Attendance Code:
XXXXX

Artificial Intelligence and Machine Learning (AIML)

2023–24

- **Last ML lectures**:
  - **Unsupervised learning**: clustering and the k-means algorithm
  - **Supervised learning**:
    - Linear regression using SGD
    - Classification using the perceptron algorithm
- **This lecture**: neural networks and deep learning

AIML

#Code

What we have accomplished so far

- Supervised Learning

Labeled training data

Data point (i)	Feature 1 (x^1)	Feature 2 (x^2)	…	Feature D (x^D)	Output (y)
1					
2					
3					
4					
…					
…					
N					

Linear models for regression/classification

#Code

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$$f(w, x) = \hat{y} = f\left(\sum_{j=1}^{D} w_j x^j\right)$$

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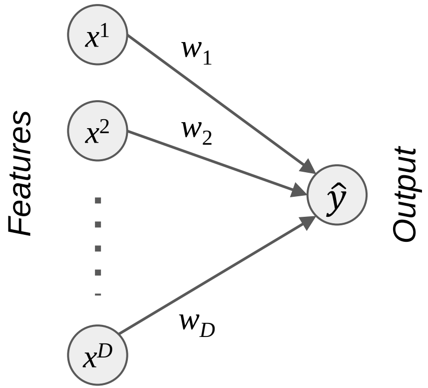
Pictorial Representation



$$f(w,x) = \hat{y} = f(w_1x^1 + w_2x^2 + \cdots + w_Dx^D)$$

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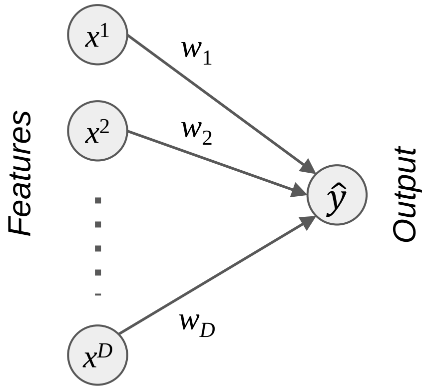


$$f(w,x) = \hat{y} = f(w_1 x^1 + w_2 x^2 + \cdots + w_D x^D)$$

$$\hat{y} = f(w^T x)$$

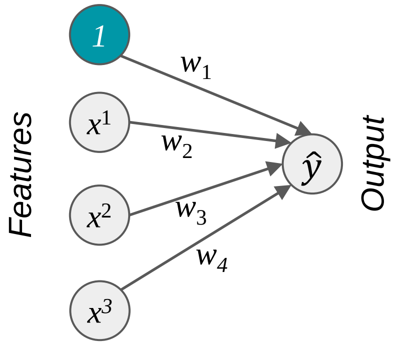
parameters vector

features vector

Classification Intuition: Housing Market

#Code

- Data on 3 features: square footage, # bedrooms, house age
- Classification label: location (UK vs. non-UK house)



$$f(w, x) = \hat{y} = \text{sign}(w_1 1 + w_2 x^1 + w_3 x^2 + w_4 x^3)$$

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#Code

How can we extend this to 3 classes?

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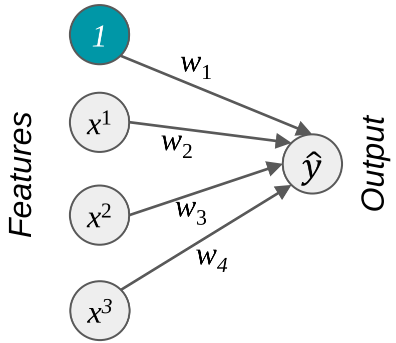
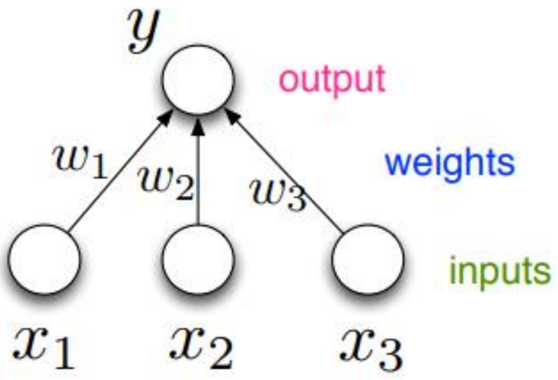


