assigns the probability to a sequence of words, in a way that more "likely" sequences receive higher scores. For example,

"I have a pen" is expected to have a higher probability than "I am a pen" since the first one seems to be a more natural sentence in the real world.

You can take advantage of this probability calculation to develop an auto-complete system.

Suppose the user typed

"I eat scrambled" Then you can find a word x such that "I eat scrambled x" receives the highest probability. If x = "eggs", the sentence would be "I eat scrambled eggs"

While a variety of language models have been developed, this assignment uses **N-grams**, a simple but powerful method for language modeling.

• N-grams are also used in machine translation and speech recognition.

Here are the steps of this assignment:

- 1. Load and preprocess data
  - · Load and tokenize data.
  - Split the sentences into train and test sets.
  - Replace words with a low frequency by an unknown marker <unk>.
- 2. Develop N-gram based language models
  - Compute the count of n-grams from a given data set.
  - Estimate the conditional probability of a next word with k-smoothing.
- 3. Evaluate the N-gram models by computing the perplexity score.
- 4. Use your own model to suggest an upcoming word given your sentence.

```
In [1]: import math
import random
import numpy as np
import pandas as pd
import nltk
nltk.data.path.append('.')
```

## Part 1: Load and Preprocess Data

#### Part 1.1: Load the data

You will use twitter data. Load the data and view the first few sentences by running the next cell.

Notice that data is a long string that contains many many tweets. Observe that there is a line break "\n" between tweets.

```
In [2]: | with open("en_US.twitter.txt", "r") as f:
            data = f.read()
        print("Data type:", type(data))
        print("Number of letters:", len(data))
        print("First 300 letters of the data")
        print("----")
        display(data[0:300])
        print("----")
        print("Last 300 letters of the data")
        print("----")
        display(data[-300:])
        print("----")
        Data type: <class 'str'>
        Number of letters: 3335477
        First 300 letters of the data
        "How are you? Btw thanks for the RT. You gonna be in DC anytime soon? Love to see you. Been way, way too long.\nWhen y
        ou meet someone special... you'll know. Your heart will beat more rapidly and you'll smile for no reason.\nthey've dec
        ided its more fun if I don't.\nSo Tired D; Played Lazer Tag & Ran A "
        Last 300 letters of the data
        "ust had one a few weeks back....hopefully we will be back soon! wish you the best yo∖nColombia is with an 'o'...": We
        now ship to 4 countries in South America (fist pump). Please welcome Columbia to the Stunner Family"\n#GutsiestMovesYo
        uCanMake Giving a cat a bath.\nCoffee after 5 was a TERRIBLE idea.\n"
```

## Part 1.2 Pre-process the data

Preprocess this data with the following steps:

- 1. Split data into sentences using "\n" as the delimiter.
- 2. Split each sentence into tokens. Note that in this assignment we use "token" and "words" interchangeably.
- 3. Assign sentences into train or test sets.
- 4. Find tokens that appear at least N times in the training data.
- 5. Replace tokens that appear less than N times by <unk>

Note: we omit validation data in this exercise.

- In real applications, we should hold a part of data as a validation set and use it to tune our training.
- We skip this process for simplicity.

### **Exercise 01**

Split data into sentences.

```
In [3]: # UNQ_C1 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
             ### GRADED_FUNCTION: split_to_sentences ###
             def split to sentences(data):
                 Split data by linebreak "\n"
                 Args:
                     data: str
                 Returns:
                     A list of sentences
                 ### START CODE HERE (Replace instances of 'None' with your code) ###
                 sentences = data.split('\n')
                 ### END CODE HERE ###
                 # Additional clearning (This part is already implemented)
                 # - Remove leading and trailing spaces from each sentence
                 # - Drop sentences if they are empty strings.
                 sentences = [s.strip() for s in sentences]
                 sentences = [s \text{ for } s \text{ in sentences if } len(s) > 0]
                 return sentences
    In [4]: | # test your code
             x = """
             I have a pen.\nI have an apple. \nAh\nApple pen.\n
             print(x)
             split_to_sentences(x)
            I have a pen.
            I have an apple.
             Αh
             Apple pen.
    Out[4]: ['I have a pen.', 'I have an apple.', 'Ah', 'Apple pen.']
Expected answer:
   ['I have a pen.', 'I have an apple.', 'Ah', 'Apple pen.']
```

## Exercise 02

The next step is to tokenize sentences (split a sentence into a list of words).

- Convert all tokens into lower case so that words which are capitalized (for example, at the start of a sentence) in the original text are treated the same as the lowercase versions of the words.
- Append each tokenized list of words into a list of tokenized sentences.

```
In [5]: # UNO C2 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
        ### GRADED FUNCTION: tokenize sentences ###
        def tokenize sentences(sentences):
            Tokenize sentences into tokens (words)
            Args:
                sentences: List of strings
             Returns:
                List of lists of tokens
            # Initialize the list of lists of tokenized sentences
            tokenized_sentences = []
            ### START CODE HERE (Replace instances of 'None' with your code) ###
            # Go through each sentence
            for sentence in sentences:
                # Convert to Lowercase Letters
                sentence = sentence.lower()
                # Convert into a list of words
                tokenized = nltk.word_tokenize(sentence)
                # append the list of words to the list of lists
                tokenized_sentences.append(tokenized)
             ### END CODE HERE ###
            return tokenized_sentences
```

```
In [6]: # test your code
    sentences = ["Sky is blue.", "Leaves are green.", "Roses are red."]
    tokenize_sentences(sentences)

Out[6]: [['sky', 'is', 'blue', '.'],
        ['leaves', 'are', 'green', '.'],
```

['roses', 'are', 'red', '.']]

```
[['sky', 'is', 'blue', '.'],
['leaves', 'are', 'green', '.'],
['roses', 'are', 'red', '.']]
```

#### **Exercise 03**

Use the two functions that you have just implemented to get the tokenized data.

