



I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

INFO 251: Applied Machine Learning

Regression and Impact Evaluation, part 2

Announcements

- Enrollment updates
 - Roster is close to final!

Outline

- Regression and Impact Evaluation
- Heterogeneous treatment effects
- Double-Difference via Regression
- Fixed effects and Normalization

Key Concepts (last lecture)

- Randomization pitfalls
 - Spillovers
 - Non-compliance
 - Non-random attrition
- Progresa
 - Motivation
 - Program design and roll-out
 - Different approaches to impact evaluation
- Interpreting regression coefficients
- Dummy variables, "one-hot" vectors

Key Concepts (this lecture)

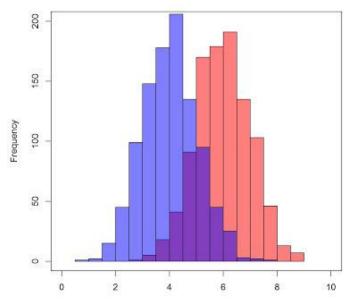
- Regression for impact evaluation
- Control variables
- Interaction variables
- Heterogeneous treatment effects
- Double difference estimation
- Fixed effects (revisited)

Regression and Impact: Basics

- How to measure effect of a binary treatment T on outcome Y via regression?
 - The regression equation:

$$Y_i = \alpha + \beta T_i + \epsilon_i$$

- Example:
 - Y_i indicates # classes attended (1-10)
 - T_i indicates whether the student received an email encouraging attendance
 - We estimate $\hat{\alpha} = 4.1, \hat{\beta} = 1.3$. What does this mean?
- If T is not randomly assigned, $\hat{\beta}$ tells us about the correlation between T on Y
- If T is randomly assigned, $\hat{\beta}$ is an estimate of the *causal impact* of T on Y



Regression: "Control" variables

How to simultaneously estimate the effect of a randomized treatment T and a non-random control variable X on an outcome Y?

$$Y_i = \alpha_1 + \beta_1 T_i + \gamma X_i + e_i$$

How is this different from a version without control variables?

$$Y_i = \alpha_2 + \beta_2 T_i + \epsilon_i$$

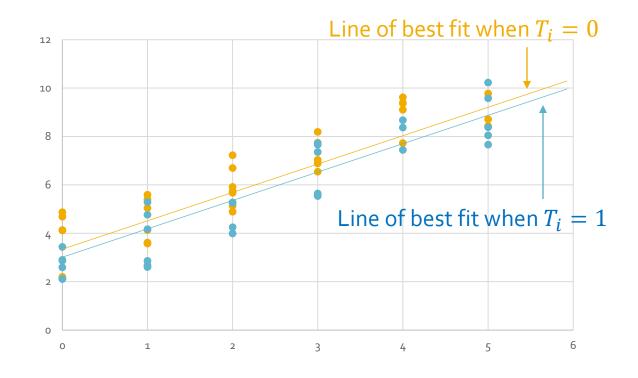
- If T is perfectly randomized...
 - What, if anything, can we say about $Cor(T_i, X_i)$?
 - What, if anything, can we say about our estimates of β_1 and β_2 ?
 - What, if anything, can we say about γ ?

Control variables: Example

- Example: We are estimating the effect of receiving an email (T_i) on attendance (Y_i) , while controlling for years in grad school (X_i)
- Regression equation:

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$$Y_i = \alpha + \beta T_i + \gamma X_i + \epsilon_i$$

- Coefficient estimates
 - $\hat{\alpha} = 3.4, \hat{\beta} = -0.5, \hat{\gamma} = 1.2$
 - What do these results mean?
- How to visualize?



