Tutorial Sections 5-6

1. You are managing a small gardening business that specialises in cultivating a rare type of plant. These plants can be cut and sold in segments, and each segment's length can vary. The unique aspect of these plants is that the price of each segment depends not just on its length but also on demand for specific lengths. Your task is to determine how to cut a plant of a given length to maximise your total revenue.

   The selling prices for different lengths of plant segments are as follows:

Length (units)	Price (£)
1	2
2	4
3	7
4	3
5	9

   (a) Let $P(n)$ denote the maximum price achievable by cutting a plant of length n. Which recursion problem should you solve to ascertain the optimal strategy for cutting the plant?

   (b) What would be the optimal cutting length if you had a plant that is 5 units long if you want to maximise revenue?

   (c) How many different calculations you would need to perform to solve this problem using a fully exhaustive approach? How does that compare to the above strategy?

2. Imagine that you are a logistics coordinator of an online store planning the route for a delivery truck. The truck needs to make deliveries to three different locations in a city. Your task is to plan the most efficient route that minimises the total distance travelled until the last city. The distances (in miles) between the locations (including the starting point) are as follows:

	Starting Point	Location 1	Location 2	Location 3
Starting Point	-	10	20	30
Location 1	10	-	25	15
Location 2	20	25	-	10
Location 3	30	15	10	-

   (a) Explain how recursion can be used to find the shortest path considering all previous steps, and use this approach to find the shortest path for the truck.

   (b) Consider the following recursion to find the minimum path:

$$C(S, i) = \min_{j \in S, j \neq i} (C(S \backslash \{i\}, j) + d_{j,i}),$$

   which represents the minimum cost of a path that starts at the starting point, visits each location in set S, and ends at location i. How this strategy can be used to simplify the problem?

3. You are helping to organise a multi-cultural food festival and need to select a diverse menu featuring four different dishes from various cuisines. The selection of dishes is guided by their popularity ratings, determined through surveys of the local community prior to the event. This year, the popularity ratings for each dish are as follows:

Food	Local Popularity
Pizza	4.2
Sushi	3.9
Curry	4.4
Tacos	3.7
Pad Thai	4.3
Dim Sum	3.8
Crepes	4.1
Falafel	4.0

(a) Last year, the festival featured falafel, tacos, sushi, and dim sum on its menu. Starting with the menu from last year, your goal is to create a menu with the highest combined popularity rating. Due to the festival's theme of cultural diversity, you must ensure a wide representation of cuisines. You can only swap out one dish at a time to maintain a balance in the menu.

(b) How much more effective would the algorithm be if using a 2-Hamming neighbour? (Note: you can answer this question with reference to the size of the full configuration space without going through any computation.)