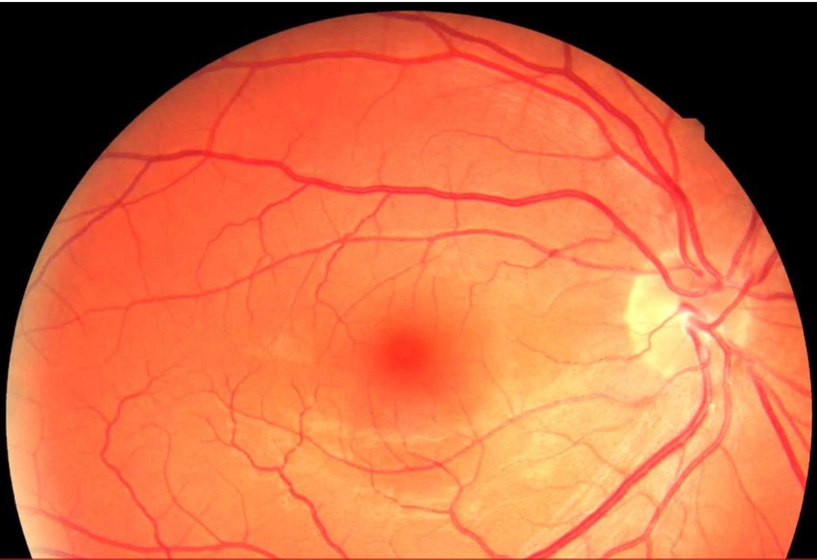
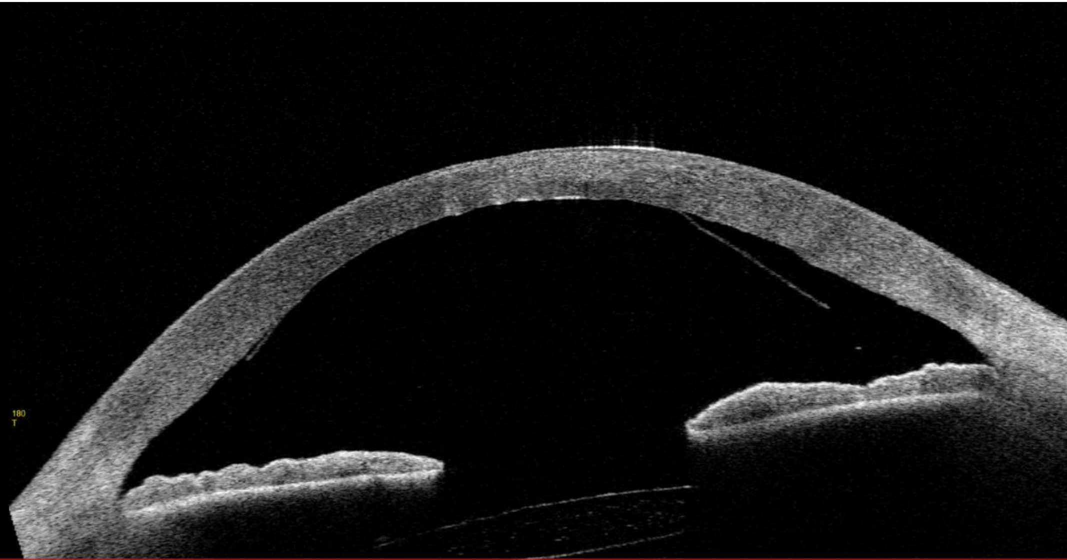
EINDHOVEN UNIVERSITY OF TECHNOLOGY

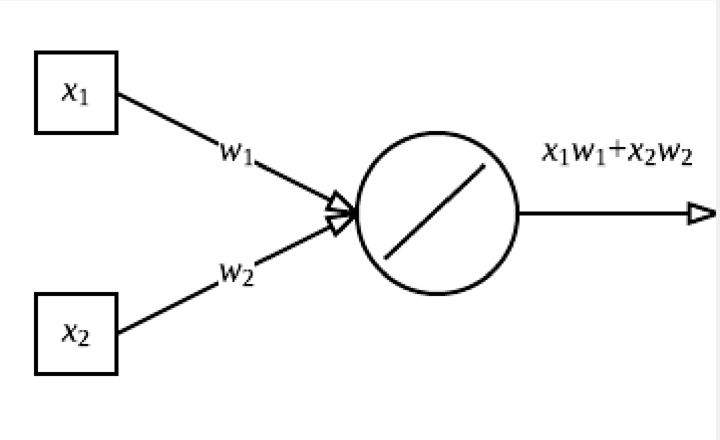




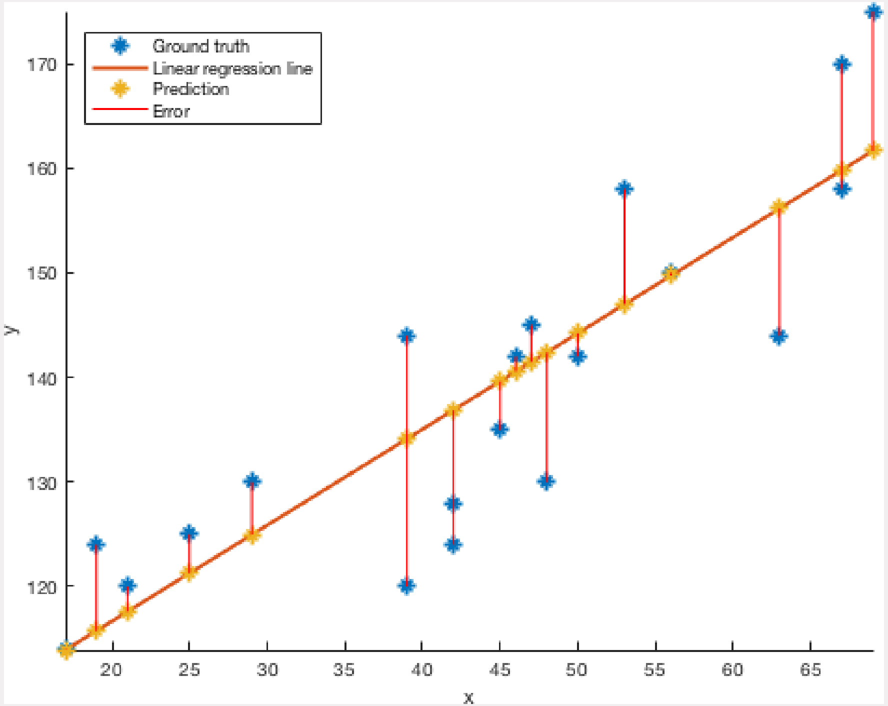
Convolutional Neural Networks (8DC00)

Cian Scannell (slides from Friso Heslinga)

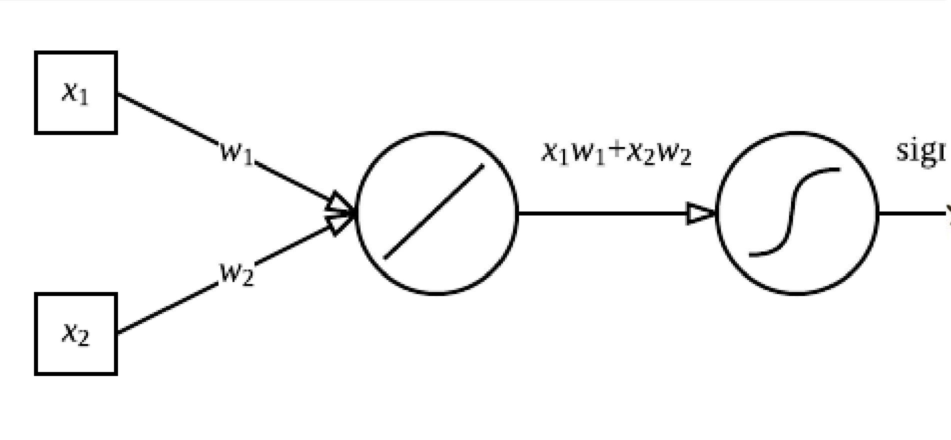
Previously – Linear regression



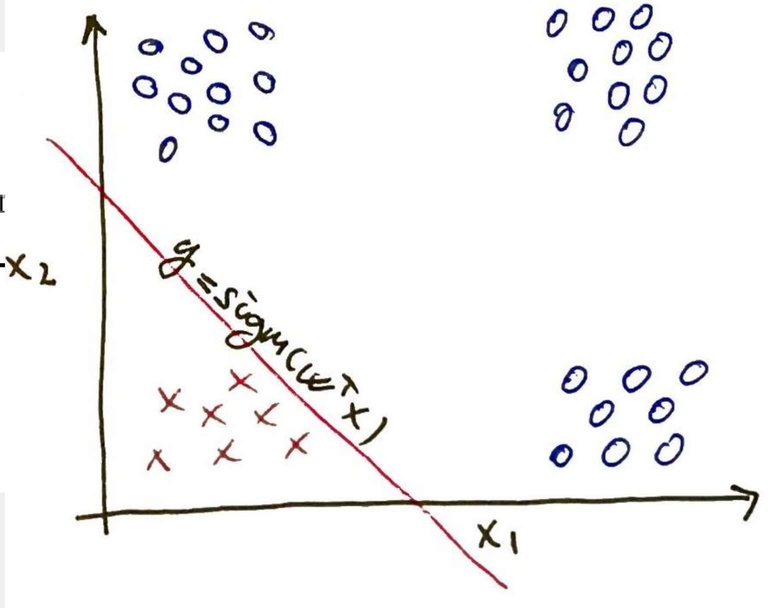
$$\hat{y} = \theta^{\mathsf{T}}\mathbf{x}$$



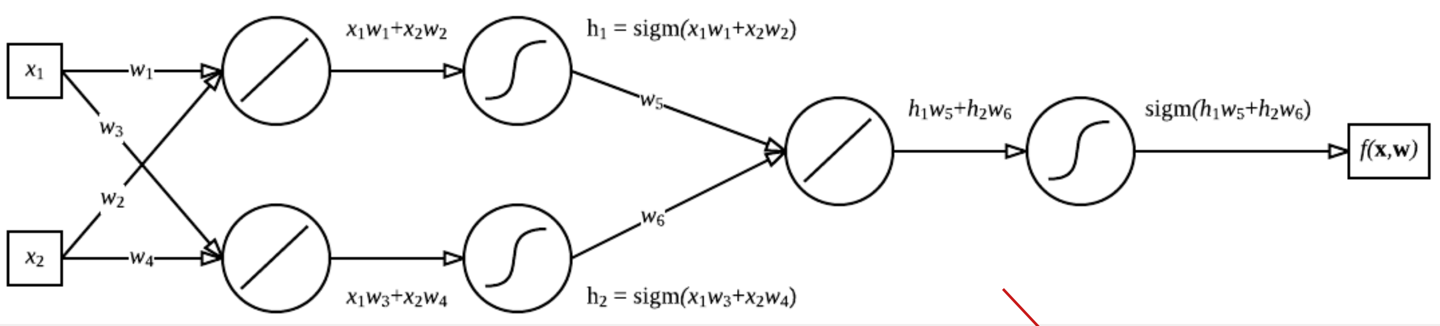
Previously – Logistic regression

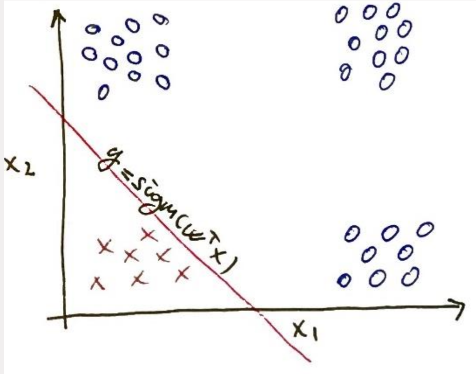


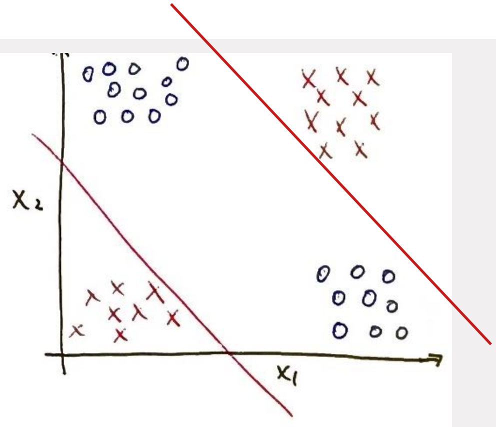
$$p(y = 1 \mid \mathbf{x}) = \sigma(\boldsymbol{\theta}^\intercal \mathbf{x})$$