- Extra data labeled as Dubai houses
- How would you build a classification model to classify between the UK, Dubai, and other locations using the same features?

$$f(w, x) = \hat{y} = \text{sign}(w_1 1 + w_2 x^1 + w_3 x^2 + w_4 x^3)$$

$$\hat{y} = \text{sign}(w^T x)$$

- Recall the simple neuron-like unit:



$$\overset{\text{output}}{y} = \underset{\text{nonlinearity}}{g}\left(\overset{\text{bias}}{b} + \sum_i \underset{\text{i'th input}}{x_i}\,\overset{\text{i'th weight}}{w_i}\right)$$

- These units are much more powerful if we connect many of them into a neural network.

How can we extend this to 3 classes?

#Code

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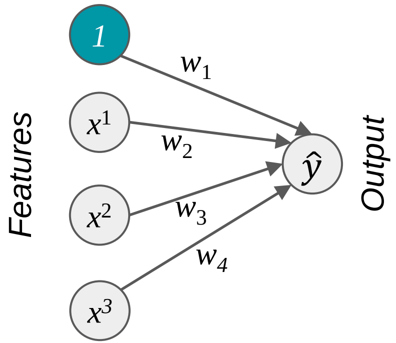

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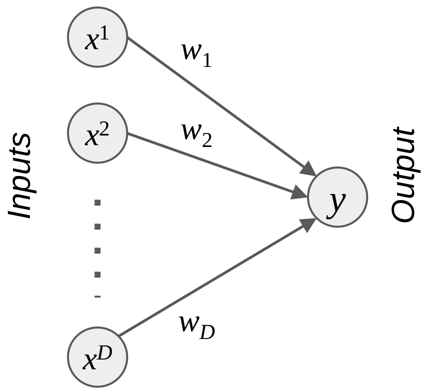
#Code

Neural networks: single layer perceptron

- Can view the simple linear perceptron with its loss function, in the form of a **weighted linear combination** with nonlinear **activation function**

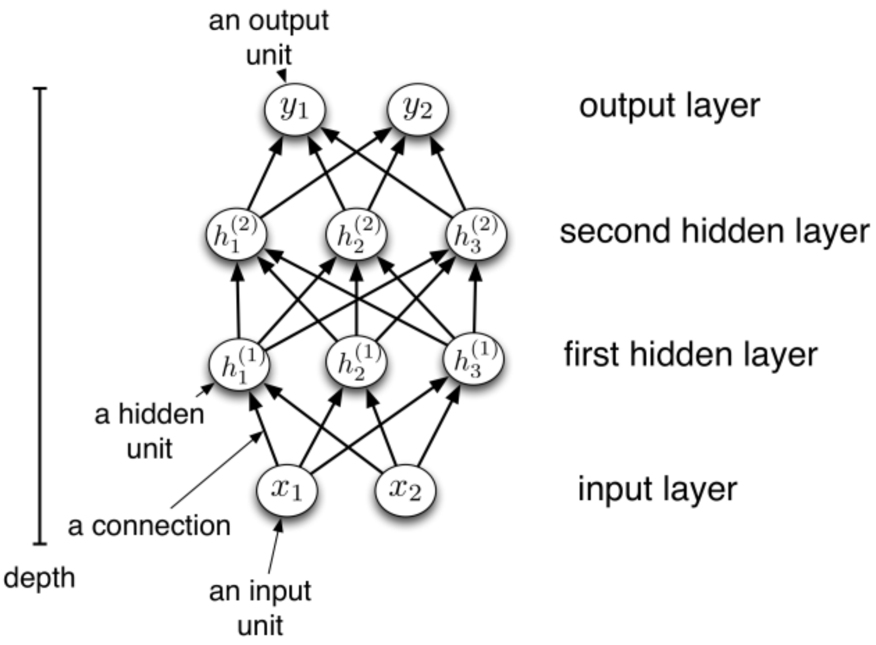


$$y = \max(0, w^T x)$$

- Inspired by a **biological neuron**, which has axons, dendrites and a nucleus, which fires when a certain threshold is crossed

Multi-layer perceptron

- We can connect lots of units together into a directed acyclic graph.
- This gives a feed-forward neural network. That's in contrast to recurrent neural networks, which can have cycles. (We'll talk about those later.)
- Typically, units are grouped together into layers.



