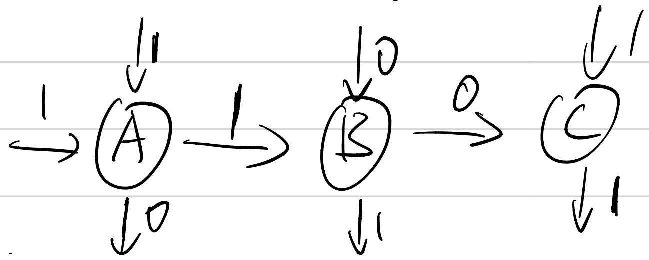
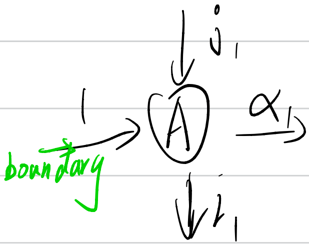
Hw 2

problem 1(a)

Right-shift :





$j_1+1 \to i_1$
$\downarrow \alpha_1$

output

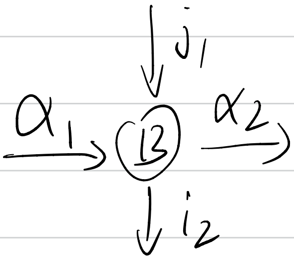
Entry	j_1	i_1	α_1
1	0	1	0
1	1	0	1

$$S^0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad S^- = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad S^+ = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$A[0\,1\,0] = 1$

$A[1,0,1] = 1$

$$A = [\, S^+ \quad S^- \,]$$



$\alpha_1 + j_1 \to i_2$
$\downarrow \alpha_2$

α_1	j_1	i_2	α_2
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

output

$B[0\,0\,0\,0] = 1$

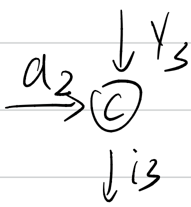
$B[0\,1\,1\,0] = 1$

$B[1\,0\,1\,0] = 1$

$B[1\,1\,0\,1] = 1$

order 4 : $(2\times2\times2\times2)$

$$B = \begin{bmatrix} S^0 & S^+ \\ S^+ & S^- \end{bmatrix}$$



$\alpha_2 + \gamma_3 \to i_3$

α_2	γ_3	i_3
0	0	0
0	1	1
1	0	1
1	1	0

boundary condition

$(2\times2\times2)$

$\Rightarrow$

$C[0\,0\,0] = 1$

$C[0\,1\,1] = 1$

$C[1\,0\,1] = 1$

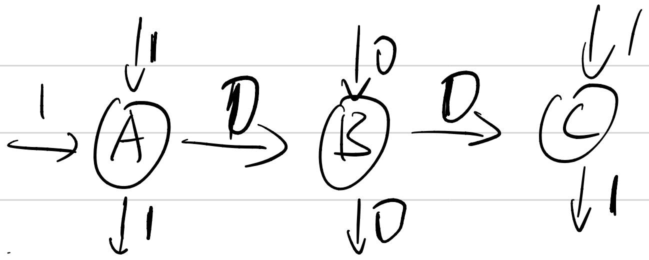
$C[1\,1\,0] = 1$

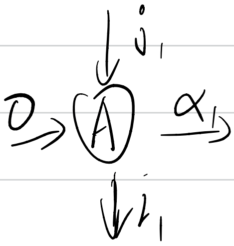
$$C = [\, I \quad S^+ + S^- \,]$$

$$H_{\gamma s} = \sum H^{n_1} \cdots H^{n_N} = \sum A^{\gamma_1}_{\alpha_1\beta_1} B^{\gamma_2 n_2}_{\alpha_2\beta_2} \cdots B^{\gamma_{N-1} n_{N-1}}_{\alpha_{N-1}\beta_{N-1}} C^{n_N}_{\alpha_N \beta_N}$$

problem 1(b)

Idenity matrix :





