# Tutorial Sections 1-2

1. Consider the following sets: $A = \{a, b, c, d\}$, $B = \{c, d, e, f, g\}$, and $C = \{b, d, e, g\}$.
   (a) Is $A \backslash B = B \backslash A$?
   (b) What is the result of $(A \cup C) \backslash B$?
   (c) What is the result of $(A \cup B) \backslash (A \cap C)$?

> **Solution:** This is a direct practice of operations with sets.
>
> (a) $A \backslash B$ is the set of elements that are in $A$ but not in $B$. So, $A \backslash B = \{a, b\}$. Similarly, $B \backslash A = \{e, f, g\}$. Therefore, $A \backslash B$ is not equal to $B \backslash A$.
>
> (b) $A \cup C = \{a, b, c, d, e, g\}$, as it combines all unique elements from $A$ and $C$. Therefore, $(A \cup C) \backslash B = \{a, b\}$.
>
> (c) $A \cup B = \{a, b, c, d, e, f, g\}$ and $A \cap C = \{b, d\}$, as they are the only common elements in both $A$ and $C$. Therefore, $(A \cup B) \backslash (A \cap C) = \{a, c, e, f, g\}$.

2. If A is the set of people who got jobs in the IT sector and B is the set of people who got jobs. Describe the people in each of the following sets:
   (a) $A \cap B$
   (b) $A \cup B$
   (c) $A - B$
   (d) $B - A$

> **Solution:** This is a more abstract problem dealing with set operations, which may be more appropriate for part of the students specific to this class. If A is the set of people who got jobs in the IT sector and B is the set of people who got jobs, then A is a subset of B, i.e., $A \subseteq B$. Therefore,
>
> (a) $A \cap B = A$, because all elements in A are also elements of B. This means that $A \cap B$ is the set of people who got jobs in the IT sector.
>
> (b) $A \cup B = B$, i.e., the set of people who got jobs. For the same reason as above, A will not add any new element in the union set.
>
> (c) $A - B = \emptyset$, i.e., an empty set, because all elements of A are in B.
>
> (d) $B - A$ will be a set of people who got jobs in any other sector but the IT sector.

3. Solve the following relations:
   (a) $2x = 2^3 + 2 \times (2 \times 5 - 4)^2 - 30$
   (b) $x^2 - 25 = 0$
   (c) $x - 3 < 2x + 15$

> **Solution:** This is a direct application of expressions, equations and inequalities for practice.

(a) To solve this equation, we need to apply the rules of arithmetic and evaluate the order of operations. Therefore,

$$\begin{aligned}2x &= 8 + 2 \times (10 - 4)^2 - 30 \\ &= 8 + 2 \times (6)^2 - 30 \\ &= 8 + 2 \times 36 - 30 \\ &= 8 + 72 - 30 \\ &= 50\end{aligned}$$

Dividing both sides of the equation by 2 results in $x = 25$.

(b) If $x^2 - 25 = 0$, then $x^2 = 25$. Taking the square root of both sides, we have that $|x| = 5$, as $\sqrt{x^2} = |x|$. Therefore, $x = \pm 5$. (Indeed, you can check that both $x = +5$ and $x = -5$ will satisfy the equation $x^2 - 25 = 0$.)

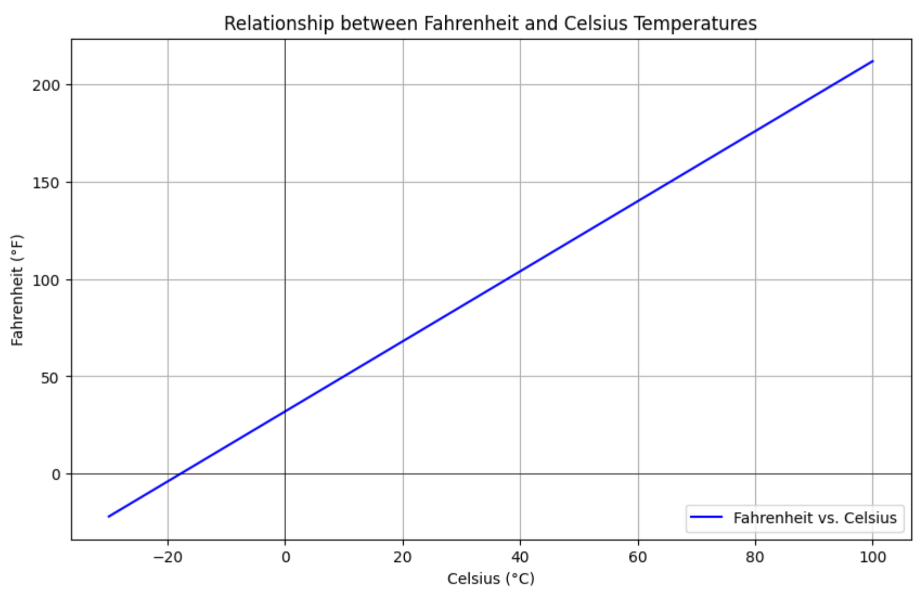
(c) Rearranging the terms, we have that $-x < 18$ or, equivalently, $x > -18$. This means that any value of $x$ greater than $-18$ will satisfy the inequality relation.

4. The relationship between Fahrenheit and Celsius can be expressed as $5T_F - 9T_C = 160$, where $T_F$ and $T_C$ represent the temperature in Fahrenheit and Celsius, respectively. Show that this is a linear function by putting it in $y = mx + b$ format with $T_C = y$. Graph the function indicating slope and intercept.