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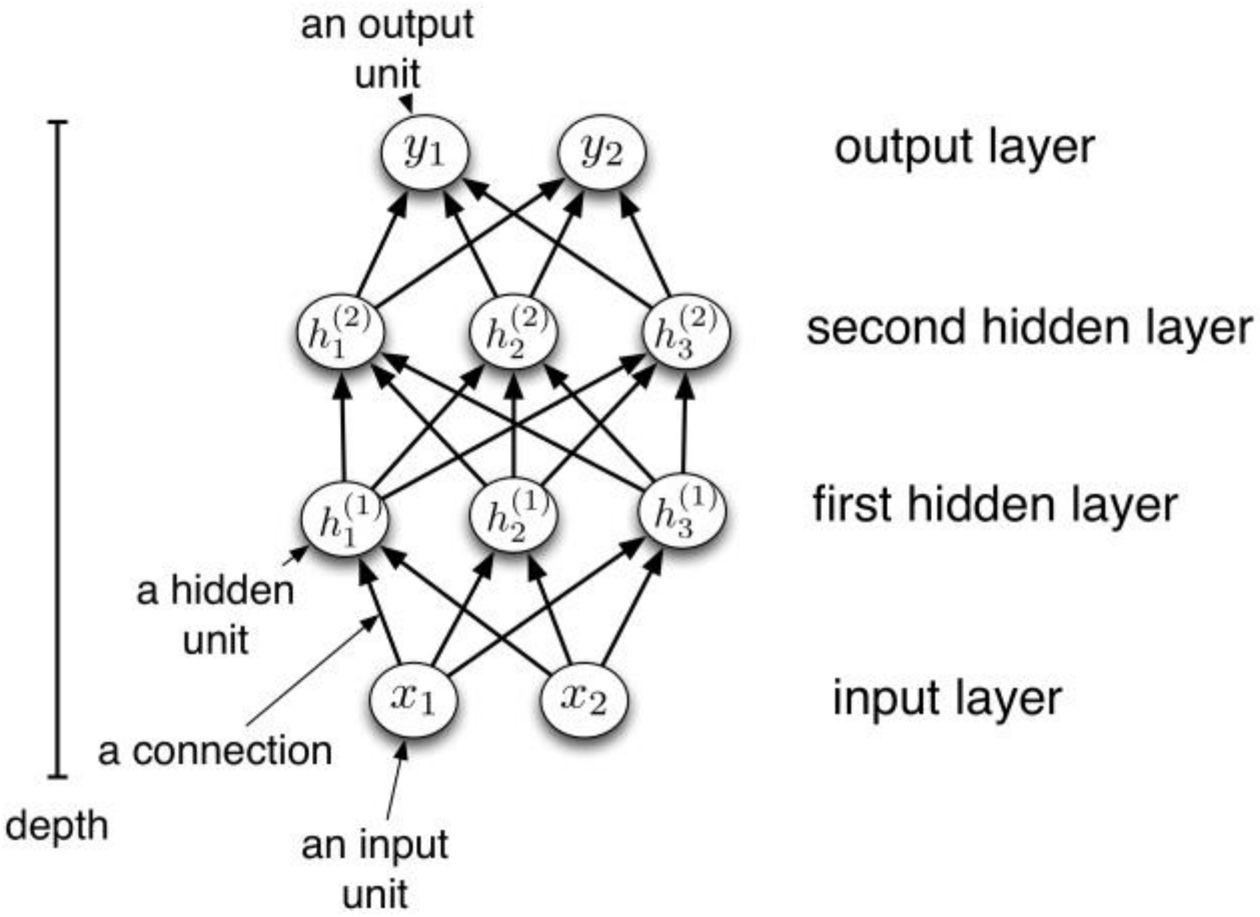


an output unit
y_1
y_2
output layer
$h_1^{(2)}$
$h_2^{(2)}$
$h_3^{(2)}$
second hidden layer
$h_1^{(1)}$
$h_2^{(1)}$
$h_3^{(1)}$
first hidden layer
a hidden unit
x_1
x_2
input layer
a connection
depth
an input unit