- split the data into sentences
- tokenize those sentences

```
In [8]: # test your function
x = "Sky is blue.\nLeaves are green\nRoses are red."
get_tokenized_data(x)

Out[8]: [['sky', 'is', 'blue', '.'],
        ['leaves', 'are', 'green'],
        ['roses', 'are', 'red', '.']]
```

#### **Expected outcome**

```
[['sky', 'is', 'blue', '.'],
['leaves', 'are', 'green'],
['roses', 'are', 'red', '.']]
```

## Split into train and test sets

Now run the cell below to split data into training and test sets.

```
In [9]: tokenized_data = get_tokenized_data(data)
    random.seed(87)
    random.shuffle(tokenized_data)

    train_size = int(len(tokenized_data) * 0.8)
    train_data = tokenized_data[0:train_size]
    test_data = tokenized_data[train_size:]
```

#### **Exercise 04**

You won't use all the tokens (words) appearing in the data for training. Instead, you will use the more frequently used words.

- You will focus on the words that appear at least N times in the data.
- First count how many times each word appears in the data.

You will need a double for-loop, one for sentences and the other for tokens within a sentence.

```
In [11]: # UNQ_C4 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         ### GRADED_FUNCTION: count_words ###
         def count_words(tokenized_sentences):
             Count the number of word appearence in the tokenized sentences
             Args:
                 tokenized_sentences: List of lists of strings
             Returns:
                 dict that maps word (str) to the frequency (int)
             word_counts = {}
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # Loop through each sentence
             for sentence in tokenized_sentences: # complete this line
                 # Go through each token in the sentence
                 for token in sentence: # complete this line
                     # If the token is not in the dictionary yet, set the count to 1
                     if token not in word_counts: # complete this line
                         word_counts[token] = 1
                     # If the token is already in the dictionary, increment the count by 1
                     else:
                         word_counts[token] += 1
             ### END CODE HERE ###
             return word_counts
```

Note that the order may differ.

```
{ sky : 1, is : 1, is
```

## Handling 'Out of Vocabulary' words

If your model is performing autocomplete, but encounters a word that it never saw during training, it won't have an input word to help it determine the next word to suggest. The model will not be able to predict the next word because there are no counts for the current word.

- This 'new' word is called an 'unknown word', or out of vocabulary (OOV) words.
- The percentage of unknown words in the test set is called the **OOV** rate.

To handle unknown words during prediction, use a special token to represent all unknown words 'unk'.

- Modify the training data so that it has some 'unknown' words to train on.
- Words to convert into "unknown" words are those that do not occur very frequently in the training set.
- Create a list of the most frequent words in the training set, called the closed vocabulary .
- Convert all the other words that are not part of the closed vocabulary to the token 'unk'.

#### **Exercise 05**

You will now create a function that takes in a text document and a threshold <code>count\_threshold</code> .

- Any word whose count is greater than or equal to the threshold count\_threshold is kept in the closed vocabulary.
- Returns the word closed vocabulary list.

```
In [13]: # UNO C5 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         ### GRADED_FUNCTION: get_words_with_nplus_frequency ###
         def get_words_with_nplus_frequency(tokenized_sentences, count_threshold):
             Find the words that appear N times or more
             Args:
                 tokenized_sentences: List of lists of sentences
                 count threshold: minimum number of occurrences for a word to be in the closed vocabulary.
             Returns:
                 List of words that appear N times or more
             # Initialize an empty list to contain the words that
             # appear at least 'minimum_freq' times.
             closed_vocab = []
             # Get the word couts of the tokenized sentences
             # Use the function that you defined earlier to count the words
             word_counts = count_words(tokenized_sentences)
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # for each word and its count
             for word, cnt in word_counts.items(): # complete this line
                 # check that the word's count
                 # is at least as great as the minimum count
                 if cnt >= count_threshold:
                     # append the word to the list
                     closed_vocab.append(word)
             ### END CODE HERE ###
             return closed_vocab
```

#### **Exercise 06**

The words that appear count\_threshold times or more are in the closed vocabulary.

- All other words are regarded as unknown.
- Replace words not in the closed vocabulary with the token <unk>.

```
In [15]: # UNO C6 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         ### GRADED FUNCTION: replace oov words by unk ###
         def replace oov words by unk(tokenized sentences, vocabulary, unknown token="<unk>"):
             Replace words not in the given vocabulary with '<unk>' token.
             Args:
                 tokenized sentences: List of lists of strings
                 vocabulary: List of strings that we will use
                 unknown token: A string representing unknown (out-of-vocabulary) words
             Returns:
                 List of lists of strings, with words not in the vocabulary replaced
             # Place vocabulary into a set for faster search
             vocabulary = set(vocabulary)
             # Initialize a list that will hold the sentences
             # after less frequent words are replaced by the unknown token
             replaced tokenized sentences = []
             # Go through each sentence
             for sentence in tokenized sentences:
                 # Initialize the list that will contain
                 # a single sentence with "unknown token" replacements
                 replaced_sentence = []
                 ### START CODE HERE (Replace instances of 'None' with your code) ###
                 # for each token in the sentence
                 for token in sentence: # complete this line
                     # Check if the token is in the closed vocabulary
                     if token in vocabulary: # complete this line
                         # If so, append the word to the replaced sentence
                         replaced sentence.append(token)
                     else:
                         # otherwise, append the unknown token instead
                         replaced_sentence.append(unknown_token)
                 ### END CODE HERE ###
                 # Append the list of tokens to the list of lists
                 replaced tokenized sentences.append(replaced sentence)
             return replaced tokenized sentences
```

```
In [16]: tokenized_sentences = [["dogs", "run"], ["cats", "sleep"]]
    vocabulary = ["dogs", "sleep"]
    tmp_replaced_tokenized_sentences = replace_oov_words_by_unk(tokenized_sentences, vocabulary)
    print(f"Original sentence:")
    print(tokenized_sentences)
    print(f"tokenized_sentences with less frequent words converted to '<unk>':")
    print(tmp_replaced_tokenized_sentences)

Original sentence:
    [['dogs', 'run'], ['cats', 'sleep']]
    tokenized_sentences with less frequent words converted to '<unk>':
    [['dogs', '<unk>'], ['<unk>', 'sleep']]
```