$j_1+1 \to i_1$

$\downarrow \alpha_1$

		output	
Entry	j_1	i_1	α_1
0	0	0	0
0	1	1	0

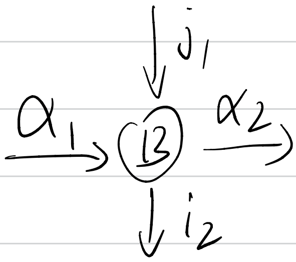
$A[000]=1$

$A[110]=1$

$A = [0 \;\; s^+]$

$\alpha_1+j_1 \to i_2$

$\downarrow \alpha_2$

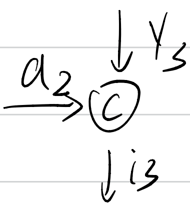


		output	
α_1	j_1	i_2	α_2
0	0	0	0
0	1	1	0

$$B = \begin{bmatrix} s^0 & s^+ \\ 0 & 0 \end{bmatrix}$$

$B[0000]=1$

$B[0110]=1$ order 4 : $(2\times2\times2\times2)$



$a_2+\gamma_3 \to i_3$

α_2	γ_3	i_3
0	0	0
0	1	1

$(2\times2\times2)$

$\Rightarrow$ $C[000]=1$

$C[011]=1$

$C = [I \;\; 0]$

$$H_{rs} = \sum H^{n_1}\cdots H^{n_N} = \sum A^{r_1}_{\alpha_1\beta_1} B^{\gamma_2 n_2}_{\alpha_2\beta_2}\cdots B^{\gamma_{N-1} n_{N-1}}_{\alpha_{N-1}\beta_{N-1}} C^{n_N}_{\alpha_N\beta_N}$$

problem 3

(a) For $N=4$

$$H = W^{[1]} W^{[2]} W^{[3]} W^{[4]}$$

$$= \begin{bmatrix} -hS_1^x & J_1 S^z & J_2 S^z & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ S^z & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ -hS_2^x & J_1 S^z & J_2 S^z & I \end{bmatrix} W^{[3]} W^{[4]}$$

$$= \begin{bmatrix} -hS_1^x + J_1 S_1^z S_2^z - hS_2^x & J_2 S_1^z + J_1 S_2^z & J_2 S_2^z & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ S^z & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ -hS_3^x & J_1 S^z & J_2 S^z & I \end{bmatrix} W^{[4]}$$

$$= \begin{bmatrix} J_1 S_1^z S_2^z - h(S_1^x + S_2^x + S_3^x) + J_2 S_1^z S_3^z + J_1 S_2^z S_3^z & J_2 S_2^z + J_1 S_3^z & J_2 S_3^z & I \end{bmatrix} \begin{bmatrix} I \\ S_4^z \\ 0 \\ -hS_4^z \end{bmatrix}$$

$$= J_1 S_1^z S_2^z + J_1 S_2^z S_3^z + J_1 S_3^z S_4^z$$

$$+ J_2 S_1^z S_3^z + J_2 S_2^z S_4^z$$

$$- h(S_1^x + S_2^x + S_3^x + S_4^x)$$

$$= J_1 \sum_{i=1}^{3} S_i^z S_{i+1}^z + J_2 \sum_{i=1}^{2} S_i^z S_{i+2}^z - h \sum_{i=1}^{4} S_i^x$$

1b) $H = \sum_{i=1}^{N-1} \sum_{r=1}^{N-i} J(r)\, S_i^z\, S_{i+r}^z$

$J(r) = J e^{-\frac{r}{\xi}}$

Example $N = 4$

$$W^1 = [\,0 \quad J\lambda S^z \quad I\,]$$

$$W^4 = \begin{bmatrix} I \\ S^z \\ 0 \end{bmatrix}$$

$$W^{2,3} = \begin{bmatrix} I & 0 & 0 \\ S^z & \lambda I & 0 \\ 0 & J\lambda S^z & I \end{bmatrix}$$

$$H = [\,0 \;\; J\lambda S_1^z \;\; I\,] \begin{bmatrix} I & 0 & 0 \\ S_2^z & \lambda I & 0 \\ 0 & J\lambda S_2^z & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ S_3^z & \lambda I & 0 \\ 0 & J\lambda S_3^z & I \end{bmatrix} \begin{bmatrix} I \\ S_4^z \\ 0 \end{bmatrix}$$

$$= [\,J\lambda S_1^z S_2^z \quad J\lambda^2 S_1^z + J\lambda S_2^z \quad I\,] \begin{bmatrix} I & 0 & 0 \\ S_3^z & \lambda I & 0 \\ 0 & J\lambda S_3^z & I \end{bmatrix} \begin{bmatrix} I \\ S_4^z \\ 0 \end{bmatrix}$$

$$= [\,J\lambda S_1^z S_2^z + J\lambda^2 S_1^z S_3^z + J\lambda S_2^z S_3^z \quad J\lambda^3 S_1^z + J\lambda^2 S_2^z + J\lambda S_3^z \quad I\,] \begin{bmatrix} I \\ S_4^z \\ 0 \end{bmatrix}$$

$$= J\lambda S_1^z S_2^z + J\lambda^2 S_1^z S_3^z + J\lambda^3 S_1^z S_4^z + J\lambda S_2^z S_3^z + J\lambda^2 S_2^z S_4^z + J\lambda S_3 S_4$$

$$= \sum_{i=1}^{4} \sum_{j=i+1}^{4} J \lambda^{j-i} S_i^z S_j^z$$