UNIVERSITY OF BIRMINGHAM

School of Computer Science

Artificial Intelligence and Machine Learning

Main Summer Examinations 2024

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1

(a) List the main, formal mathematical elements of a combinatorial optimization problem. **[3 marks]**

(b) Explain the essential differences between an exact and an approximate algorithm for solving a combinatorial optimization problem. **[2 marks]**

(c) Describe the steps in an exhaustive algorithm for solving an optimization problem. Is this exhaustive algorithm exact or approximate? Justify your answer. **[2 marks]**

(d) Consider the problem of finding the minimum sum subset of a set of N items.

   (i) What is the size of the configuration space of this problem, in terms of N? **[1 mark]**

   (ii) Give the time complexity class of the exhaustive algorithm for solving this. **[1 mark]**

   (iii) Is this exhaustive algorithm tractable or intractable? Justify your answer. **[1 mark]**

(e) You are given a set of $N = 3$ items with values $x = [5, -1, 6]$. You must find the minimum sum subset of this set.

   (i) Give the steps in an exhaustive SDP algorithm for solving this problem in N iterations. **[5 marks]**

   (ii) Apply your algorithm to compute the optimal solution, $X^\star$ and its value, $F(X^\star)$, giving the explicit results on each step of your algorithm. **[2 marks]**

   (iii) Now, give a greedy reduction step as part of your SDP algorithm which solves the problem exactly in worst-case linear time, $O(N)$. Justify why this reduction step is valid. **[3 marks]**

Question 2

(a) List, provide the usual mathematical symbols, and give the truth tables for, the five basic logical connectives of propositional calculus. **[5 marks]**

(b) Translate the following natural language proposition into a formal logical proposition over two propositional variables: "If it is winter but it is not cold, then the climate has changed". **[2 marks]**

(c) A *tautology* is a logical proposition which is always true, regardless of the truth value of any propositional variables it uses. Using a truth table, show that the proposition $((P \Rightarrow Q) \vee R) \vee (P \wedge \neg Q \wedge \neg R)$ over the logical variables P, Q, R, is a tautology. **[5 marks]**

(d) A simple logical knowledge database contains the following facts: $P =$ "Finley is President", $Q =$ "Presidents are elected", along with the rule $R =$ "It is not true that both Finley is President and Presidents are elected". You must determine whether the query $Y = \neg P \vee \neg Q$ is entailed by this database.

   (i) Give the steps in the logical model checking algorithm for testing entailment, $R \models Y$ in the given knowledge database. **[3 marks]**

   (ii) Apply the steps in the model checking algorithm to compute whether $R \models Y$. Show your calculations in full, including any truth tables. **[4 marks]**

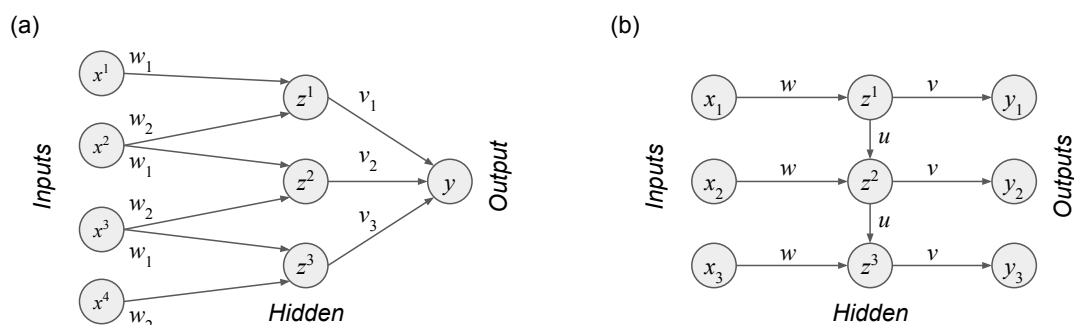   (iii) Is this entailment tautological? Justify your answer. **[1 mark]**

Question 3

Figure 1: Deep learning networks for Q3.

(a) List and provide the mathematical symbols and/or formulae for, three widely used nonlinear activation functions used to construct deep neural networks. **[3 marks]**

(b) For each of the deep neural networks in Figures 1(a) and 1(b) above:

   (i) Identify what is special about the network. **[2 marks]**

   (ii) Give an example of the kind of data which would be most suitable for this network to process. **[2 marks]**

   (iii) List all the parameters (weights) for the network. **[2 marks]**

(c) For the deep neural network in Figure 1(a) above, write down the formulae for each of the hidden neurons z^1, z^2, z^3 with ReLU nonlinear activation functions, in terms of the inputs x^1, x^2, x^3, x^4, and the formula for the output y in terms of the hidden neurons z^1, z^2, z^3. **[4 marks]**

(d) Assume that, for the deep neural network in Figure 1(b) above, the formulae for computing the values of the hidden and output neurons are:

$$z^1 = \max(0, x_1 w) \qquad y_1 = \max(0, z^1 v)$$
$$z^2 = \max(0, x_2 w + z^1 u) \quad y_2 = \max(0, z^2 v)$$
$$z^3 = \max(0, x_3 w + z^2 u) \quad y_3 = \max(0, z^3 v)$$

For the input data $x_1 = 0.3$, $x_2 = -0.5$, $x_3 = 1.1$ and where $w = 0.6$, $u = -0.7$, $v = 1.5$, compute the values of all hidden and output nodes. Show all your working. **[4 marks]**

(e) Give the diagram for a novel deep network design with two hidden layers, where the first layer is convolutional, the second hidden layer is recurrent, and the last layer is fully connected. Provide unambiguous mathematical labels for all nonlinear nodes and parameters. **[3 marks]**

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.

- Unauthorised materials (e.g. notes or Tippex) <u>must</u> be placed in the designated area.

- Check that you <u>do not</u> have any unauthorised materials with you (e.g. in your pockets, pencil case).

- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.

- You are <u>not</u> permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.

- You are <u>not</u> permitted to have writing on your hand, arm or other body part.

- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately

- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.