Homework 4
Statistics 201B

None of these problems will be turned in.

1. Let $X_1, \ldots, X_n \overset{iid}{\sim} F$ and let $\hat{F}_n$ be the empirical distribution function. Let $a < b$ be fixed numbers and define $\theta = T(F) = F(b) - F(a)$. Let $\hat{\theta} = T(\hat{F}_n) = \hat{F}_n(b) - \hat{F}_n(a)$. Find the estimated standard error of $\hat{\theta}$. Find an expression for an approximate $1 - \alpha$ confidence interval for θ.

2. Let $X_1, \ldots, X_n$ be i.i.d. distinct observations (no ties) from a distribution F. Let $X_1^*, \ldots, X_n^*$ denote an i.i.d sample from the empirical CDF, i.e.

$$X_i^* \overset{i.i.d.}{\sim} \hat{F}_n.$$

   $X_1^*, \ldots, X_n^*$ is called a *bootstrap sample*. Let $\bar{X}_n^* = \frac{1}{n}\sum_{i=1}^n X_i^*$. Find the conditional and unconditional mean and variance of $\bar{X}_n^*$:

$$E_{\hat{F}_n}\bar{X}_n^* = E(\bar{X}_n^*|X_1, \ldots, X_n)$$
$$Var_{\hat{F}_n}(\bar{X}_n^*) = Var(\bar{X}_n^*|X_1, \ldots, X_n)$$
$$E(\bar{X}_n^*)$$
$$Var(\bar{X}_n^*)$$