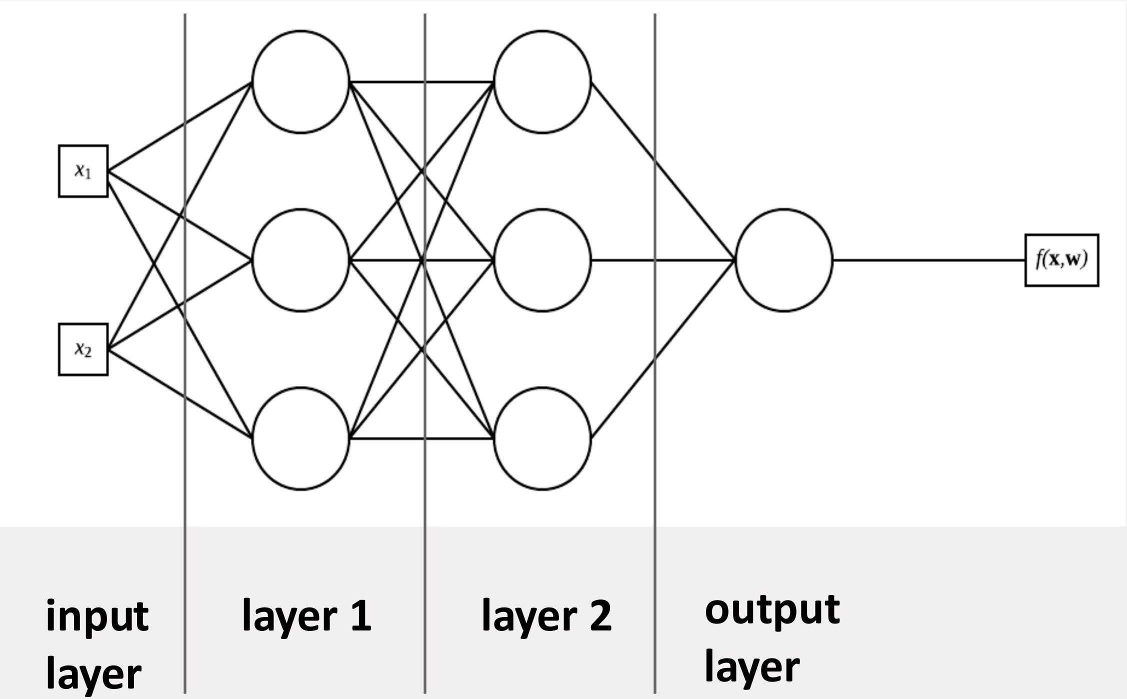
Previously – Neural networks

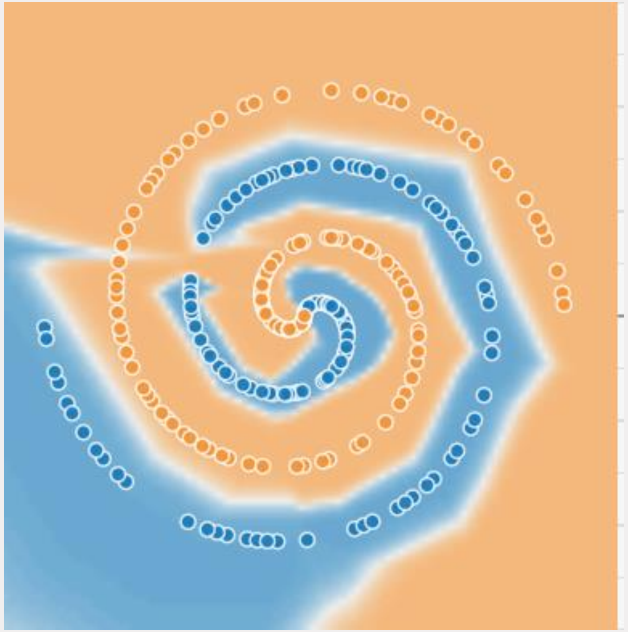






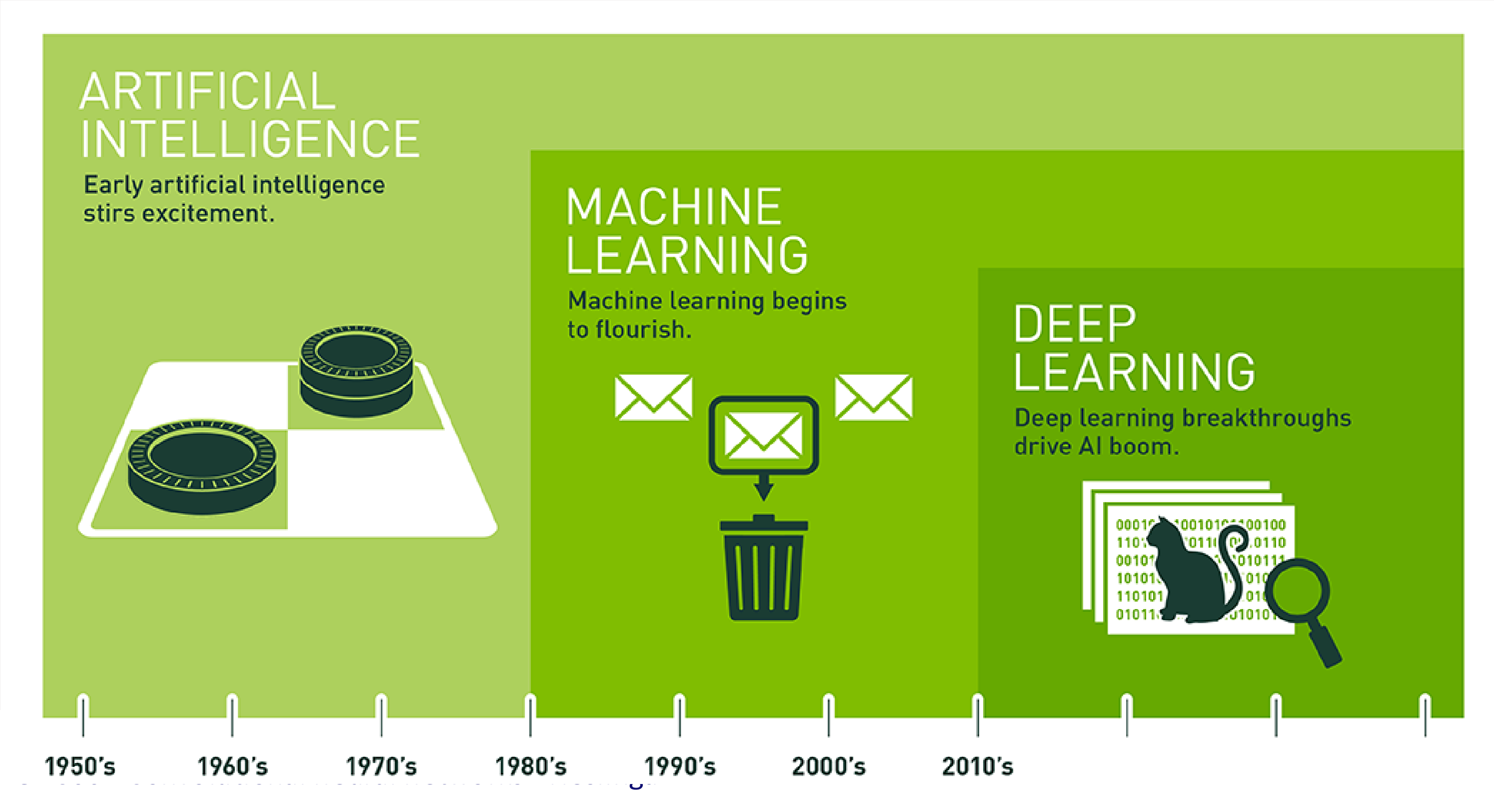
Previously – Neural networks





Previously – Machine learning

Figure source: nvidia.com



Today: Convolutional neural networks (CNN)

- Neural networks → Convolutional neural networks

Building blocks for deep learning models for image analysis:

- Convolutional layer
- Max-pooling layer

- Not needed for the project, but will be on the exam

Learning outcomes

- Student can explain the concept of **convolutions** in a neural network
- Student can describe why we can use a **convolutional approach** for (medical) **images**
- Student can explain why convolutions enable development of **deep** (and large) **neural networks**
- Students can explain and apply the **max-pooling layer** in a convolutional neural network
- Students can motivate the choice for a **kernel size**

Lecture outline

- Images as input to neural network
- Reducing # of weights
- 1D convolutions
- 2D convolutions
- Kernels
- Max-pooling
- Interactive example

Images as input to neural networks

24 pixels

24 pixels

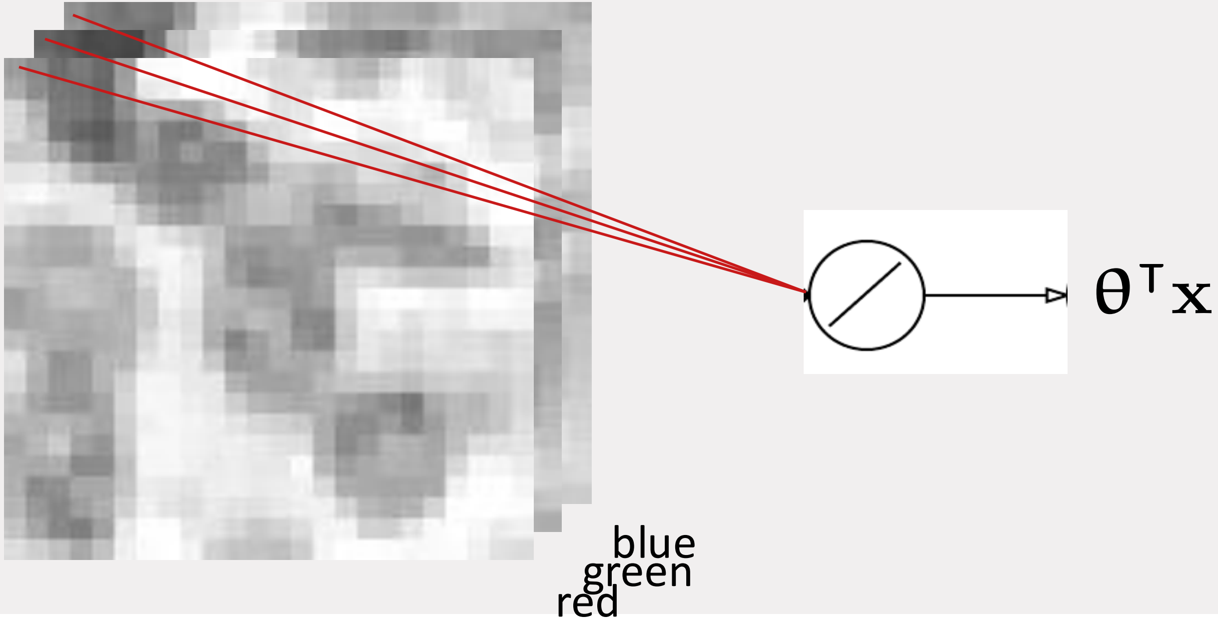
3 color channels
(red, green, blue)

= 1728 features ($\mathbf{x}_i$ are 1728 dimensional vectors)

If we train linear regression with these inputs (such as in the first practical), we will have 1728 weights w and a bias b.