#Code

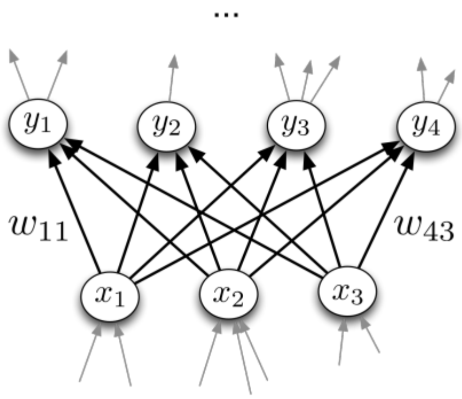
Multi-layer perceptron

- Each layer connects N input units to M output units.
- In the simplest case, all input units are connected to all output units. We call this a fully connected layer. We'll consider other layer types later.
- Note: the inputs and outputs for a layer are distinct from the inputs and outputs to the network.

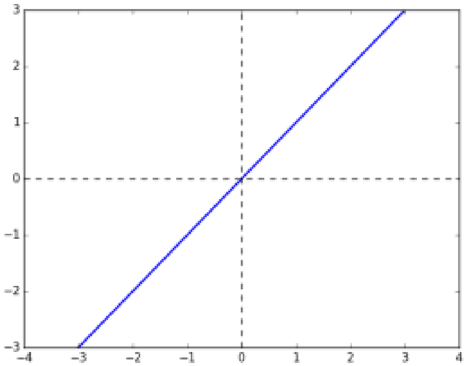
- Recall from multiway logistic regression: this means we need an $M \times N$ weight matrix.
- The output units are a function of the input units:

$$\mathbf{y} = f(\mathbf{x}) = \phi(\mathbf{W}\mathbf{x} + \mathbf{b})$$

- A multilayer network consisting of fully connected layers is called a multilayer perceptron. Despite the name, it has nothing to do with perceptrons!

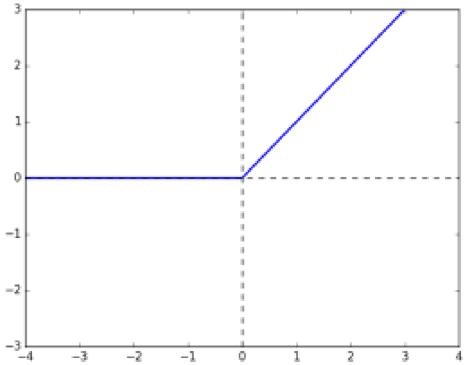


Some activation functions:



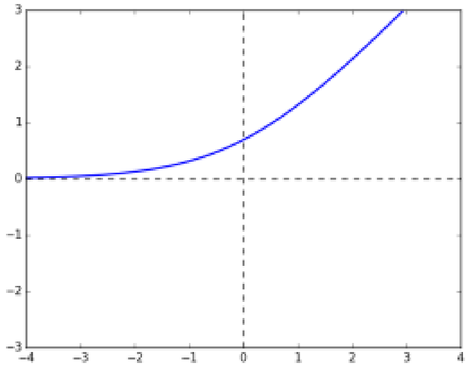
Linear

$$y = z$$



Rectified Linear Unit (ReLU)

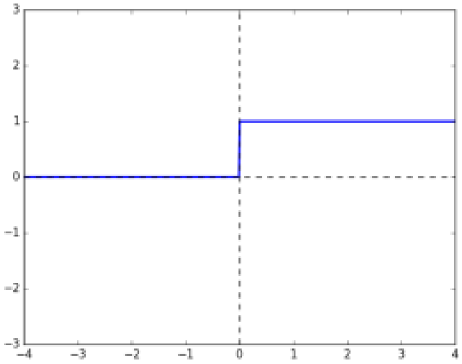
$$y = \max(0, z)$$



Soft ReLU

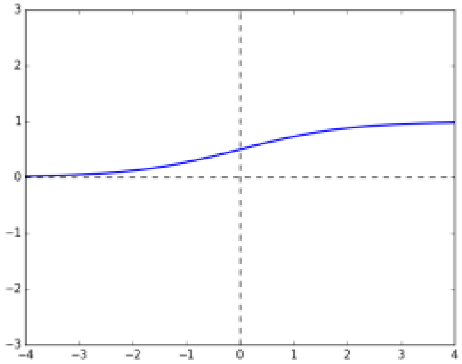
$$y = \log 1 + e^z$$

Some activation functions:



