Practice Problems for Exam
Statistics 201B

1. Let $X_1, ..., X_n$ be iid exponential with density $f(x; \beta) = \beta e^{-\beta x}$ for $x > 0$ and $\beta > 0$.

   (a) Find the asymptotic (large sample) likelihood ratio test of size α for $H_0 : \beta = \beta_0$ versus $H_1 : \beta \neq \beta_0$.

   (b) Find a Wald test of size α for the same null hypothesis.

2. (Wasserman, Question 13) Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$.

   (a) Find the asymptotic (large sample) likelihood ratio test of size α for $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$.

   (b) Find a Wald test of size α for the same null hypothesis.

3. Suppose that instead of 0-1 loss, you had a loss that was not symmetric, so that different mistakes counted differently,

$$L(Y, \hat{Y}) = \begin{cases} 0 & \hat{Y} = Y \\ a & \hat{Y} = 0 \text{ and } Y = 1 \\ b & \hat{Y} = 1 \text{ and } Y = 0 \end{cases}$$

   Show that the following is true for the optimal (Bayes) classification rule $h^*(x)$ in this case:

   (a)

$$h^*(x) = \begin{cases} 1 & \text{if } r(x) > \frac{b}{a+b} \\ 0 & \text{otherwise} \end{cases}$$

   (b) The decision boundary is given by

$$\{x : P(Y = 1|X) = \frac{b}{a} P(Y = 0|X)\}$$

   where $r(x) = P(Y = 1|X) = \frac{\pi_1 f_1(x)}{\pi_1 f_1(x) + (1-\pi_1) f_0(x)}$

4. Show that decision boundary for logistic regression is linear.