Every pixel is an input

Every pixel from every color channel is multiplied by a weight



Post-training: visualizing what model has learned

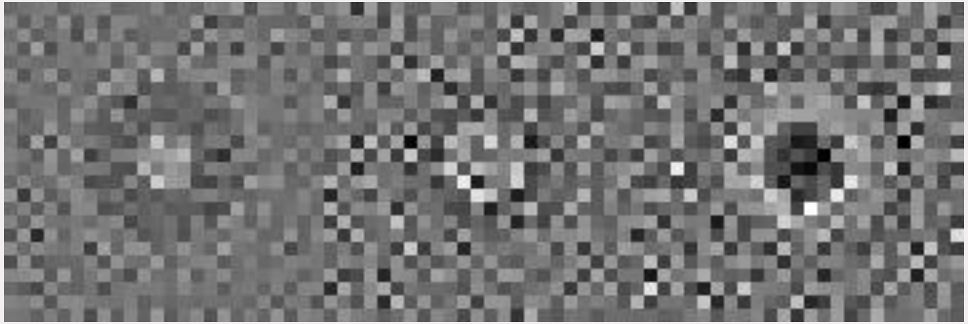
Reshape vector of parameters into 24x24x3 image

24 pixels

24 pixels

3 color channels
(red, green, blue)

24 pixels 24 pixels 24 pixels



Post-training: visualizing what model has learned

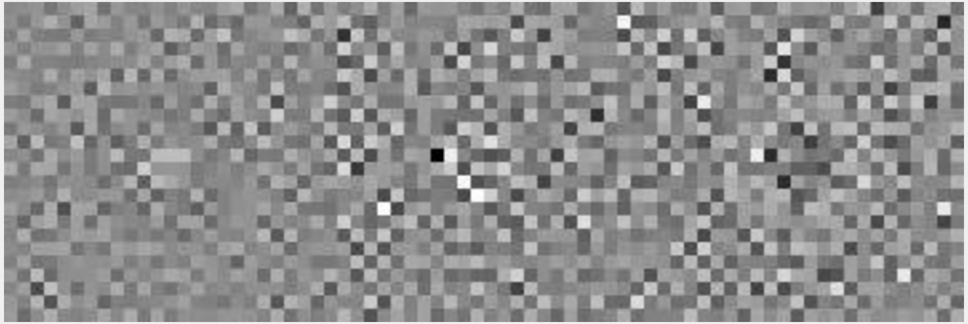
Only 25% of training samples.. Looks noisy!

24 pixels

24 pixels

3 color channels
(red, green, blue)

24 pixels 24 pixels 24 pixels



Post-training: visualizing what model has learned

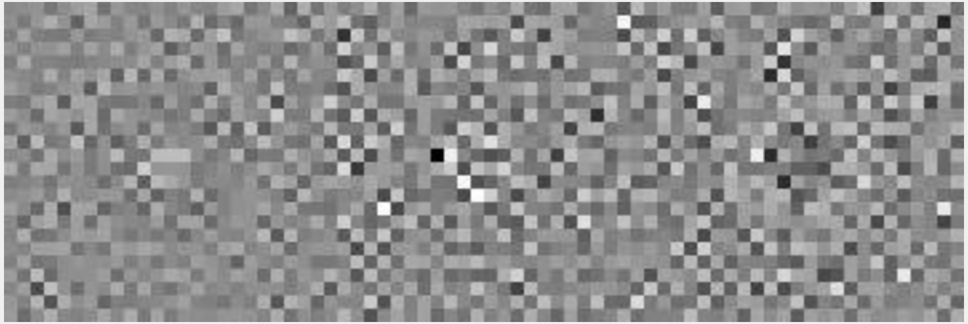
You can think of it as “there is not enough training data to reliably estimate all model weights”.

24 pixels

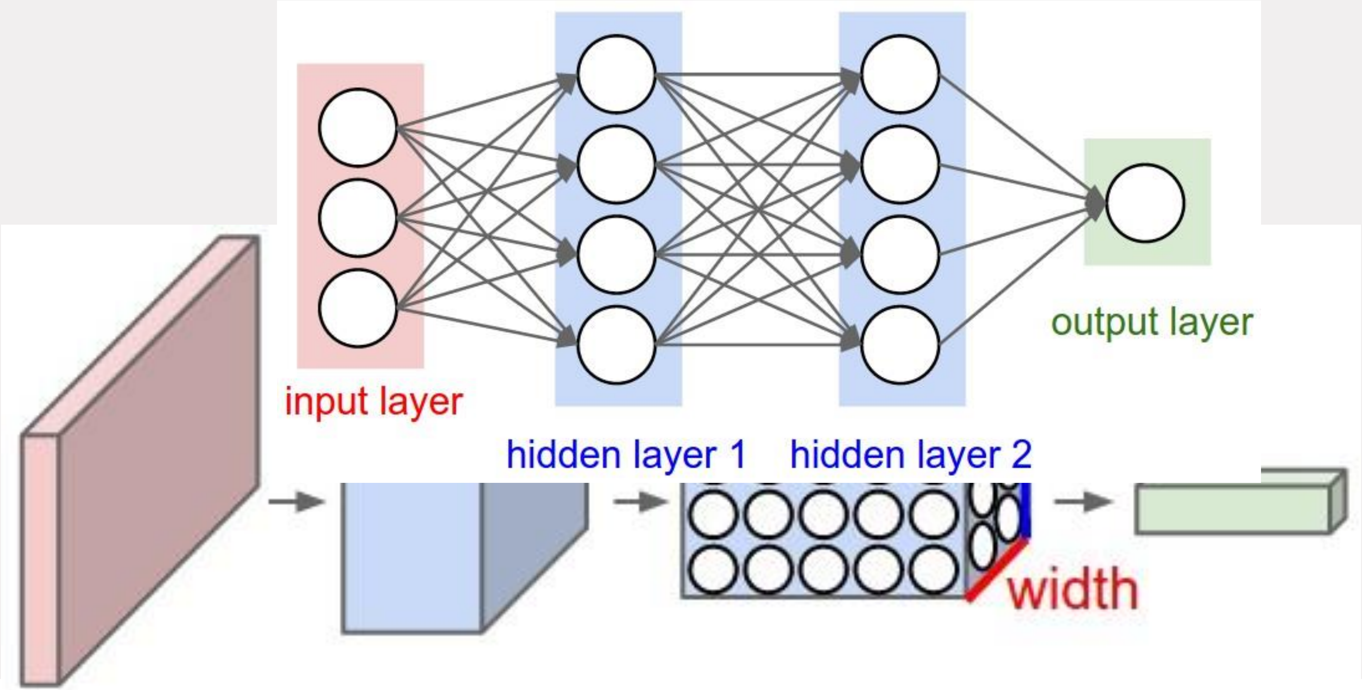
24 pixels

3 color channels
(red, green, blue)

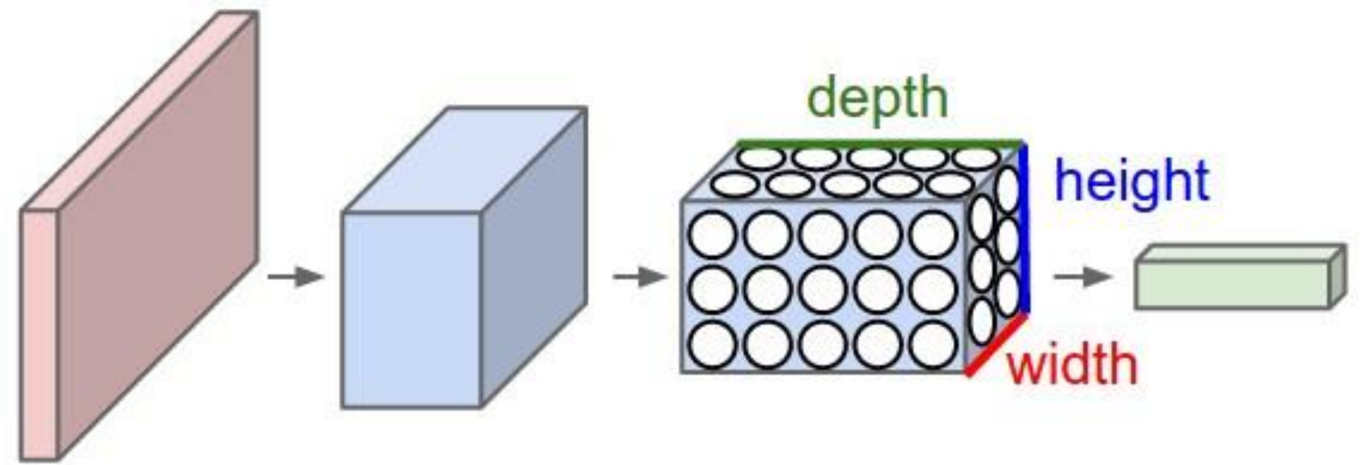
24 pixels 24 pixels 24 pixels



Many weights



With large inputs such as images and deep networks, the number of weights ”explodes”. We need a way to reduce the number of weights, without sacrificing performance.



