MATH 6602 HOMEWORK #21 (APRIL 10, 2019)

Consider the inviscid Burger's equation

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial}{\partial x}\left(\dfrac{u^2}{2}\right) = 0, & 0 \le x \le 2\pi, \\ u(x,0) = 1 + \dfrac{1}{2}\sin x, \end{cases}$$

with periodic boundary condition.

(1) Determine the critical time t^* such that the solution develops shock afterward. Express the analytic solution before and after the critical time t^*.

(2) Solve the equation using a 3rd-order numerical method, which has

- 3rd-order finite volume approximation in space;
- 3rd-order Runge-Kutta integration in time.

Use equi-distance grids with 50, 100, 200, 400 grid points, and solve the equation up to $T_1 = t^*/2$ and also $T_2 = 3$.

- Plots the numerical solutions (against analytical solution).
- Report the numerical errors *vs* number of grid points for T_1 case.
- Comment on what you observe.

Reference:
Third-order finite-volume approximation

$$v_{j+\frac{1}{2}} = -\frac{1}{6}\bar{v}_{j-1} + \frac{5}{6}\bar{v}_j + \frac{1}{3}\bar{v}_{j+1}$$