**Solution:** This is a too simple application of functions with plotting. It's a problem from Gill's book. Rearranging the terms, we have that $9T_C = 5T_F - 160$. Dividing the entire equation by 9,

$$T_C = \frac{5}{9}T_F - \frac{160}{9}.$$

In this equation, $T_C = y, T_F = x$, the slope is $m = 5/9$, and the y-intercept is $b = -160/9 \simeq -17.8$. The graph for this function is shown below.




5. Consider two functions, $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, defined as $f(x) = 2x + 1$ and $g(x) = x^2 + 3x$.

(a) What is the value of $f(-2) - \frac{1}{g(2)}$?

(b) Plot the two functions in a graph paper and determine in which point $f(x)$ intersects $g(x)$. Show this point in your graph.

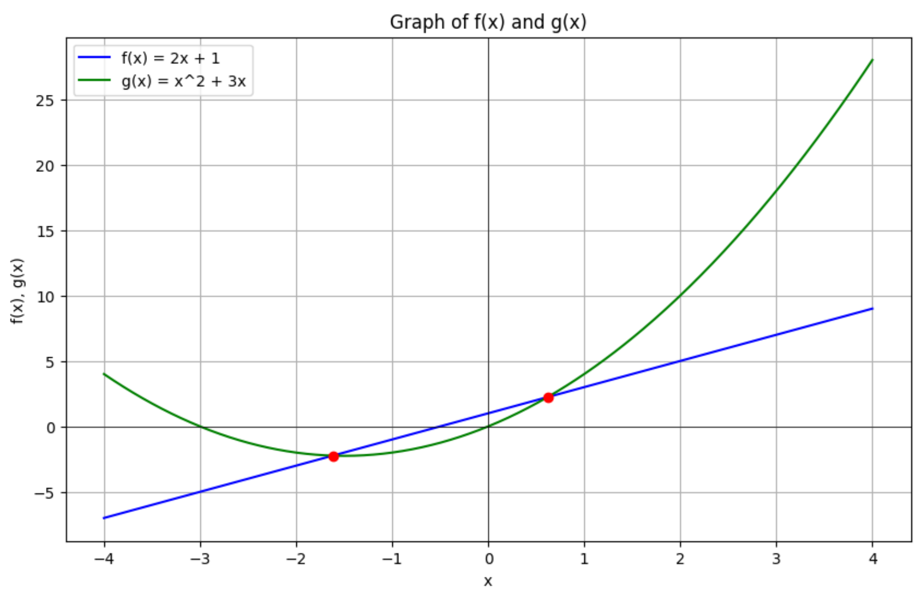
(c) Is the statement $f(g(x)) = g(f(x))$ true?

**Solution:** This is a more abstract problem with function but more generic and deals with several topics related to functions.

(a) From the definition above, $f(-2) = 2(-2)+1 = -4+1 = -3$ and $g(2) = (2)^2+3(2) = 4+6 = 10$. Therefore,

$$f(-2) - \frac{1}{g(2)} = -3 - \frac{1}{10} = \frac{-30-1}{10} = -\frac{31}{10} = -3.1$$

(b) The point of intersection happens when $f(x) = g(x)$, which means that $2x+1 = x^2+3x$. Rearranging the terms, we have $x^2 + x - 1 = 0$. Solving for $x$, we find that the two curves intersect at $x = -1.62$ and $x = 0.62$. Calculating the $y$-value for each $x$ using any of the two functions (since, at these points, both $f(x)$ and $g(x)$ are equal, we found the two points of intersection are $(-1.62, -2.24)$ and $(0.62, 2.24)$. The plot with the functions and the intersection points is shown below:




(c) To verify the statement, we need to compute both compositions:

$$f(g(x)) = f(x^2+3x) = 2(x^2+3x) + 1 = 2x^2 + 6x + 1.$$

Similarly,

$$g(f(x)) = g(2x+1) = (2x+1)^2 + 3(2x+1) = 4x^2 + 4x + 1 + 6x + 3 = 4x^2 + 10x + 4.$$

Since $2x^2 + 6x + 1$ is not equal to $4x^2 + 10x + 4$ for all $x$, the statement is not true.

6. Consider a module whose assessment is entirely based on coursework. This module consists of five pieces of coursework, which we will denote as $x_n$, where $n = 1, 2, ..., 5$. A student enrolled in this module

achieved scores of 8 in the first piece of coursework, 10 in the second, 6 in the third, 9 in the fourth, and 8 in the fifth.

(a) Suppose the final mark for the module, $G$, is simply the average mark obtained across all pieces of coursework, i.e.,

$$G = \frac{1}{N} \sum_{n=1}^{N} x_n,$$

where $N = 5$. What would be the student's final mark under this criterion?

(b) Another method to calculate the final mark could be to use the geometric mean, expressed as

$$G = \left( \prod_{n=1}^{N} x_n \right)^{1/N}.$$

What would be the student's final mark in this scenario?

**Solution:** This is just a contextualised problem that applies the concepts of summation and product operators. In this problem, we have $x_1 = 8, x_2 = 10, x_3 = 6, x_4 = 9, x_5 = 8$.

(a) Expanding the summation, we have

$$G = \frac{1}{5}(x_1 + x_2 + x_3 + x_4 + x_5) = \frac{1}{5}(8 + 10 + 6 + 9 + 8) = \frac{1}{5}(41) = 8.2$$

(b) Expanding the product operator, we have

$$G = (x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5)^{1/5} = (8 \times 10 \times 6 \times 9 \times 8)^{1/5} = (34,560)^{1/5} = \sqrt[5]{34560} \simeq 8.1$$