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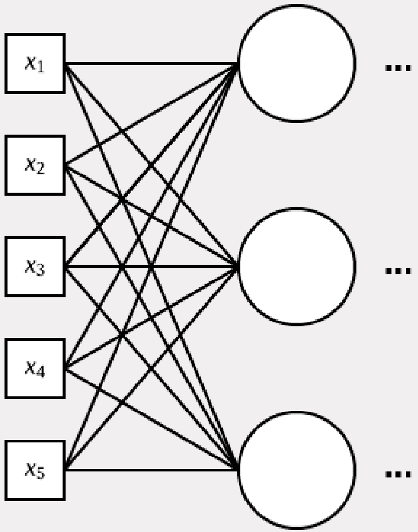
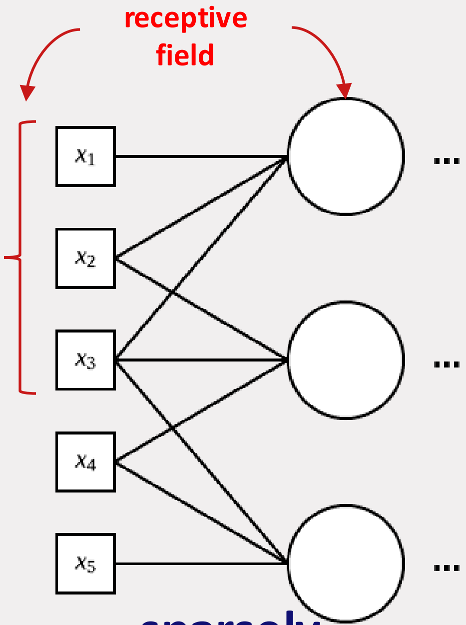
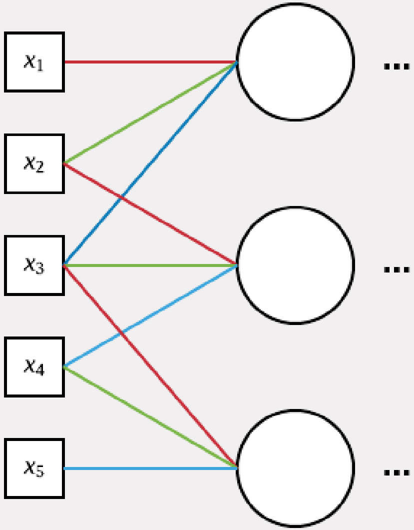
How many weights?



"regular" NN
15 weights

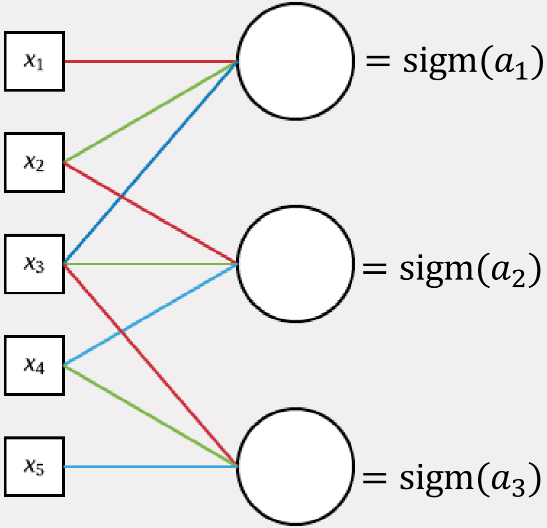


sparsely connected NN
9 weights



shared weights
3 weights

Shared weights



$$a_1 = x_1 w_1 + x_2 w_2 + x_3 w_3$$
$$a_2 = x_2 w_1 + x_3 w_2 + x_4 w_3$$
$$a_3 = x_3 w_1 + x_4 w_2 + x_5 w_3$$

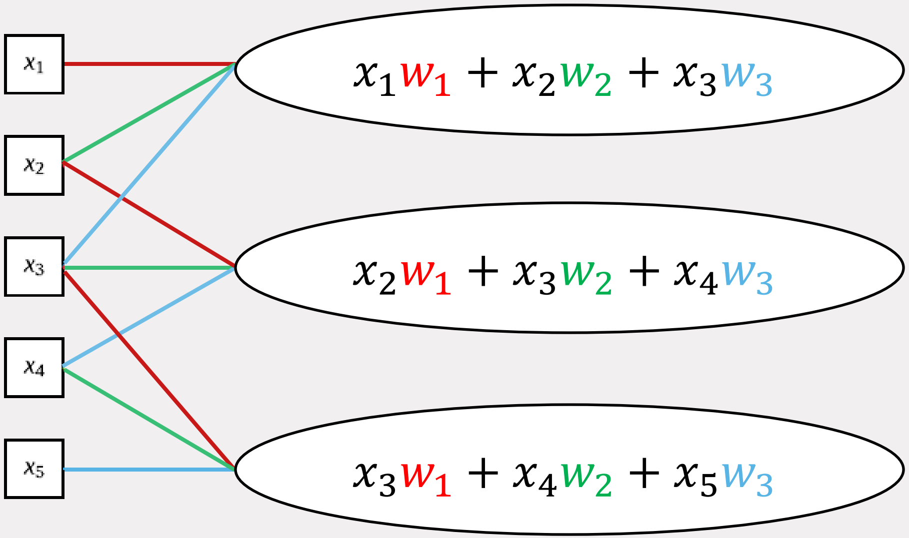
$$[a_1 \quad a_2 \quad a_3] =$$
$$[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] * [w_1 \quad w_2 \quad w_3]$$

shared weights

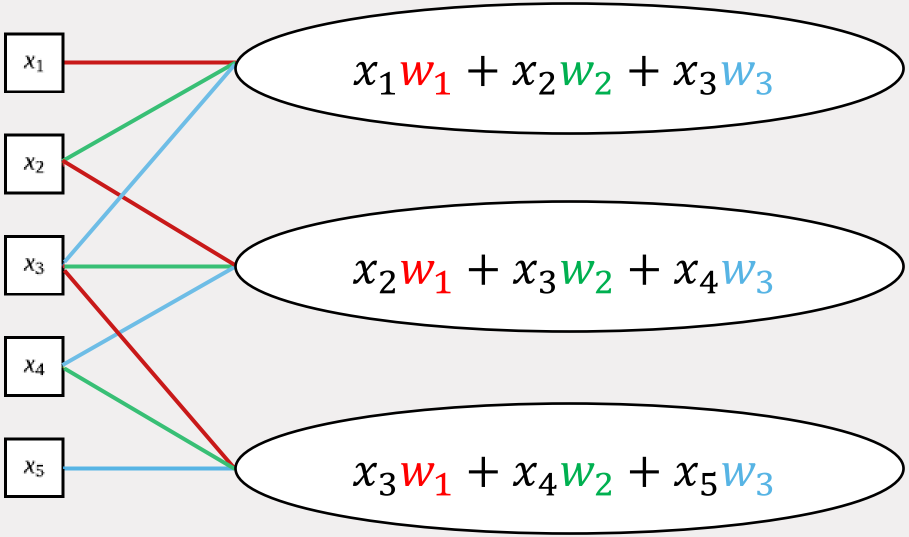
3 weights

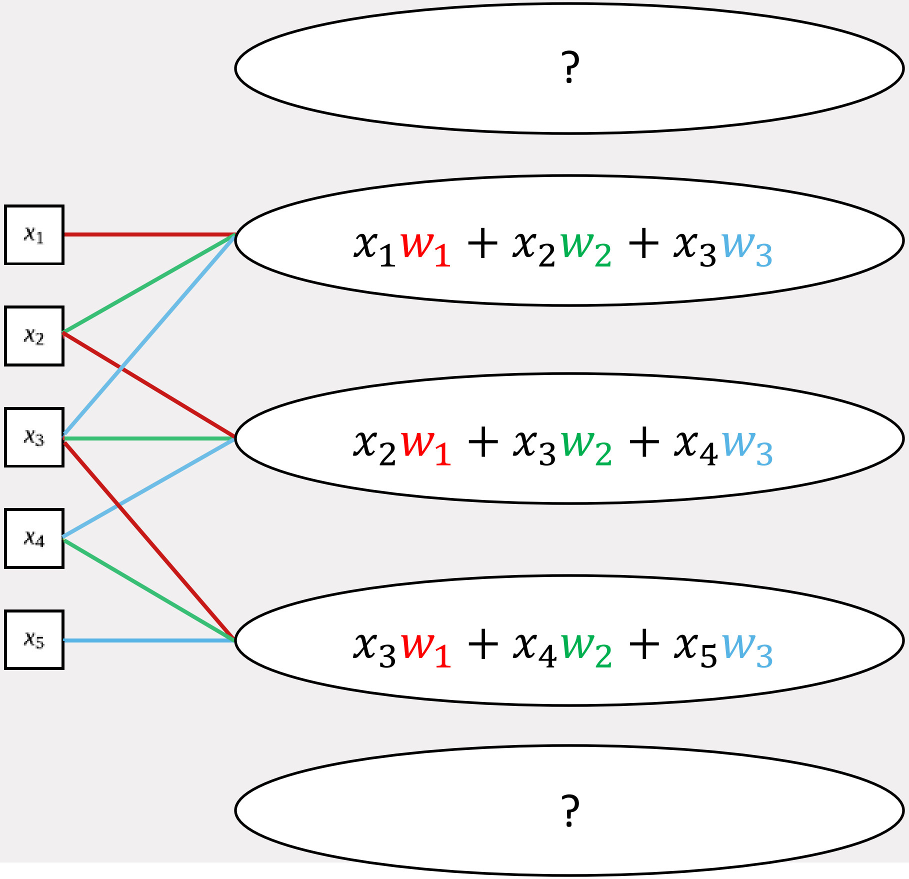
convolution, thus convolutional NN

1-D Convolution



1-D Convolution



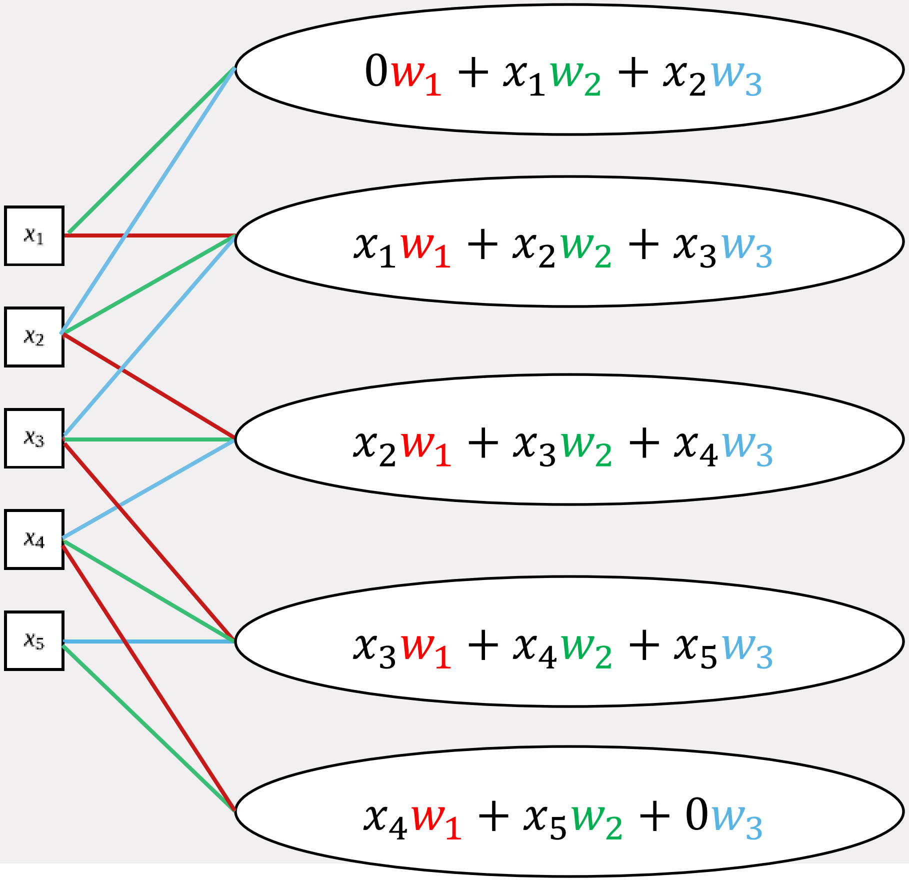


1-D Convolution

How can we keep the same number of features in hidden layer 1?