### **Expected answer**

```
Original sentence:

[['dogs', 'run'], ['cats', 'sleep']]

tokenized_sentences with less frequent words converted to '<unk>':

[['dogs', '<unk>'], ['<unk>', 'sleep']]
```

## **Exercise 07**

Now we are ready to process our data by combining the functions that you just implemented.

- 1. Find tokens that appear at least count threshold times in the training data.
- 2. Replace tokens that appear less than count threshold times by "<unk>" both for training and test data.

```
In [17]: # UNO C7 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         ### GRADED FUNCTION: preprocess data ###
         def preprocess data(train data, test data, count threshold):
             Preprocess data, i.e.,
                 - Find tokens that appear at Least N times in the training data.
                 - Replace tokens that appear less than N times by "<unk>" both for training and test data.
             Args:
                 train data, test data: List of lists of strings.
                 count threshold: Words whose count is less than this are
                               treated as unknown.
             Returns:
                 Tuple of
                 - training data with low frequent words replaced by "<unk>"
                 - test data with low frequent words replaced by "<unk>"
                 - vocabulary of words that appear n times or more in the training data
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # Get the closed vocabulary using the train data
             vocabulary = get words with nplus frequency(train data, count threshold)
             # For the train data, replace less common words with "<unk>"
             train data replaced = replace oov words by unk(train data, vocabulary)
             # For the test data, replace less common words with "<unk>"
             test data replaced = replace oov words by unk(test data, vocabulary)
             ### END CODE HERE ###
             return train data replaced, test data replaced, vocabulary
```

```
In [18]: # test your code
         tmp_train = [['sky', 'is', 'blue', '.'],
              ['leaves', 'are', 'green']]
         tmp_test = [['roses', 'are', 'red', '.']]
         tmp_train_repl, tmp_test_repl, tmp_vocab = preprocess_data(tmp_train,
                                                                     tmp test,
                                                                     count_threshold = 1)
         print("tmp_train_repl")
         print(tmp_train_repl)
         print()
         print("tmp_test_repl")
         print(tmp_test_repl)
         print()
         print("tmp_vocab")
         print(tmp_vocab)
         tmp_train_repl
         [['sky', 'is', 'blue', '.'], ['leaves', 'are', 'green']]
         tmp_test_repl
         [['<unk>', 'are', '<unk>', '.']]
```

#### **Expected outcome**

tmp\_vocab

```
tmp_train_repl
[['sky'], 'is', 'blue', '.'], ['leaves', 'are', 'green']]

tmp_test_repl
[['<unk>', 'are', '<unk>', '.']]

tmp_vocab
['sky'], 'is', 'blue', '.', 'leaves', 'are', 'green']
```

['sky', 'is', 'blue', '.', 'leaves', 'are', 'green']

#### Preprocess the train and test data

Run the cell below to complete the preprocessing both for training and test sets.

```
In [19]: minimum freq = 2
        train data processed, test data processed, vocabulary = preprocess data(train data,
                                                                        test data,
                                                                        minimum freq)
In [20]: print("First preprocessed training sample:")
        print(train_data_processed[0])
        print()
        print("First preprocessed test sample:")
        print(test_data_processed[0])
        print()
        print("First 10 vocabulary:")
        print(vocabulary[0:10])
        print()
        print("Size of vocabulary:", len(vocabulary))
        First preprocessed training sample:
        ['i', 'personally', 'would', 'like', 'as', 'our', 'official', 'glove', 'of', 'the', 'team', 'local', 'company', 'and',
        'quality', 'production']
        First preprocessed test sample:
        First 10 vocabulary:
        ['i', 'personally', 'would', 'like', 'as', 'our', 'official', 'glove', 'of', 'the']
        Size of vocabulary: 14821
```

You are done with the preprocessing section of the assignment. Objects train\_data\_processed, test\_data\_processed, and vocabulary will be used in the rest of the exercises.

## Part 2: Develop n-gram based language models

In this section, you will develop the n-grams language model.

- Assume the probability of the next word depends only on the previous n-gram.
- The previous n-gram is the series of the previous 'n' words.

The conditional probability for the word at position 't' in the sentence, given that the words preceding it are  $w_{t-1}, w_{t-2} \cdots w_{t-n}$  is:

$$P(w_t|w_{t-1}\dots w_{t-n}) \tag{1}$$

You can estimate this probability by counting the occurrences of these series of words in the training data.

- The probability can be estimated as a ratio, where
- The numerator is the number of times word 't' appears after words t-1 through t-n appear in the training data.
- The denominator is the number of times word t-1 through t-n appears in the training data.

$$\hat{P}(w_t|w_{t-1}\dots w_{t-n}) = \frac{C(w_{t-1}\dots w_{t-n}, w_n)}{C(w_{t-1}\dots w_{t-n})}$$
(2)

- The function  $C(\cdots)$  denotes the number of occurrence of the given sequence.
- $\hat{P}$  means the estimation of P.
- Notice that denominator of the equation (2) is the number of occurrence of the previous n words, and the numerator is the same sequence followed by the word  $w_t$ .

