Tutorial Sections 7-8

1. In financial data analysis, data scientists are often tasked with analyzing historical stock market data to identify periods that would have maximized investment returns in specific stocks. This involves examining daily stock price changes (which could be positive, negative, or zero).

   Suppose you are analyzing a particular company to identify the time segment (i.e., a continuous sequence of days) where investing in this company's stock would have yielded the highest return. Consider a series of daily stock price changes, such as $[-2, 1, -3, 4, -1, 2, 1, -5, 4]$.

   (a) Utilize the Bellman recursion defined by

   $$R_n^\star = \left(\max\left(T_n, R_{n-1}^{\star 1}\right), T_n\right),$$

   with base case $R_0^\star = (0, 0)$, where $T_n = \max\left(0, x_n + R_{n-1}^{\star 2}\right)$. Here, R^{*1} refers to the first element in a pair and $R^{\star 2}$ refers to the second element in a pair. Employ this recursion to find the time segment that offers the maximum return and calculate the maximum value for the stock within this timeframe.

   (b) What would be the computational complexity required to solve this problem using a fully exhaustive approach? Contrast this result with the recursive strategy outlined above.

2. Working in a company, consider you were given a set of tasks, each with a start time, an end time, and its importance measured on a $0 - 10$ scale: As you can see, there is some overlap in the tasks, which

Task	Start Time	End Time	Importance
1	1	3	3
2	2	5	5
3	4	6	2
4	6	7	4
5	5	8	6

   makes it not feasible to perform all tasks on a given day. We can assign a list segment, p, indicating the last non-conflicting task with the current one. For example, the segment $p = [0, 0, 1, 3, 2]$ implies that task 5 can follow task 2 (as they don't overlap), task 4 can follow task 3, and so on.

   (a) Use the given recursion:

   $$M_0^\star = 0$$
   $$M_n^\star = \max\left(M_{n-1}^\star, M_{p_n}^\star + w_n\right)$$

   to select a subset of non-overlapping tasks that maximizes the sum of their importance.

   (b) What would be the computational complexity required to solve this problem using a fully exhaustive approach? Contrast this result with the recursive strategy outlined above.

3. Suppose P and Q are the statements: P: "Jack passed math". Q: "Jill passed math".

   (a) Translate "Jack and Jill both passed math" into symbols.

   (b) Translate "If Jack passed math, then Jill did not" into symbols.

   (c) Translate "$P \vee Q$" into English.

   (d) Translate "$\neg(P \wedge Q) \implies Q$" into English.

   (e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that (i) Jill passed math? (ii) Jill did not pass math?

4. Consider the statement about a party, "If it's your birthday or there will be cake, then there will be cake."
   (a) Translate the above statement into symbols. Clearly state which statement is P and which is Q.
   (b) Make a truth table for the statement.
   (c) Assuming the statement is true, what (if anything) can you conclude if there will be cake?
   (d) Assuming the statement is true, what (if anything) can you conclude if there will not be cake?
   (e) Suppose you found out that the statement was a lie. What can you conclude?

5. Consider a community health program with the following facts:

   1. Individual is vaccinated
   2. Individual exercises regularly
   3. Individual is considered to have a high level of health

   Assume the program has the following rule: if an individual is vaccinated and exercises regularly, then they are considered to have a high level of health.
   (a) What is the proposition knowledge base for this case?
   (b) Does the knowledge base entail that if an individual does not exercise regularly, they are not considered to have a high level of health?

6. Consider a neighbourhood safety program with the following facts and rule:

   1. Neighbourhood watch is active
   2. Street lighting is adequate
   3. Neighbourhood is safe
   4. If the neighbourhood watch is active and street lighting is adequate, then the neighbourhood is considered safe.

   Does the knowledge base entail that if the neighbourhood is not safe, then either the neighbourhood watch is not active or the street lighting is not adequate?