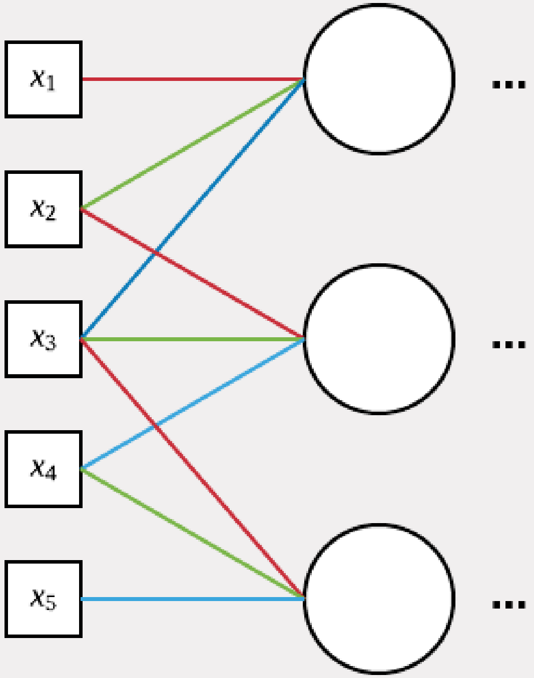
1-D Convolution

Zero-padding!



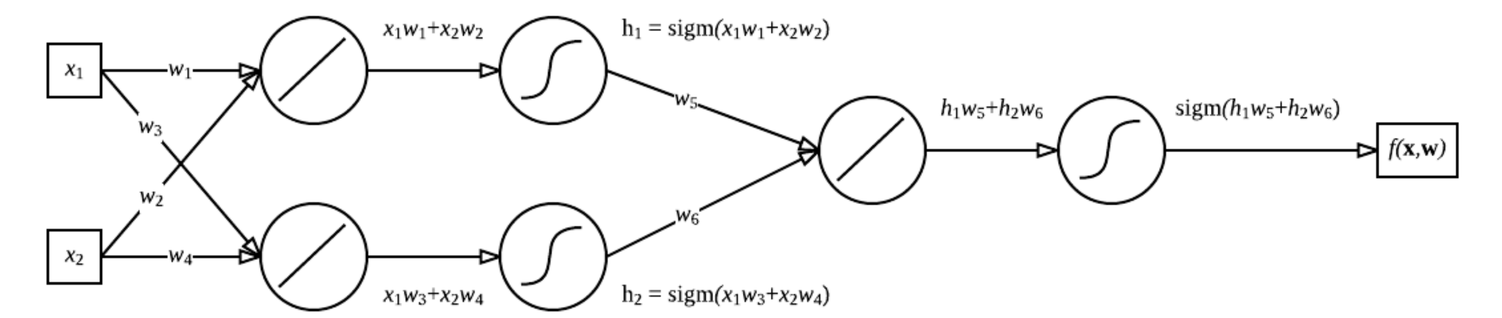
Properties of convolutional neural networks

- Sparse connectivity
- Weight sharing
- Parallel computations



Why does this work?

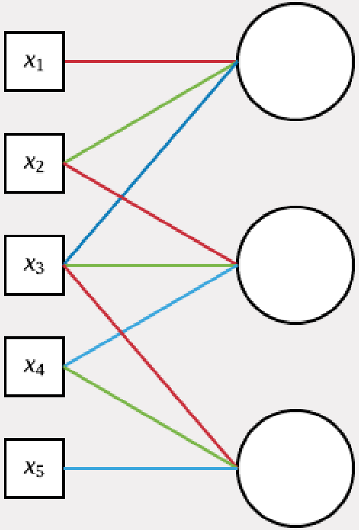
- Multiple layers
- Hidden layers contain features calculated from previous layers



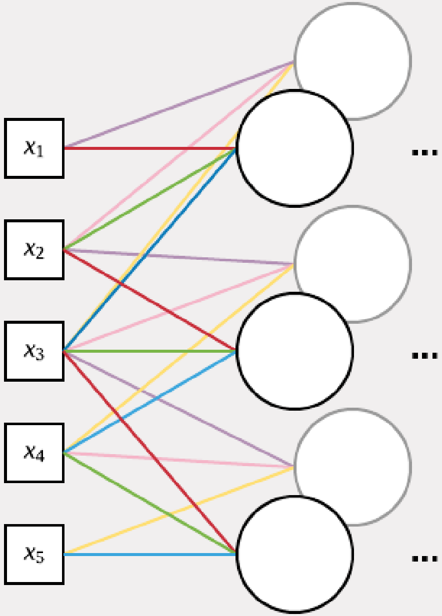
Why does this work?

- Different layers contain different transformation
  - Simple (e.g. edges, colors)
  - Complex (final layers)

One added benefit is that the learned transformations will be equivariant with translation (if the features/image is shifted up/down the features will still be detected).



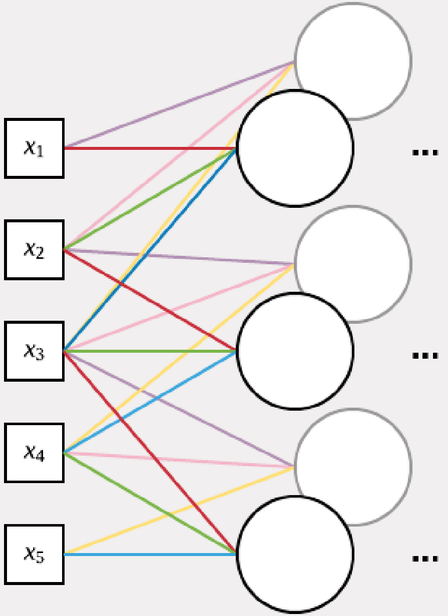
shared weights
3 weights



two sets shared weights
6 weights

We can add additional sets of weights that can learn additional interesting transformation of the input.

Note that the added neurons are not a new layer. They are part of layer 1.



two sets shared weights

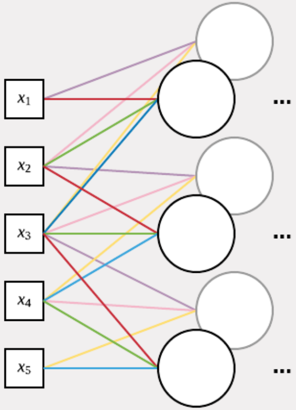
6 weights

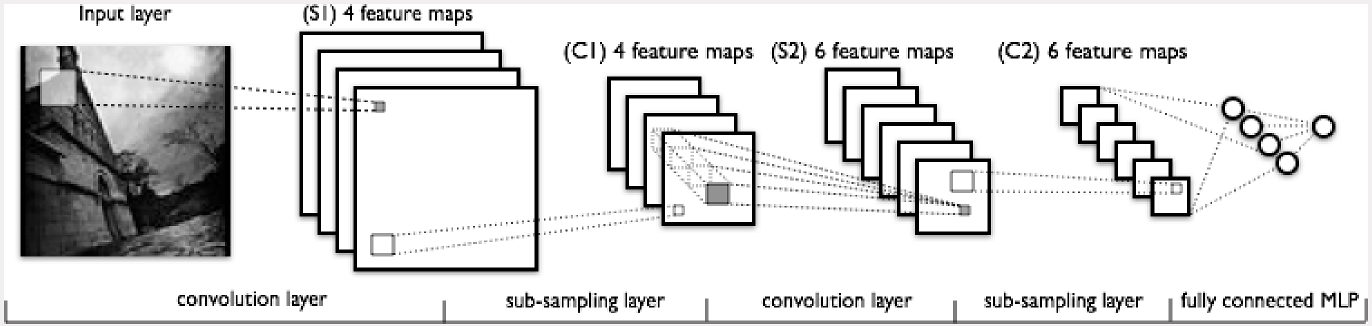
$$[a_{1,1} \quad a_{1,2} \quad a_{1,3}] =$$
$$[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] * [w_{1,1} \quad w_{1,2} \quad w_{1,3}]$$

$$[a_{2,1} \quad a_{2,2} \quad a_{2,3}] =$$
$$[x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] * [w_{2,1} \quad w_{2,2} \quad w_{2,3}]$$

$[w_{1,1} \quad w_{1,2} \quad w_{1,3}]$, and $[w_{2,1} \quad w_{2,2} \quad w_{2,3}]$ are **convolution kernels**. They extract features. However, they are not hand-designed features – they were learned by the neural network.

Convolutional neural networks are ideal for images





1D NN ↔ **2D NN (images)**

Because of the weight sharing, convolutional neural networks only work with structured data (such as images) as inputs.

$$\begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \\ I_6 & I_7 & I_8 & I_9 & I_{10} \\ I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} =$$

$$\begin{bmatrix} \begin{matrix} I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9 \end{matrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \\ I_6 & I_7 & I_8 & I_9 & I_{10} \\ I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} =$$

$$\begin{bmatrix} \begin{array}{c} I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9 \end{array} & \boxed{\begin{array}{c} I_2w_1+I_3w_2+I_4w_3 \\ + \\ I_7w_4+I_8w_5+I_9w_6 \\ + \\ I_{12}w_7+I_{13}w_8+I_{14}w_9 \end{array}} & \\ & & \\ & & \end{bmatrix}$$

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$$
\begin{array}{|c|c|c|c|c|}
\hline
I_1 & I_2 & I_3 & I_4 & I_5 \\
\hline
I_6 & I_7 & I_8 & I_9 & I_{10} \\
\hline
I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\
\hline
I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\
\hline
I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \\
\hline
\end{array}
\;*\;
\begin{array}{|c|c|c|}
\hline
w_1 & w_2 & w_3 \\
\hline
w_4 & w_5 & w_6 \\
\hline
w_7 & w_8 & w_9 \\
\hline
\end{array}
\;=
$$

$$
\begin{array}{|c|c|c|}
\hline
\begin{matrix} I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9 \end{matrix} &
\begin{matrix} I_2w_1+I_3w_2+I_4w_3 \\ + \\ I_7w_4+I_8w_5+I_9w_6 \\ + \\ I_{12}w_7+I_{13}w_8+I_{14}w_9 \end{matrix} &
\begin{matrix} I_3w_1+I_4w_2+I_5w_3 \\ + \\ I_8w_4+I_9w_5+I_{10}w_6 \\ + \\ I_{13}w_7+I_{14}w_8+I_{15}w_9 \end{matrix} \\
\hline
 & & \\
\hline
 & & \\
\hline
\end{array}
$$

$$
\begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \\ I_6 & I_7 & I_8 & I_9 & I_{10} \\ I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} =
$$

$$
\begin{bmatrix}
\begin{array}{c} I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9 \end{array} & \begin{array}{c} I_2w_1+I_3w_2+I_4w_3 \\ + \\ I_7w_4+I_8w_5+I_9w_6 \\ + \\ I_{12}w_7+I_{13}w_8+I_{14}w_9 \end{array} & \begin{array}{c} I_3w_1+I_4w_2+I_5w_3 \\ + \\ I_8w_4+I_9w_5+I_{10}w_6 \\ + \\ I_{13}w_7+I_{14}w_8+I_{15}w_9 \end{array} \\
\begin{array}{c} I_6w_1+I_7w_2+I_8w_3 \\ + \\ I_{11}w_4+I_{12}w_5+I_{13}w_6 \\ + \\ I_{16}w_7+I_{17}w_8+I_{18}w_9 \end{array} & & \\
 & &
\end{bmatrix}
$$

$$
\begin{bmatrix}
I_1 & I_2 & I_3 & I_4 & I_5 \\
I_6 & I_7 & I_8 & I_9 & I_{10} \\
I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\
I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\
I_{21} & I_{22} & I_{23} & I_{24} & I_{25}
\end{bmatrix}
*
\begin{bmatrix}
w_1 & w_2 & w_3 \\
w_4 & w_5 & w_6 \\
w_7 & w_8 & w_9
\end{bmatrix}
=
$$