Later, you will modify the equation (2) by adding k-smoothing, which avoids errors when any counts are zero.

The equation (2) tells us that to estimate probabilities based on n-grams, you need the counts of n-grams (for denominator) and (n+1)-grams (for numerator).

#### **Exercise 08**

Next, you will implement a function that computes the counts of n-grams for an arbitrary number n.

When computing the counts for n-grams, prepare the sentence beforehand by prepending n-1 starting markers "<s>" to indicate the beginning of the sentence.

- For example, in the bi-gram model (N=2), a sequence with two start tokens "<s><s>" should predict the first word of a sentence.
- So, if the sentence is "I like food", modify it to be "<s><s> I like food".
- Also prepare the sentence for counting by appending an end token "<e>" so that the model can predict when to finish a sentence.

Technical note: In this implementation, you will store the counts as a dictionary.

- The key of each key-value pair in the dictionary is a **tuple** of n words (and not a list)
- The value in the key-value pair is the number of occurrences.
- The reason for using a tuple as a key instead of a list is because a list in Python is a mutable object (it can be changed after it is first created). A tuple is "immutable", so it cannot be altered after it is first created. This makes a tuple suitable as a data type for the key in a dictionary.

```
In [21]: # UNO C8 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         ### GRADED FUNCTION: count n grams ###
         def count_n_grams(data, n, start_token='<s>', end_token = '<e>'):
             Count all n-grams in the data
             Args:
                 data: List of lists of words
                 n: number of words in a sequence
             Returns:
                 A dictionary that maps a tuple of n-words to its frequency
             # Initialize dictionary of n-grams and their counts
             n grams = \{\}
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # Go through each sentence in the data
             for sentence in data: # complete this line
                 # prepend start token n times, and append <e> one time
                 sentence = [start_token] * n + sentence + [end_token]
                 # convert list to tuple
                 # So that the sequence of words can be used as
                 # a key in the dictionary
                 sentence = tuple(sentence)
                 # Use 'i' to indicate the start of the n-gram
                 # from index 0
                 # to the last index where the end of the n-gram
                 # is within the sentence.
                 for i in range(len(sentence) - n + 1): # complete this line
                     # Get the n-gram from i to i+n
                     n gram = sentence[i:i+n]
                     # check if the n-gram is in the dictionary
                     if n gram in n grams: # complete this line
                         # Increment the count for this n-gram
                         n_grams[n_gram] += 1
                     else:
                         # Initialize this n-gram count to 1
```

```
n_grams[n_gram] = 1

### END CODE HERE ###

return n_grams
```

{('<s>', '<s>'): 2, ('<s>', 'i'): 1, ('i', 'like'): 1, ('like', 'a'): 2, ('a', 'cat'): 2, ('cat', '<e>'): 2, ('<s>',

#### Expected outcome:

```
Uni-gram:
{('\s>\infty,): 2, ('i',): 1, (\infty)ike\infty,): 2, (\infty)ike\infty,): 2, (\infty)ike\infty,): 2, (\infty)ike\infty,): 1, (\infty)ike\infty,): 1, (\infty)ike\infty,): 1}
Bi-gram:
{(\infty\s>\infty): 2, (\infty\s>\infty): 2, (\infty\s>\infty): 1, (\infty\infty): 1, (\infty\infty): 1, (\infty\infty): 2, (\infty\cat\infty): 1}
his \infty): 1, (\infty\this\infty): 1, (\infty\dog\infty): 1, (\infty\infty\infty): 1, (\infty\infty\infty): 1}
```

'this'): 1, ('this', 'dog'): 1, ('dog', 'is'): 1, ('is', 'like'): 1}

#### **Exercise 09**

Next, estimate the probability of a word given the prior 'n' words using the n-gram counts.

$$\hat{P}(w_t|w_{t-1}\dots w_{t-n}) = rac{C(w_{t-1}\dots w_{t-n}, w_n)}{C(w_{t-1}\dots w_{t-n})}$$
 (2)

This formula doesn't work when a count of an n-gram is zero..

- Suppose we encounter an n-gram that did not occur in the training data.
- Then, the equation (2) cannot be evaluated (it becomes zero divided by zero).

A way to handle zero counts is to add k-smoothing.

• K-smoothing adds a positive constant k to each numerator and  $k \times |V|$  in the denominator, where |V| is the number of words in the vocabulary.

$$\hat{P}(w_t|w_{t-1}\dots w_{t-n}) = \frac{C(w_{t-1}\dots w_{t-n}, w_n) + k}{C(w_{t-1}\dots w_{t-n}) + k|V|}$$
(3)

For n-grams that have a zero count, the equation (3) becomes  $\frac{1}{|V|}$  .

• This means that any n-gram with zero count has the same probability of  $\frac{1}{|V|}$  .

Define a function that computes the probability estimate (3) from n-gram counts and a constant k.