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Treatment effect heterogeneity

- How to simultaneously measure the effect on an outcome Y of: (i) a randomized treatment T; and (ii) a non-random control variable X; and (iii) allow for the treatment effect to vary with the control variable?
 - Example: Phd students may respond differently to emails than MA students
 - The treatment effect is different for different types of people (where "type" is measured with X_i)
 - We can estimate this with a regression:

$$Y_i = \alpha + \beta T_i + \gamma X_i + \delta (T_i * X_i) + \epsilon_i$$

Treatment effect heterogeneity: Binary X

- Example 1: We want to estimate the effect of receiving and email (T) on attendance (Y), while controlling for PhD (X) and the interaction (T*X)
 - $Y_i = \alpha + \beta T_i + \gamma X_i + \delta (T_i * X_i) + \epsilon_i$
 - $\hat{\alpha} = 1.4, \hat{\beta} = -1.0, \hat{\gamma} = 0.8, \hat{\delta} = 0.6$
 - What do these results mean?
 - What is the expected happiness for:
 - A PhD student with a cookie
 - An MA student with a cookie
 - A PhD student without a cookie
 - An MA student without a cookie

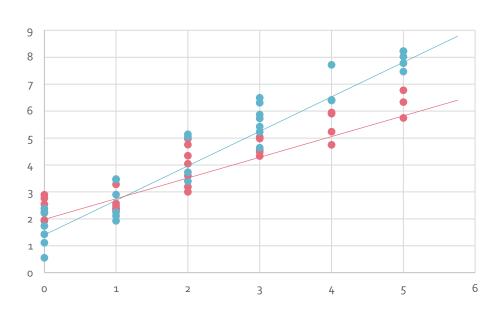
Treatment effect heterogeneity: Continuous X

- Example 2: We want to estimate the effect of receiving an email (T) on attendance (Y), while controlling for years in grad school (X) (a continuous variable), and the interaction effect (T*X)
 - Note: this is identical to the last example, except now our X variable is continuous, not binary

$$Y_i = \alpha + \beta T_i + \gamma X_i + \delta (T_i * X_i) + \epsilon_i$$

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$$\hat{\alpha} = 1.4, \hat{\beta} = -1.0, \hat{\gamma} = 0.8, \hat{\delta} = 0.6$$

- What do these results mean?
- How to visualize?



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Regression and Impact: Diff-in-Diff

- In a typical double-difference setting, data is collected at baseline (pretreatment) and at endline (post-treatment)
- Very similar to a situation where you have a treatment T and a nonexperimental binary control variable X and a differential effect of treatment by a binary control variable
- In this case, the control variable is time!
 - Instead of a binary X_i (in previous example, X=PhD), we have a binary $Post_i$

Regression and Impact: Diff-in-Diff

For example, our data look like this:

ID	Υ	Post?	Treatment?	Post * Treat	
1	12	o (PRE)	ı (Treat)	0	
1	14	1 (POST)	ı (Treat)	1	
2	11	o (PRE)	o (Control)	0	
2	11	1 (POST)	o (Control)	0	
3	24	o (PRE)	o (Control)	0	
	•••				
12419					•••

	Control	Treat	
Pre	Α	В	
Post	C	D	

This is the regression equation:

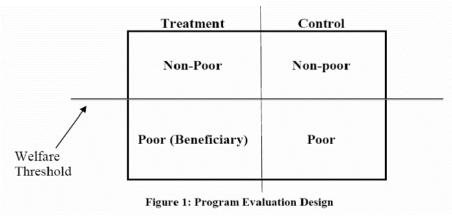
$$Y_i = \alpha + \beta T_i + \gamma Post_i + \delta (T_i * Post_i) + \epsilon_i$$

Regression and Impact: Diff-in-Diff

Let's return to Progresa (Shultz, 2004: page 209)

$$S_i = \alpha_0 + \alpha_1 P_i + \alpha_2 E_i + \alpha_3 P_i E_i + \sum_{k=1}^K \gamma_{ki} C_{ki} + \sum_{j=1}^J \beta_j X_{ji} + e_i \quad i = 1, 2 \dots, n$$
 (1)

- P_i = Progresa village ("T")
- E_i = Eligible (poor) household ("X")
- $P_i E_i$ = Difference-in-difference estimator
- C_{ki} = Dummy variable for grade of child
 - Controls for fact that enrollment rates vary across grades
- X_{jj} = Other control variables (I think?)