$I_1w_1+I_2w_2+I_3w_3$ + $I_6w_4+I_7w_5+I_8w_6$ + $I_{11}w_7+I_{12}w_8+I_{13}w_9$	$I_2w_1+I_3w_2+I_4w_3$ + $I_7w_4+I_8w_5+I_9w_6$ + $I_{12}w_7+I_{13}w_8+I_{14}w_9$	$I_3w_1+I_4w_2+I_5w_3$ + $I_8w_4+I_9w_5+I_{10}w_6$ + $I_{13}w_7+I_{14}w_8+I_{15}w_9$
$I_6w_1+I_7w_2+I_8w_3$ + $I_{11}w_4+I_{12}w_5+I_{13}w_6$ + $I_{16}w_7+I_{17}w_8+I_{18}w_9$	$I_7w_1+I_8w_2+I_9w_3$ + $I_{12}w_4+I_{13}w_5+I_{14}w_6$ + $I_{17}w_7+I_{18}w_8+I_{19}w_9$	

$$\begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \\ I_6 & I_7 & I_8 & I_9 & I_{10} \\ I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} =$$

$I_1w_1+I_2w_2+I_3w_3$ + $I_6w_4+I_7w_5+I_8w_6$ + $I_{11}w_7+I_{12}w_8+I_{13}w_9$	$I_2w_1+I_3w_2+I_4w_3$ + $I_7w_4+I_8w_5+I_9w_6$ + $I_{12}w_7+I_{13}w_8+I_{14}w_9$	$I_3w_1+I_4w_2+I_5w_3$ + $I_8w_4+I_9w_5+I_{10}w_6$ + $I_{13}w_7+I_{14}w_8+I_{15}w_9$
$I_6w_1+I_7w_2+I_8w_3$ + $I_{11}w_4+I_{12}w_5+I_{13}w_6$ + $I_{16}w_7+I_{17}w_8+I_{18}w_9$	$I_7w_1+I_8w_2+I_9w_3$ + $I_{12}w_4+I_{13}w_5+I_{14}w_6$ + $I_{17}w_7+I_{18}w_8+I_{19}w_9$	$I_8w_1+I_9w_2+I_{10}w_3$ + $I_{13}w_4+I_{14}w_5+I_{15}w_6$ + $I_{18}w_7+I_{19}w_8+I_{20}w_9$

$$
\begin{bmatrix}
I_1 & I_2 & I_3 & I_4 & I_5 \\
I_6 & I_7 & I_8 & I_9 & I_{10} \\
I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\
I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\
I_{21} & I_{22} & I_{23} & I_{24} & I_{25}
\end{bmatrix}
*
\begin{bmatrix}
w_1 & w_2 & w_3 \\
w_4 & w_5 & w_6 \\
w_7 & w_8 & w_9
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
\substack{I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9} & \substack{I_2w_1+I_3w_2+I_4w_3 \\ + \\ I_7w_4+I_8w_5+I_9w_6 \\ + \\ I_{12}w_7+I_{13}w_8+I_{14}w_9} & \substack{I_3w_1+I_4w_2+I_5w_3 \\ + \\ I_8w_4+I_9w_5+I_{10}w_6 \\ + \\ I_{13}w_7+I_{14}w_8+I_{15}w_9} \\
\substack{I_6w_1+I_7w_2+I_8w_3 \\ + \\ I_{11}w_4+I_{12}w_5+I_{13}w_6 \\ + \\ I_{16}w_7+I_{17}w_8+I_{18}w_9} & \substack{I_7w_1+I_8w_2+I_9w_3 \\ + \\ I_{12}w_4+I_{13}w_5+I_{14}w_6 \\ + \\ I_{17}w_7+I_{18}w_8+I_{19}w_9} & \substack{I_8w_1+I_9w_2+I_{10}w_3 \\ + \\ I_{13}w_4+I_{14}w_5+I_{15}w_6 \\ + \\ I_{18}w_7+I_{19}w_8+I_{20}w_9} \\
\substack{I_{11}w_1+I_{12}w_2+I_{13}w_3 \\ + \\ I_{16}w_4+I_{17}w_5+I_{18}w_6 \\ + \\ I_{21}w_7+I_{22}w_8+I_{23}w_9} & &
\end{bmatrix}
$$

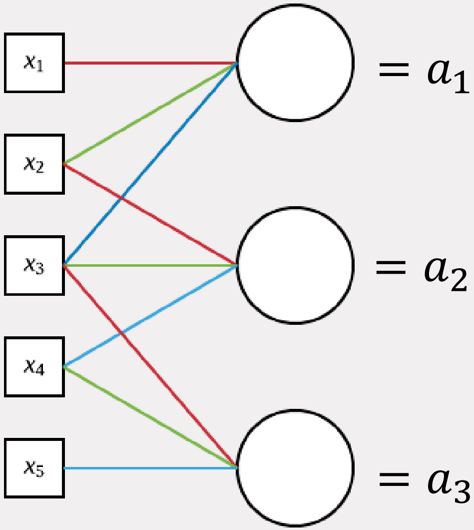
$$\begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \\ I_6 & I_7 & I_8 & I_9 & I_{10} \\ I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ I_{16} & I_{17} & I_{18} & I_{19} & I_{20} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} =$$

$$\begin{bmatrix}
\begin{array}{c} I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9 \end{array} &
\begin{array}{c} I_2w_1+I_3w_2+I_4w_3 \\ + \\ I_7w_4+I_8w_5+I_9w_6 \\ + \\ I_{12}w_7+I_{13}w_8+I_{14}w_9 \end{array} &
\begin{array}{c} I_3w_1+I_4w_2+I_5w_3 \\ + \\ I_8w_4+I_9w_5+I_{10}w_6 \\ + \\ I_{13}w_7+I_{14}w_8+I_{15}w_9 \end{array} \\
\begin{array}{c} I_6w_1+I_7w_2+I_8w_3 \\ + \\ I_{11}w_4+I_{12}w_5+I_{13}w_6 \\ + \\ I_{16}w_7+I_{17}w_8+I_{18}w_9 \end{array} &
\begin{array}{c} I_7w_1+I_8w_2+I_9w_3 \\ + \\ I_{12}w_4+I_{13}w_5+I_{14}w_6 \\ + \\ I_{17}w_7+I_{18}w_8+I_{19}w_9 \end{array} &
\begin{array}{c} I_8w_1+I_9w_2+I_{10}w_3 \\ + \\ I_{13}w_4+I_{14}w_5+I_{15}w_6 \\ + \\ I_{18}w_7+I_{19}w_8+I_{20}w_9 \end{array} \\
\begin{array}{c} I_{11}w_1+I_{12}w_2+I_{13}w_3 \\ + \\ I_{16}w_4+I_{17}w_5+I_{18}w_6 \\ + \\ I_{21}w_7+I_{22}w_8+I_{23}w_9 \end{array} &
\begin{array}{c} I_{12}w_1+I_{13}w_2+I_{14}w_3 \\ + \\ I_{17}w_4+I_{18}w_5+I_{19}w_6 \\ + \\ I_{22}w_7+I_{23}w_8+I_{24}w_9 \end{array} &
\end{bmatrix}$$

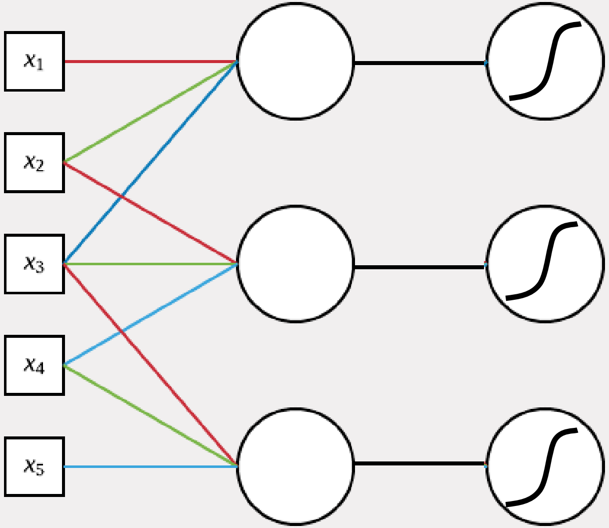
$$
\begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \\ I_6 & I_7 & I_8 & I_9 & I_{10} \\ I_{11} & I_{12} & \boxed{I_{13}} & \boxed{I_{14}} & \boxed{I_{15}} \\ I_{16} & I_{17} & \boxed{I_{18}} & \boxed{I_{19}} & \boxed{I_{20}} \\ I_{21} & I_{22} & \boxed{I_{23}} & \boxed{I_{24}} & \boxed{I_{25}} \end{bmatrix} * \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix} + b =
$$

$$
\begin{bmatrix}
\begin{matrix} I_1w_1+I_2w_2+I_3w_3 \\ + \\ I_6w_4+I_7w_5+I_8w_6 \\ + \\ I_{11}w_7+I_{12}w_8+I_{13}w_9 \end{matrix} & \begin{matrix} I_2w_1+I_3w_2+I_4w_3 \\ + \\ I_7w_4+I_8w_5+I_9w_6 \\ + \\ I_{12}w_7+I_{13}w_8+I_{14}w_9 \end{matrix} & \begin{matrix} I_3w_1+I_4w_2+I_5w_3 \\ + \\ I_8w_4+I_9w_5+I_{10}w_6 \\ + \\ I_{13}w_7+I_{14}w_8+I_{15}w_9 \end{matrix} \\
\begin{matrix} I_6w_1+I_7w_2+I_8w_3 \\ + \\ I_{11}w_4+I_{12}w_5+I_{13}w_6 \\ + \\ I_{16}w_7+I_{17}w_8+I_{18}w_9 \end{matrix} & \begin{matrix} I_7w_1+I_8w_2+I_9w_3 \\ + \\ I_{12}w_4+I_{13}w_5+I_{14}w_6 \\ + \\ I_{17}w_7+I_{18}w_8+I_{19}w_9 \end{matrix} & \begin{matrix} I_8w_1+I_9w_2+I_{10}w_3 \\ + \\ I_{13}w_4+I_{14}w_5+I_{15}w_6 \\ + \\ I_{18}w_7+I_{19}w_8+I_{20}w_9 \end{matrix} \\
\begin{matrix} I_{11}w_1+I_{12}w_2+I_{13}w_3 \\ + \\ I_{16}w_4+I_{17}w_5+I_{18}w_6 \\ + \\ I_{21}w_7+I_{22}w_8+I_{23}w_9 \end{matrix} & \begin{matrix} I_{12}w_1+I_{13}w_2+I_{14}w_3 \\ + \\ I_{17}w_4+I_{18}w_5+I_{19}w_6 \\ + \\ I_{22}w_7+I_{23}w_8+I_{24}w_9 \end{matrix} & \boxed{\begin{matrix} I_{13}w_1+I_{14}w_2+I_{15}w_3 \\ + \\ I_{18}w_4+I_{19}w_5+I_{20}w_6 \\ + \\ I_{23}w_7+I_{24}w_8+I_{25}w_9 \end{matrix}}
\end{bmatrix} + b
$$