- The function takes in a dictionary 'n\_gram\_counts', where the key is the n-gram and the value is the count of that n-gram.
- The function also takes another dictionary n\_plus1\_gram\_counts, which you'll use to find the count for the previous n-gram plus the current word.

```
In [35]: # UNO C9 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         ### GRADED FUNCTION: estimate probability ###
         def estimate_probability(word, previous_n_gram,
                                   n gram counts, n plus1 gram counts, vocabulary size, k=1.0):
             Estimate the probabilities of a next word using the n-gram counts with k-smoothing
             Args:
                 word: next word
                 previous n gram: A sequence of words of Length n
                 n gram counts: Dictionary of counts of n-grams
                 n plus1 gram counts: Dictionary of counts of (n+1)-grams
                 vocabulary size: number of words in the vocabulary
                 k: positive constant, smoothing parameter
             Returns:
                 A probability
             # convert list to tuple to use it as a dictionary key
             previous n gram = tuple(previous n gram)
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # Set the denominator
             # If the previous n-gram exists in the dictionary of n-gram counts,
             # Get its count. Otherwise set the count to zero
             # Use the dictionary that has counts for n-grams
             previous n gram count = n gram counts.get(previous n gram, 0)
             # Calculate the denominator using the count of the previous n gram
             # and apply k-smoothing
             denominator = previous n gram count + k * vocabulary size
             # Define n plus 1 gram as the previous n-gram plus the current word as a tuple
             n plus1 gram = tuple(list(previous n gram) + [word])
             # Set the count to the count in the dictionary,
             # otherwise 0 if not in the dictionary
             # use the dictionary that has counts for the n-gram plus current word
             n plus1 gram count = n plus1 gram counts.get(n plus1 gram, 0)
             # Define the numerator use the count of the n-gram plus current word,
             # and apply smoothing
             numerator = n plus1 gram count + k
             # Calculate the probability as the numerator divided by denominator
             probability = numerator / denominator
             ### END CODE HERE ###
```

**return** probability

The estimated probability of word 'cat' given the previous n-gram 'a' is: 0.3333

#### Expected output

The estimated probability of word cat given the previous n-gram 'a' is: 0.3333

## Estimate probabilities for all words

The function defined below loops over all words in vocabulary to calculate probabilities for all possible words.

• This function is provided for you.

```
In [37]: def estimate probabilities(previous n gram, n gram counts, n plus1 gram counts, vocabulary, k=1.0):
             Estimate the probabilities of next words using the n-gram counts with k-smoothing
             Args:
                 previous n gram: A sequence of words of length n
                 n gram counts: Dictionary of counts of (n+1)-grams
                 n_plus1_gram_counts: Dictionary of counts of (n+1)-grams
                 vocabulary: List of words
                 k: positive constant, smoothing parameter
             Returns:
                 A dictionary mapping from next words to the probability.
             # convert list to tuple to use it as a dictionary key
             previous n gram = tuple(previous n gram)
             # add <e> <unk> to the vocabulary
             # <s> is not needed since it should not appear as the next word
             vocabulary = vocabulary + ["<e>", "<unk>"]
             vocabulary size = len(vocabulary)
             probabilities = {}
             for word in vocabulary:
                 probability = estimate probability(word, previous n gram,
                                                     n_gram_counts, n_plus1_gram_counts,
                                                     vocabulary size, k=k)
                 probabilities[word] = probability
             return probabilities
```

```
{ cat : 0.27272727272727,
    'i': 0.09090909090909091,
    'this': 0.09090909090909091,
    'a': 0.09090909090909091,
    'is': 0.09090909090909091,
    'like': 0.09090909090909091,
    'dog': 0.09090909090909091,
    '<e>': 0.0909090909090909091,
    '<unk>': 0.0909090909090909091}
```

'dog': 0.09090909090909091,
'<e>': 0.09090909090909091,
'<unk>': 0.09090909090909091}

```
In [39]: # Additional test
    trigram_counts = count_n_grams(sentences, 3)
    estimate_probabilities(["<s>", "<s>"], bigram_counts, trigram_counts, unique_words, k=1)

Out[39]: {'i': 0.181818181818182,
    'cat': 0.090909090909091,
    'a': 0.090909090909091,
    'this': 0.181818181818182,
    'like': 0.090909090909091,
    'is': 0.090909090909091,
    'dog': 0.090909090909091,
    'ce>': 0.09090909090909091,
    '<e>': 0.0909090909090909091,
    'ce>': 0.09090909090909091,
    'cat': 0.090909090909090909090]
```

## **Count and probability matrices**

As we have seen so far, the n-gram counts computed above are sufficient for computing the probabilities of the next word.

- It can be more intuitive to present them as count or probability matrices.
- The functions defined in the next cells return count or probability matrices.
- This function is provided for you.

```
In [40]: def make_count_matrix(n_plus1_gram_counts, vocabulary):
             # add <e> <unk> to the vocabulary
             # <s> is omitted since it should not appear as the next word
             vocabulary = vocabulary + ["<e>", "<unk>"]
             # obtain unique n-grams
             n grams = []
             for n_plus1_gram in n_plus1_gram_counts.keys():
                 n gram = n plus1 gram[0:-1]
                 n grams.append(n gram)
             n grams = list(set(n grams))
             # mapping from n-gram to row
             row_index = {n_gram:i for i, n_gram in enumerate(n_grams)}
             # mapping from next word to column
             col_index = {word:j for j, word in enumerate(vocabulary)}
             nrow = len(n grams)
             ncol = len(vocabulary)
             count matrix = np.zeros((nrow, ncol))
             for n_plus1_gram, count in n_plus1_gram_counts.items():
                 n_gram = n_plus1_gram[0:-1]
                 word = n plus1 gram[-1]
                 if word not in vocabulary:
                     continue
                 i = row index[n gram]
                 j = col_index[word]
                 count_matrix[i, j] = count
             count matrix = pd.DataFrame(count matrix, index=n grams, columns=vocabulary)
             return count matrix
```

bigram counts

	i	cat	а	this	like	is	dog	<e></e>	<unk></unk>
(cat,)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	0.0
(is,)	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
(dog,)	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
(a,)	0.0	2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
(this,)	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
( <s>,)</s>	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
(i,)	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
(like,)	0.0	0.0	2.0	0.0	0.0	0.0	0.0	0.0	0.0

