MATH 6602 HOMEWORK #20 (APRIL 8, 2019)

Consider the simple linear advection equation

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial u}{\partial x} = 0, & -1 \le x \le 1, \\ u(x,0) = u^0(x), \end{cases}$$

with periodic boundary condition.

Consider two kinds of initial conditions:

$$u_1^0(x) = \exp(-x^2/0.04);$$

and

$$u_2^0(x) = \begin{cases} 1, & -0.4 \le x \le 0.4, \\ 0, & \text{otherwise.} \end{cases}$$

Use uniform grids with N interval of $N = 40, 80, 160, 320$ and solve the above two cases using

- Upwind
- Leap-frog
- Lax-Friedrichs
- Lax-Wendroff
- Crank-NIcolson
- Global finite difference (using all points)

Choose appropriate time step Δt and solve the equation for $T = 3$ and $T = 30$.

Plot the numerical solutions (against) the exact solution for all these cases and comment on what you observe.