Convolutions + non-linearity



Convolutions + non-linearity (sigmoid)

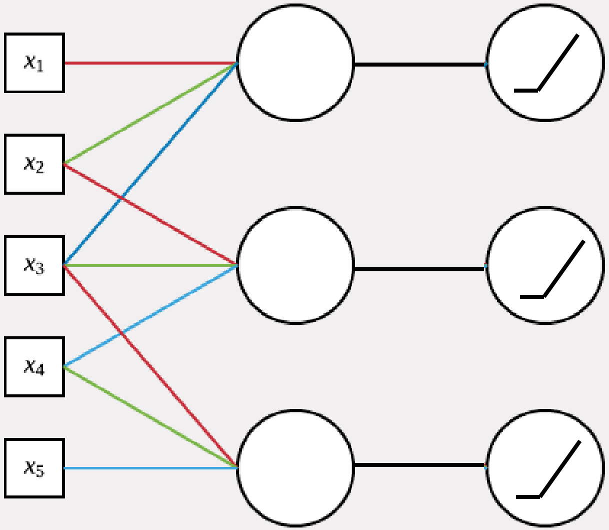


$= sigm(a_1)$

$= sigm(a_2)$

$= sigm(a_3)$

Convolutions + non-linearity (ReLU)

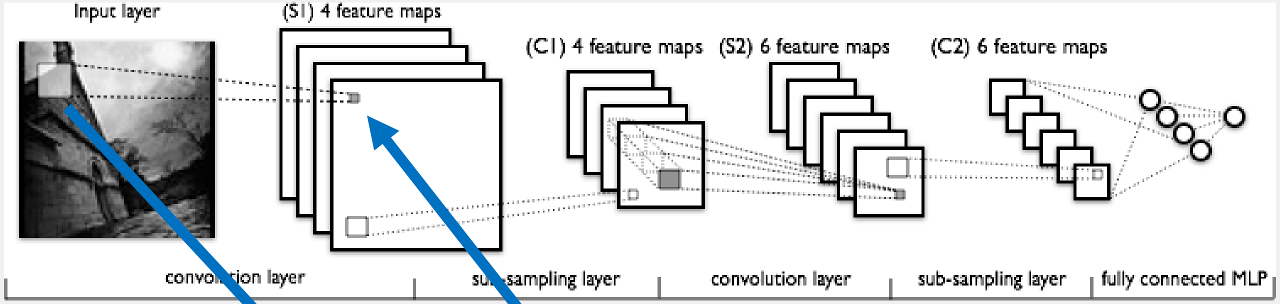


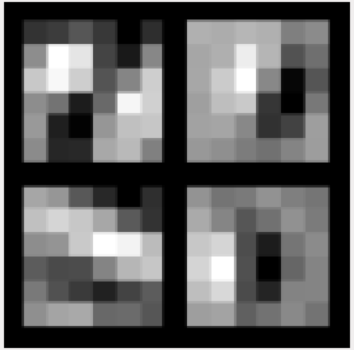
$= ReLU(a_1)$

$= ReLU(a_2)$

$= ReLU(a_3)$

Convolutional kernels



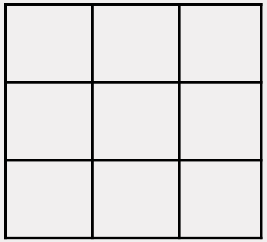


these are the weights (convolutional kernels) that produce the four feature maps

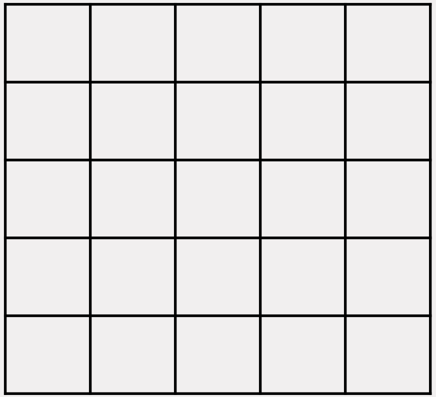
Kernel size

1 x 1

3 x 3

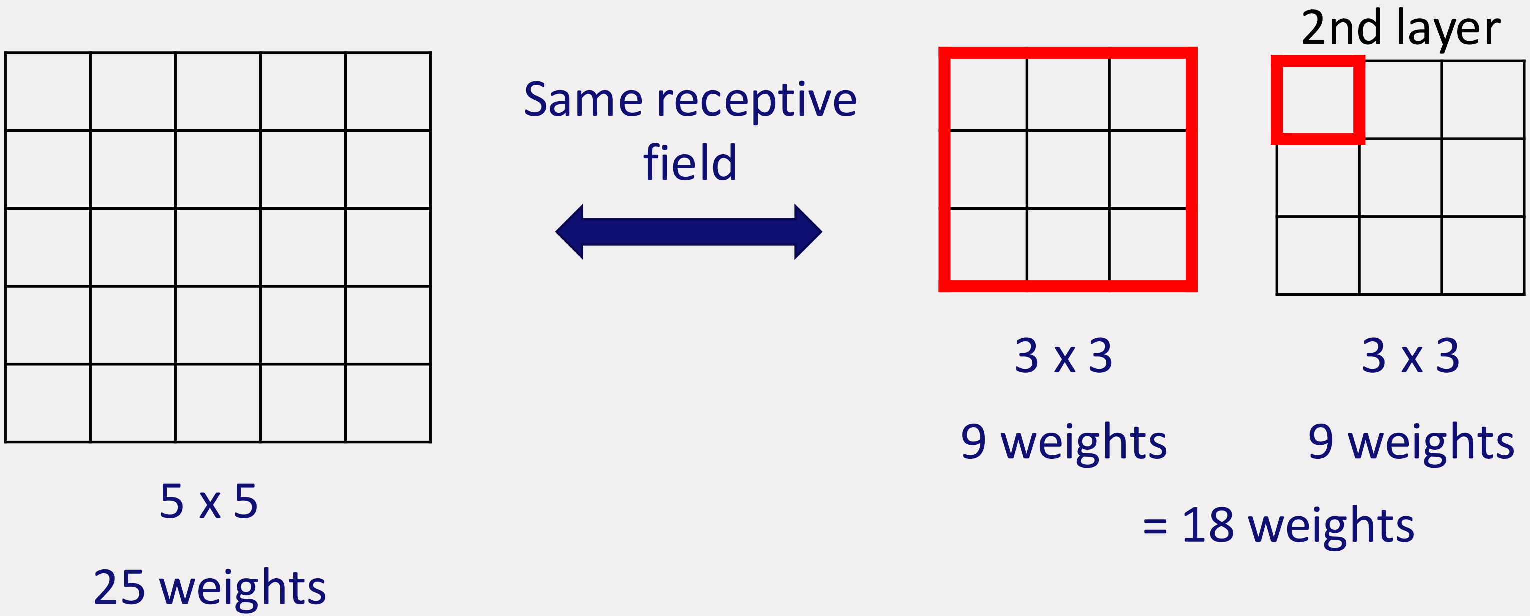


5 x 5

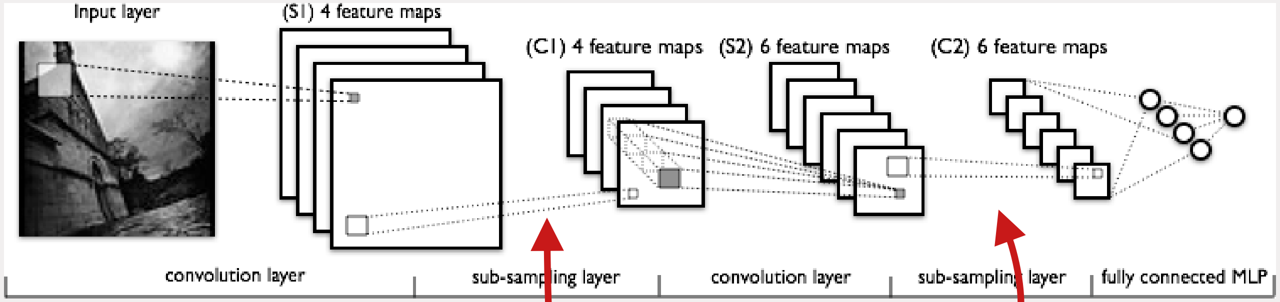


- More weights → more information
- More computations / memory
- Receptive field

Receptive field



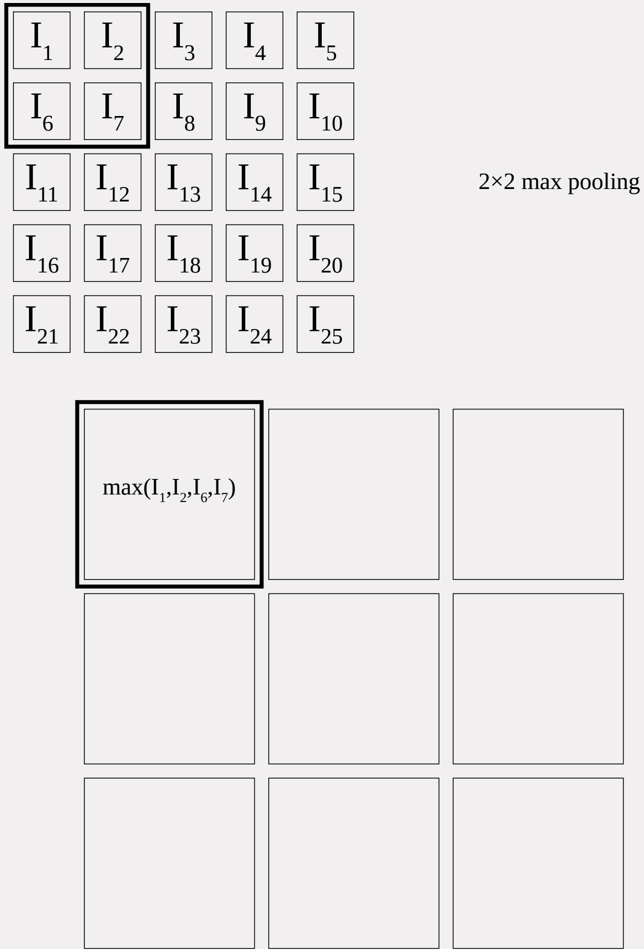
Max-pooling

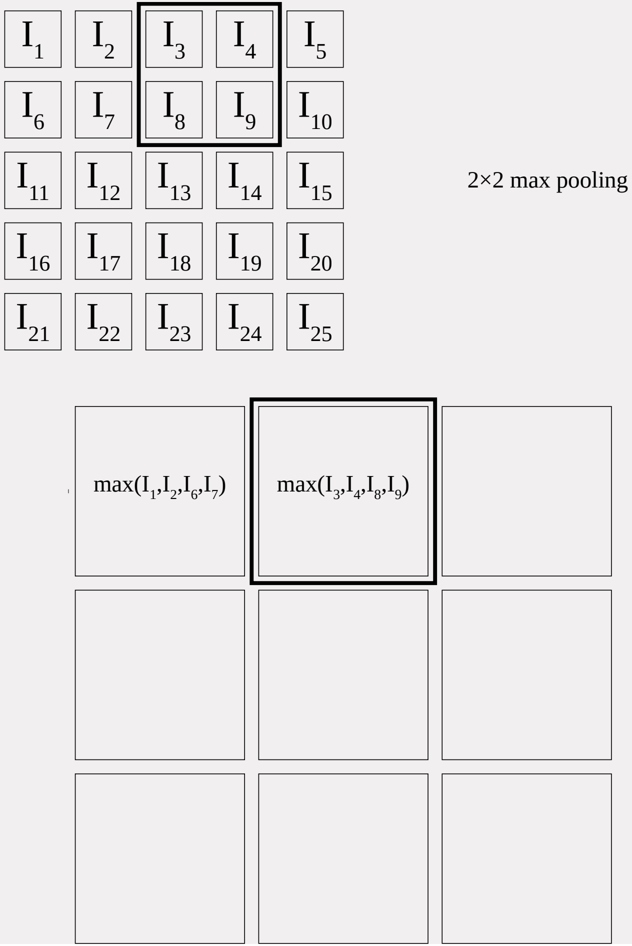


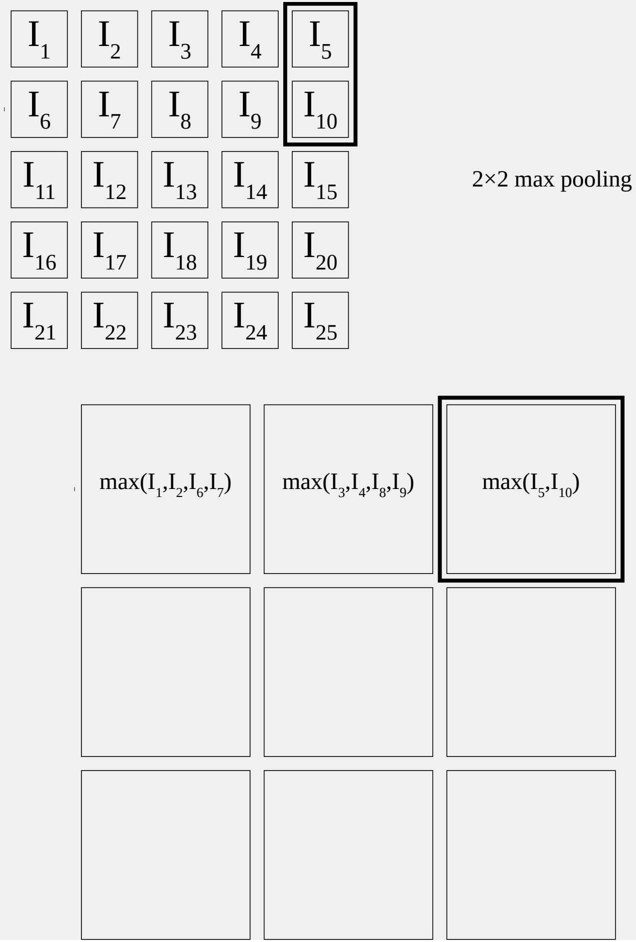
max-pooling

Max-pooling

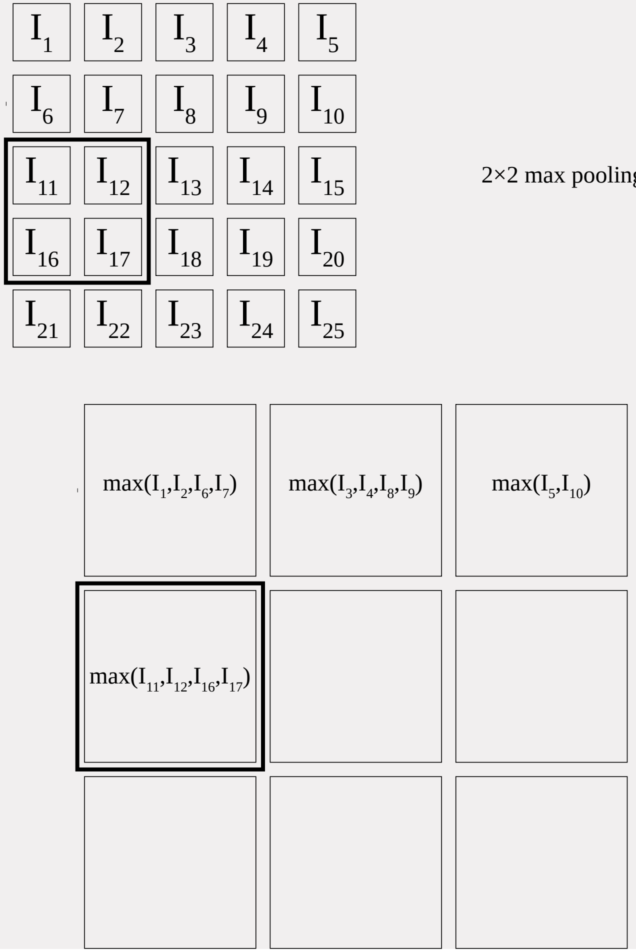
- Reduce size of feature space
- Maximum of features
- Typical kernel size = 2 x 2


I1 I2 I3 I4 I5
I6 I7 I8 I9 I10
I11 I12 I13 I14 I15
I16 I17 I18 I19 I20
I21 I22 I23 I24 I25
2×2 max pooling
max(I1,I2,I6,I7)

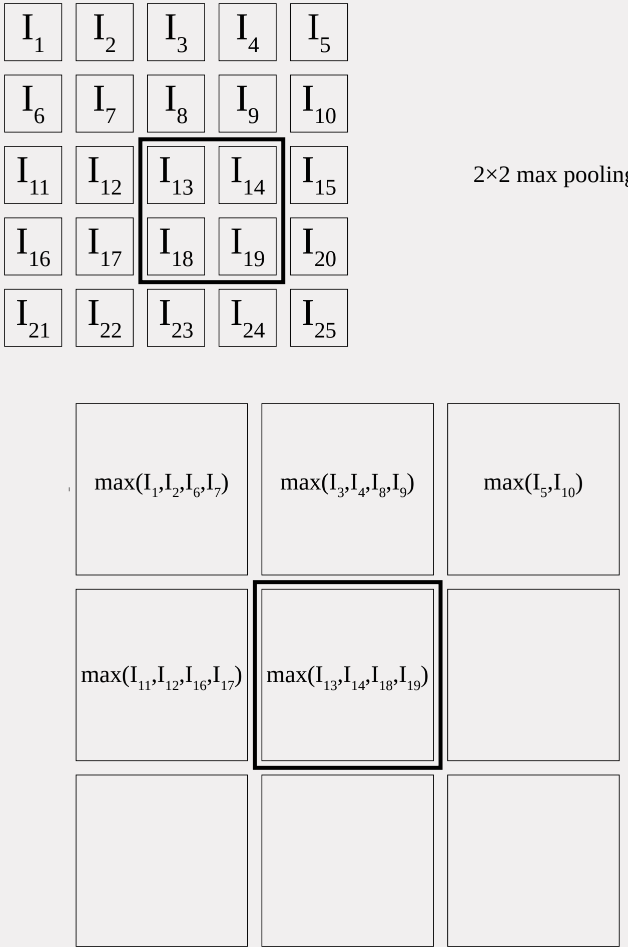

I_1 I_2 I_3 I_4 I_5
I_6 I_7 I_8 I_9 I_{10}
I_{11} I_{12} I_{13} I_{14} I_{15}
I_{16} I_{17} I_{18} I_{19} I_{20}
I_{21} I_{22} I_{23} I_{24} I_{25}
2×2 max pooling
$\max(I_1,I_2,I_6,I_7)$
$\max(I_3,I_4,I_8,I_9)$



In this example:
no zero padding



2×2 max pooling



2×2 max pooling

I_1	I_2	I_3	I_4	I_5
I_6	I_7	I_8	I_9	I_{10}
I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
$\boxed{I_{21}\ I_{22}}$		I_{23}	I_{24}	I_{25}

2×2 max pooling

$\max(I_1,I_2,I_6,I_7)$	$\max(I_3,I_4,I_8,I_9)$	$\max(I_5,I_{10})$
$\max(I_{11},I_{12},I_{16},I_{17})$	$\max(I_{13},I_{14},I_{18},I_{19})$	$\max(I_{15},I_{20})$
$\boxed{\max(I_{21},I_{22})}$		

I_1	I_2	I_3	I_4	I_5
I_6	I_7	I_8	I_9	I_{10}
I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
I_{21}	I_{22}	I_{23}	I_{24}	I_{25}