## Expected output

bigram counts

```
this a is
                                like dog <e>
         cat
                                                <unk>
              1.0 1.0 0.0 0.0 0.0
                                     0.0 0.0
(<s>,)
         0.0
                                                0.0
(a,)
              0.0 0.0 0.0 0.0 0.0
                                     0.0
                                          0.0
         2.0
                                                0.0
(this,)
              0.0 0.0 0.0 0.0
                                0.0
                                     1.0
         0.0
                                          0.0
                                                0.0
(like,)
              0.0 0.0 2.0 0.0 0.0
                                     0.0 0.0
         0.0
                                                0.0
(dog,)
              0.0 0.0 0.0 1.0 0.0
                                     0.0 0.0
                                                0.0
         0.0
(cat,)
              0.0 0.0 0.0 0.0 0.0
                                     0.0 2.0
         0.0
                                                0.0
(is,)
              0.0 0.0 0.0 0.0 1.0
         0.0
                                      0.0 0.0
                                                0.0
(i,)
              0.0 0.0 0.0 0.0 1.0
         0.0
                                      0.0 0.0
                                                0.0
```

```
In [42]: # Show trigram counts
print('\ntrigram counts')
trigram_counts = count_n_grams(sentences, 3)
display(make_count_matrix(trigram_counts, unique_words))
```

trigram counts

	i	cat	а	this	like	is	dog	<e></e>	<unk></unk>
( <s>, this)</s>	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0
(this, dog)	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
(i, like)	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
( <s>, i)</s>	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
(is, like)	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
(a, cat)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	0.0
(dog, is)	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
( <s>, <s>)</s></s>	1.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
(like, a)	0.0	2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

## Expected output

 ${\tt trigram}\ {\tt counts}$ 

```
this
                            a is
                                    like dog <e>
            cat
                                                    <unk>
                      0.0 0.0 0.0 1.0
(dog, is)
            0.0
                  0.0
                                         0.0
                                              0.0
                                                    0.0
                                             0.0
(this, dog)
            0.0
                  0.0 0.0 0.0 1.0 0.0
                                         0.0
                                                    0.0
                  0.0 0.0 0.0 0.0 0.0
(a, cat)
            0.0
                                         0.0 2.0
                                                    0.0
(like, a)
            2.0
                  0.0 0.0 0.0 0.0
                                   0.0
                                         0.0
                                             0.0
                                                    0.0
(is, like)
            0.0
                  0.0 0.0 1.0 0.0 0.0
                                         0.0 0.0
                                                     0.0
(<s>, i)
                  0.0 0.0 0.0 0.0 1.0
                                                     0.0
            0.0
                                          0.0 0.0
(i, like)
            0.0
                  0.0 0.0 1.0 0.0
                                    0.0
                                         0.0
                                             0.0
                                                    0.0
                 1.0 1.0 0.0 0.0 0.0
(<s>, <s>)
                                         0.0 0.0
                                                    0.0
            0.0
(<s>, this)
                  0.0 0.0 0.0 0.0 0.0
            0.0
                                         1.0 0.0
                                                     0.0
```

The following function calculates the probabilities of each word given the previous n-gram, and stores this in matrix form.

• This function is provided for you.

bigram probabilities

	i	cat	а	this	like	is	dog	<e></e>	<unk></unk>
(cat,)	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909	0.272727	0.090909
(is,)	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000	0.100000
(dog,)	0.100000	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000
(a,)	0.090909	0.272727	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909
(this,)	0.100000	0.100000	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000
( <s>,)</s>	0.181818	0.090909	0.090909	0.181818	0.090909	0.090909	0.090909	0.090909	0.090909
(i,)	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000	0.100000
(like,)	0.090909	0.090909	0.272727	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909

```
In [45]: print("trigram probabilities")
    trigram_counts = count_n_grams(sentences, 3)
    display(make_probability_matrix(trigram_counts, unique_words, k=1))
```

trigram probabilities

	i	cat	а	this	like	is	dog	<e></e>	<unk></unk>
( <s>, this)</s>	0.100000	0.100000	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000
(this, dog)	0.100000	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000
(i, like)	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000	0.100000	0.100000	0.100000
( <s>, i)</s>	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000	0.100000
(is, like)	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000	0.100000	0.100000	0.100000
(a, cat)	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909	0.272727	0.090909
(dog, is)	0.100000	0.100000	0.100000	0.100000	0.200000	0.100000	0.100000	0.100000	0.100000
( <s>, <s>)</s></s>	0.181818	0.090909	0.090909	0.181818	0.090909	0.090909	0.090909	0.090909	0.090909
(like, a)	0.090909	0.272727	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909	0.090909

Confirm that you obtain the same results as for the <code>estimate\_probabilities</code> function that you implemented.

## **Part 3: Perplexity**

In this section, you will generate the perplexity score to evaluate your model on the test set.

- · You will also use back-off when needed.
- Perplexity is used as an evaluation metric of your language model.
- To calculate the the perplexity score of the test set on an n-gram model, use:

$$PP(W) = \sqrt[N]{\prod_{t=n+1}^{N} \frac{1}{P(w_t|w_{t-n}\cdots w_{t-1})}}$$
 (4)

- ullet where N is the length of the sentence.
- *n* is the number of words in the n-gram (e.g. 2 for a bigram).
- In math, the numbering starts at one and not zero.

In code, array indexing starts at zero, so the code will use ranges for t according to this formula:

$$PP(W) = \sqrt[N]{\prod_{t=n}^{N-1} \frac{1}{P(w_t|w_{t-n}\cdots w_{t-1})}}$$
 (4.1)

The higher the probabilities are, the lower the perplexity will be.

