

Artificial Intelligence and Machine Learning (AIML)

2023–24





- **Last lecture:** essential mathematical background
- **This lecture:** combinatorial optimization in AI, combinatorial explosion and computational complexity

Combinatorial optimization: overview

Good Old-Fashioned AI (GOFAI), or Symbolic AI

- Many **symbolic AI** (GOFAI) problems are **optimization** problems
- **Minimize objective function** over all possible **configurations**
- Exact vs. approximate methods
- Exact: **guaranteed** to find an **optimal solution**; approximate: **not guaranteed** to find an optimal solution, may **come close** ("good enough")
- Exact methods slow, approximate methods unreliable

Combinatorial optimization: overview

- Minimize objective function F with respect to configurations X from a set of possible configurations,

X^* is the optimal set $X^* = \arg \min_{X' \in \mathcal{X}} F(X')$ Minimize objective function F
 X' belongs to \mathcal{X}

- Globally optimal solution satisfies:

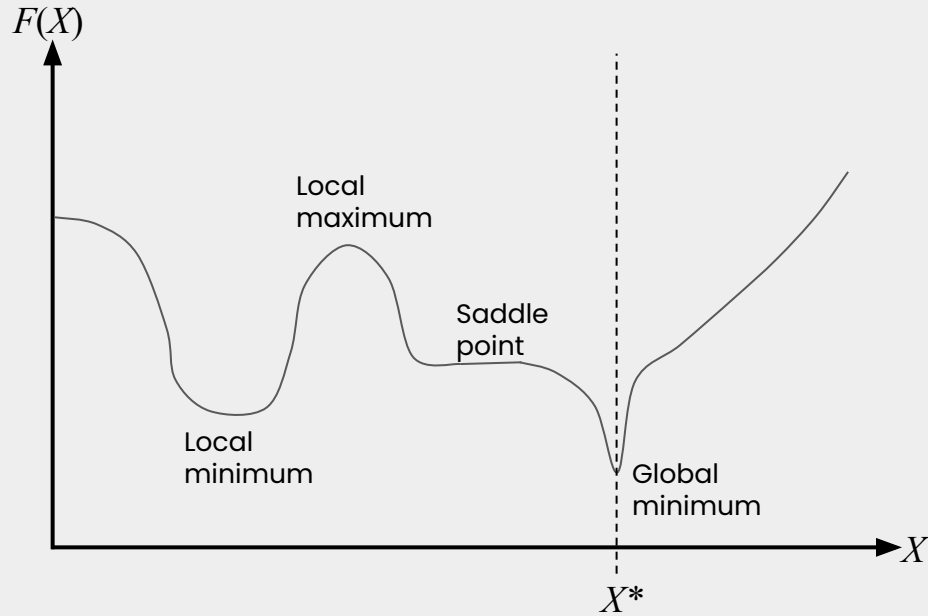
$$F(X') \geq F(X^*), \forall X' \in \mathcal{X} \quad \text{For all } X', \text{ they belong to } \mathcal{X}$$

- Locally optimal solution for neighbourhood of all configurations

$$F(X') \geq F(X^*), \forall X' \in \mathcal{N}$$

For all X' , they belong to natural numbers

Optimizing objective functions



Exhaustive solution

- Generate all possible configurations, compute objective function for each, select the best one
- Always applicable, always guaranteed to find an optimal solution
- Simple to formulate, often there are simple methods to systematically generate all possible configurations
- Often the starting point for more efficient, exact algorithms (see dynamic programming later)

Combinatorial explosion

- Size of the configuration space: typically exponential or worse complexity, number of possible configurations as a function of the **problem size N**
- Complexity orders: from constant time/space (tractable) to factorial (intractable) controllable
- In general: **polynomial and better** are considered "practical", whereas exponential and worse are considered "impractical", when taking into account realistic computational resources
- Implication that exhaustive, although generally applicable, simple and exact, is rarely practical; justifies the need for AI: smart solutions to otherwise intractable problems

Complexity classes

constant	$O(1)$	Tractable
logarithmic	$O(\log N)$	
linear	$O(N), O(Nk)$	
log-linear	$O(N \log N)$	
polynomial	$O(N^k)$	
exponential	$O(k^N)$	Intractable
factorial	$O(N!)$	

Computational complexity

- Space versus time complexity
- Trading off time for space and vice-versa

References and further reading

- **MLSP**, Section 1.8, Section 2.6
- **CLRS**, Chapter 3
 - Machine Learning for Signal Processing
 - Introduction to Algorithms
- **R&N**, Section A.1
 - Artificial Intelligence: A Modern Approach