2×2 max pooling

$\max(I_1,I_2,I_6,I_7)$	$\max(I_3,I_4,I_8,I_9)$	$\max(I_5,I_{10})$
$\max(I_{11},I_{12},I_{16},I_{17})$	$\max(I_{13},I_{14},I_{18},I_{19})$	$\max(I_{15},I_{20})$
$\max(I_{21},I_{22})$	$\max(I_{23},I_{24})$	

I_1	I_2	I_3	I_4	I_5
I_6	I_7	I_8	I_9	I_{10}
I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
I_{21}	I_{22}	I_{23}	I_{24}	$\boxed{I_{25}}$

2×2 max pooling

$\max(I_1,I_2,I_6,I_7)$	$\max(I_3,I_4,I_8,I_9)$	$\max(I_5,I_{10})$
$\max(I_{11},I_{12},I_{16},I_{17})$	$\max(I_{13},I_{14},I_{18},I_{19})$	$\max(I_{15},I_{20})$
$\max(I_{21},I_{22})$	$\max(I_{23},I_{24})$	$\boxed{I_{25}}$

Benefits of max-pooling

- "Quickly" reduces the number size of the feature maps
- Introduces translation invariance (slightly translated version of the input image will result in the same output)

Alternative: Average Pooling

Let try it out!

https://adamharley.com/nn_vis/cnn/2d.html

An Interactive Node-Link Visualization of Convolutional Neural Networks

Adam W. Harley(✉)

Department of Computer Science, Ryerson University,
Toronto, ON M5B 2K3, Canada
aharley@scs.ryerson.ca

Training of Convolutional Neural Networks

- Similar to training of ‘fully connected’ neural networks
- Choose some (random) initial values for network weights
- Optimize networks weights with respect to a loss function that describes difference between network output and label/annotation
- Update networks weights iteratively

Through a process called ‘backpropagation’. A good explanation can be found here: https://www.youtube.com/watch?v=i94OvYb6noo (5:10 - 28:00, not exam material)

- Keep track of model performance on train & validation set

Summary

- Concept of **convolutions** in a neural network
- Why can we use a **convolutional approach** for (medical) **images**
- Convolutions enable development of **deep** (and large) **neural networks**
- **Max-pooling layer** in a convolutional neural network
- **Kernel size**
- **Receptive field**