• The more the n-grams tell us about the sentence, the lower the perplexity score will be.

#### **Exercise 10**

Compute the perplexity score given an N-gram count matrix and a sentence.

```
In [59]: | # UNQ C10 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         # GRADED FUNCTION: calculate perplexity
         def calculate perplexity(sentence, n gram counts, n plus1 gram counts, vocabulary size, k=1.0):
             Calculate perplexity for a list of sentences
             Args:
                 sentence: List of strings
                 n_gram_counts: Dictionary of counts of (n+1)-grams
                 n plus1 gram counts: Dictionary of counts of (n+1)-grams
                 vocabulary size: number of unique words in the vocabulary
                 k: Positive smoothing constant
             Returns:
                 Perplexity score
             # Length of previous words
             n = len(list(n_gram_counts.keys())[0])
             # prepend <s> and append <e>
             sentence = ["<s>"] * n + sentence + ["<e>"]
             # Cast the sentence from a list to a tuple
             sentence = tuple(sentence)
             # Length of sentence (after adding <s> and <e> tokens)
             N = len(sentence)
             # The variable p will hold the product
             # that is calculated inside the n-root
             # Update this in the code below
             product pi = 1.0
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # Index t ranges from n to N - 1, inclusive on both ends
             for t in range(n, N): # complete this line
                 # get the n-gram preceding the word at position t
                 n gram = sentence[t - n : t]
                 # get the word at position t
                 word = sentence[t]
                 # Estimate the probability of the word given the n-gram
                 # using the n-gram counts, n-plus1-gram counts,
                 # vocabulary size, and smoothing constant
```

Perplexity for first train sample: 2.8040 Perplexity for test sample: 3.9654

```
Perplexity for first train sample: 2.8040 Perplexity for test sample: 3.9654
```

**Note:** If your sentence is really long, there will be underflow when multiplying many fractions.

• To handle longer sentences, modify your implementation to take the sum of the log of the probabilities.

## Part 4: Build an auto-complete system

In this section, you will combine the language models developed so far to implement an auto-complete system.

## **Exercise 11**

Compute probabilities for all possible next words and suggest the most likely one.

• This function also take an optional argument start\_with, which specifies the first few letters of the next words.

```
In [63]: | # UNO C11 (UNIQUE CELL IDENTIFIER, DO NOT EDIT)
         # GRADED FUNCTION: suggest a word
         def suggest a word(previous tokens, n gram counts, n plus1 gram counts, vocabulary, k=1.0, start with=None):
             Get suggestion for the next word
             Args:
                 previous tokens: The sentence you input where each token is a word. Must have length > n
                 n gram counts: Dictionary of counts of (n+1)-grams
                 n_plus1_gram_counts: Dictionary of counts of (n+1)-grams
                 vocabulary: List of words
                 k: positive constant, smoothing parameter
                 start_with: If not None, specifies the first few letters of the next word
             Returns:
                 A tuple of
                   - string of the most likely next word
                   - corresponding probability
             # Length of previous words
             n = len(list(n_gram_counts.keys())[0])
             # From the words that the user already typed
             # get the most recent 'n' words as the previous n-gram
             previous n gram = previous tokens[-n:]
             # Estimate the probabilities that each word in the vocabulary
             # is the next word,
             # given the previous n-gram, the dictionary of n-gram counts,
             # the dictionary of n plus 1 gram counts, and the smoothing constant
             probabilities = estimate probabilities(previous n gram,
                                                     n_gram_counts, n_plus1_gram_counts,
                                                     vocabulary, k=k)
             # Initialize suggested word to None
             # This will be set to the word with highest probability
             suggestion = None
             # Initialize the highest word probability to 0
             # this will be set to the highest probability
             # of all words to be suggested
             max_prob = 0
             ### START CODE HERE (Replace instances of 'None' with your code) ###
             # For each word and its probability in the probabilities dictionary:
```

```
for word, prob in probabilities.items(): # complete this line
    # If the optional start_with string is set
    if start_with: # complete this line
       # Check if the beginning of word does not match with the letters in 'start_with'
       if not word.startswith(start_with): # complete this line
            # if they don't match, skip this word (move onto the next word)
            continue # complete this line
    # Check if this word's probability
   # is greater than the current maximum probability
   if prob > max_prob: # complete this line
       # If so, save this word as the best suggestion (so far)
       suggestion = word
       # Save the new maximum probability
       max_prob = prob
### END CODE HERE
return suggestion, max_prob
```

```
In [64]: # test your code
         sentences = [['i', 'like', 'a', 'cat'],
                      ['this', 'dog', 'is', 'like', 'a', 'cat']]
         unique words = list(set(sentences[0] + sentences[1]))
         unigram counts = count n grams(sentences, 1)
         bigram counts = count n grams(sentences, 2)
         previous tokens = ["i", "like"]
         tmp suggest1 = suggest a word(previous tokens, unigram counts, bigram counts, unique words, k=1.0)
         print(f"The previous words are 'i like',\n\tand the suggested word is `{tmp suggest1[0]}` with a probability of {tmp su
         ggest1[1]:.4f}")
         print()
         # test your code when setting the starts with
         tmp starts with = 'c'
         tmp suggest2 = suggest a word(previous tokens, unigram counts, bigram counts, unique words, k=1.0, start with=tmp start
         s with)
         print(f"The previous words are 'i like', the suggestion must start with `{tmp starts with}`\n\tand the suggested word i
         s `{tmp suggest2[0]}` with a probability of {tmp suggest2[1]:.4f}")
         The previous words are 'i like',
```

```
The previous words are 'i like',
and the suggested word is `a` with a probability of 0.2727

The previous words are 'i like', the suggestion must start with `c`
and the suggested word is `cat` with a probability of 0.0909
```

```
The previous words are 'i like', and the suggested word is 'a' with a probability of 0.2727

The previous words are 'i like', the suggestion must start with 'c' and the suggested word is 'cat' with a probability of 0.0909
```

