MATH 6602 HOMEWORK #18 (APRIL, 2019)

Consider the following function

$$f(x) = e^{\sin(x)}, \qquad x \in [0, 2\pi].$$

Let $x_0, \ldots, x_N$ be a set of uniform grids in the domain with $h = 2\pi/(N+1)$. Let $f'(x_j)$ be its derivative at grid x_j and g_j be its numerical approximation.

1. Compute numerical derivatives g_j using 1st-order approximation (pick a direction), 2nd order central difference, and 4th-order central difference, using $N = 10, 20, 40, 60, 80$. Tabulate the errors, which are defined as

$$Error(N) = \sqrt{\frac{1}{N}\sum_{j=0}^{N}(f'(x_j) - g_j)^2}$$

Plot the errors vs N on the same plot for all three methods, using log-log scale (i.e. $\log Error(N)$ vs $\log N$. Describe what you observe about the error convergence behavior.

2. Repeat the same procedure using global approximation. Comment on the error convergence behavior. Re-plot the errors on $\log Error(N)$ vs N scale. What do you observe now?