Regression and Impact: Summary

Double Difference

$$Y_i = \alpha + \beta T_i + \gamma Post_i + \delta (T_i * Post_i) + \epsilon_i$$

Simple difference

$$Y_i = \alpha + \beta T_i + \epsilon_i$$

- (Estimated only in Post period)
- Pre vs. Post

$$Y_i = \alpha + \gamma Post_i + \epsilon_i$$

(Estimated only on treatment group)

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Fixed Effects and Normalization

- Cross-sectional data: Data from several units at a single point in time
 - Schooling outcomes in 1998
 - Happiness of students on Jan 20
 - Number of logins per person in March
- Time series (panel) data: Multiple observations for each unit over time
 - Schooling outcomes in 1997 and 1998
 - Happiness of students on Jan 20 and Jun 20
 - Number of logins per person in each month of 2010

Between and Within Variation

- Cross-sectional data:
 - Variation is between/across units

Location	Year	Price	Quantity sold (per capita)
Chicago	2003	\$75	2.0
Seattle	2003	\$50	1.0
Milwaukee	2003	\$60	1.5
Madison	2003	\$55	0.8

- What do you notice about relationship between price and quantity?
 - Across the four cities, price and quantity are positively correlated
 - This is not what we would expect omitted variables likely matter

Between and Within Variation

Panel data:

 Variation is between/across units and within units over time

- What do you notice about relationship between price and quantity?
 - Within each of the four cities, price and quantity are inversely correlated (as expected with downward sloping demand)

Location	Year	Price	Quantity
Chicago	2003	\$75	2.0
Chicago	2004	\$85	1.8
Seattle	2003	\$50	1.0
Seattle	2004	\$48	1.1
Milwaukee	2003	\$60	1.5
Milwaukee	2004	\$65	1.4
Madison	2003	\$55	0.8
Madison	2004	\$60	0.7

Exploiting Within-Unit Variation

- How to isolate changes in outcomes correlated with changes within a unit?
 - e.g., Changes in demand caused by changes in price within a given city (over time)

Exploiting Within-Unit Variation

- One (familiar?) approach: "difference" regressions
 - Isolate differences over time in X and Y
 - Instead of: $Y_{it} = \alpha + \beta X_{it} + \epsilon_{it}$
 - $(Y_{it} Y_{i(t-1)}) = \alpha + \beta(X_{it} X_{i(t-1)}) + \epsilon_{it}$
 - Are changes in Y related to changes in X?
- Another approach: normalization
 - Instead of: $Y_{it} = \alpha + \beta X_{it} + \epsilon_{it}$
 - $(Y_{it} \overline{Y}_i) = \alpha + \beta (X_{it} \overline{X}_i) + \epsilon_{it}$

Exploiting Within-Unit Variation

- A third (closely related) approach: Fixed effects
- Basic idea (refer to lecture notes for details)
 - Instead of: $(Y_{it} \overline{Y}_i) = \alpha + \beta (X_{it} \overline{X}_i) + \epsilon_{it}$
 - Add "dummy" variables for each i: $Y_{it} = \alpha + \beta X_{it} + (\mu_i + \dots + \mu_N) + \epsilon_{it}$
 - Conceptually the same as the "country fixed effect" example from previous lecture
 - More formally: $Y_{it} = \alpha + \beta X_{it} + \sum_{j=2}^{N} \mu_j \mathbf{1}(ID_i = j) + \epsilon_i$
 - Equivalent to adding an intercept for each i
 - Shorthand: $Y_{it} = \alpha + \beta X_{it} + \mu_i + \epsilon_{it}$
- Note: we can also "normalize" for time FE's
 - $Y_{it} = \alpha + \beta X_{it} + \mu_i + \pi_t + \epsilon_{it}$

Fixed Effects: Digging Deeper

- Key advantage of Fixed Effects
 - Fixed effects control for <u>unobserved heterogeneity</u>
 - They remove the effect of time-invariant characteristics to assess the net effect of the predictors on the outcome
- Extensions
 - Time trends
 - Region-specific slopes

Additional Resources

Beginner → Advanced