## **Get multiple suggestions**

The function defined below loop over varioud n-gram models to get multiple suggestions.

```
In [71]: def get_suggestions(previous_tokens, n_gram_counts_list, vocabulary, k=1.0, start_with=None):
             model counts = len(n gram counts list)
             suggestions = []
             for i in range(model_counts-1):
                 n gram counts = n gram counts list[i]
                 n_plus1_gram_counts = n_gram_counts_list[i+1]
                 suggestion = suggest_a_word(previous_tokens, n_gram_counts,
                                             n plus1_gram_counts, vocabulary,
                                             k=k, start with=start with)
                 suggestions.append(suggestion)
             return suggestions
In [74]: # test your code
         sentences = [['i', 'like', 'a', 'cat'],
                      ['this', 'dog', 'is', 'like', 'a', 'cat']]
         unique words = list(set(sentences[0] + sentences[1]))
         unigram_counts = count_n_grams(sentences, 1)
         bigram_counts = count_n_grams(sentences, 2)
         trigram_counts = count_n_grams(sentences, 3)
         quadgram_counts = count_n_grams(sentences, 4)
         qintgram_counts = count_n_grams(sentences, 5)
         n_gram_counts_list = [unigram_counts, bigram_counts, trigram_counts, quadgram_counts, qintgram_counts]
         previous tokens = ["i", "like"]
         tmp_suggest3 = get_suggestions(previous_tokens, n_gram_counts_list, unique_words, k=1.0)
         print(f"The previous words are 'i like', the suggestions are:")
         display(tmp suggest3)
         The previous words are 'i like', the suggestions are:
         [('a', 0.27272727272727),
          ('a', 0.2),
          ('i', 0.11111111111111),
          ('i', 0.111111111111111)]
```

## Suggest multiple words using n-grams of varying length

Congratulations! You have developed all building blocks for implementing your own auto-complete systems.

Let's see this with n-grams of varying lengths (unigrams, bigrams, trigrams, 4-grams...6-grams).

```
In [75]: n gram counts list = []
         for n in range(1, 6):
             print("Computing n-gram counts with n =", n, "...")
             n model counts = count n grams(train data processed, n)
             n gram counts list.append(n model counts)
         Computing n-gram counts with n = 1 \dots
         Computing n-gram counts with n = 2 ...
         Computing n-gram counts with n = 3 ...
         Computing n-gram counts with n = 4 \dots
         Computing n-gram counts with n = 5 \dots
In [76]: previous_tokens = ["i", "am", "to"]
         tmp suggest4 = get suggestions(previous tokens, n gram counts list, vocabulary, k=1.0)
         print(f"The previous words are {previous tokens}, the suggestions are:")
         display(tmp suggest4)
         The previous words are ['i', 'am', 'to'], the suggestions are:
         [('be', 0.027665685098338604),
          ('have', 0.00013487086115044844),
          ('have', 0.00013490725126475548),
          ('i', 6.746272684341901e-05)]
In [77]: previous_tokens = ["i", "want", "to", "go"]
         tmp suggest5 = get suggestions(previous tokens, n gram counts list, vocabulary, k=1.0)
         print(f"The previous words are {previous tokens}, the suggestions are:")
         display(tmp suggest5)
         The previous words are ['i', 'want', 'to', 'go'], the suggestions are:
         [('to', 0.014051961029228078),
          ('to', 0.004697942168993581),
          ('to', 0.0009424436216762033),
          ('to', 0.0004044489383215369)]
```

```
In [78]: previous tokens = ["hey", "how", "are"]
         tmp suggest6 = get suggestions(previous tokens, n gram counts list, vocabulary, k=1.0)
         print(f"The previous words are {previous tokens}, the suggestions are:")
         display(tmp suggest6)
         The previous words are ['hey', 'how', 'are'], the suggestions are:
         [('you', 0.023426812585499317),
          ('you', 0.003559435862995299),
          ('you', 0.00013491635186184566),
          ('i', 6.746272684341901e-05)]
In [79]: previous_tokens = ["hey", "how", "are", "you"]
         tmp_suggest7 = get_suggestions(previous_tokens, n_gram_counts_list, vocabulary, k=1.0)
         print(f"The previous words are {previous tokens}, the suggestions are:")
         display(tmp suggest7)
         The previous words are ['hey', 'how', 'are', 'you'], the suggestions are:
         [("'re", 0.023973994311255586),
          ('?', 0.002888465830762161),
          ('?', 0.0016134453781512605),
          ('<e>', 0.00013491635186184566)]
In [80]: previous_tokens = ["hey", "how", "are", "you"]
         tmp_suggest8 = get_suggestions(previous_tokens, n_gram_counts_list, vocabulary, k=1.0, start_with="d")
         print(f"The previous words are {previous_tokens}, the suggestions are:")
         display(tmp_suggest8)
         The previous words are ['hey', 'how', 'are', 'you'], the suggestions are:
         [('do', 0.009020723283218204),
          ('doing', 0.0016411737674785006),
          ('doing', 0.00047058823529411766),
          ('dvd', 6.745817593092283e-05)]
```

# **Congratulations!**

You've completed this assignment by building an autocomplete model using an n-gram language model!

Please continue onto the fourth and final week of this course!