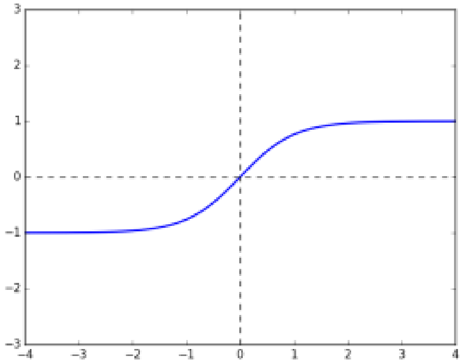
Hard Threshold

$$y = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z \leq 0 \end{cases}$$



Logistic

$$y = \frac{1}{1 + e^{-z}}$$



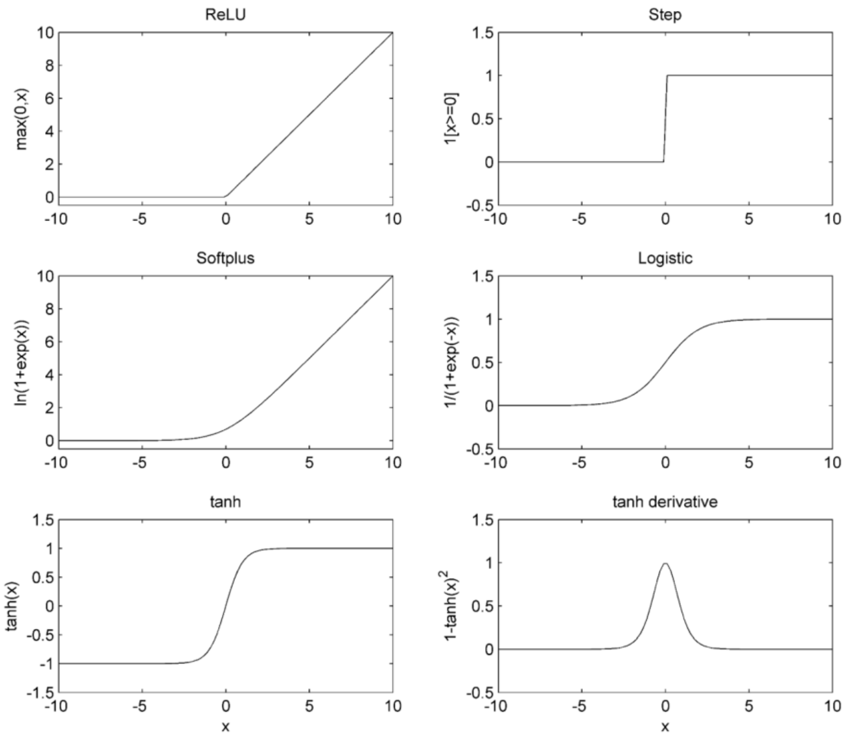
Hyperbolic Tangent (tanh)

$$y = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

#Code

Activation nonlinearities

Activation Function | Derivative



- Wide range of activation functions in use (**logistic**, **tanh**, **softplus**, **ReLU**): only criteria is that they must be **nonlinear** and should ideally be **differentiable** (almost everywhere)
- ReLU is perceptron loss, sigmoid is logistic regression loss
- ReLU most widely used activation; exactly zero for half of its input range (many outputs will be zero)

#Code

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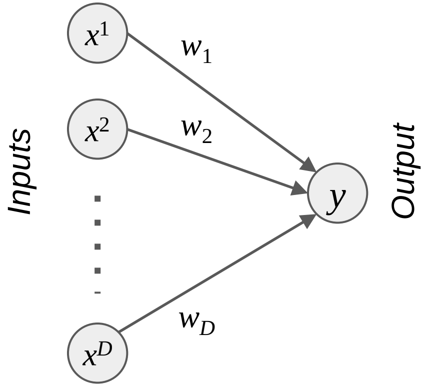


$$y = \max(0, w^T x)$$

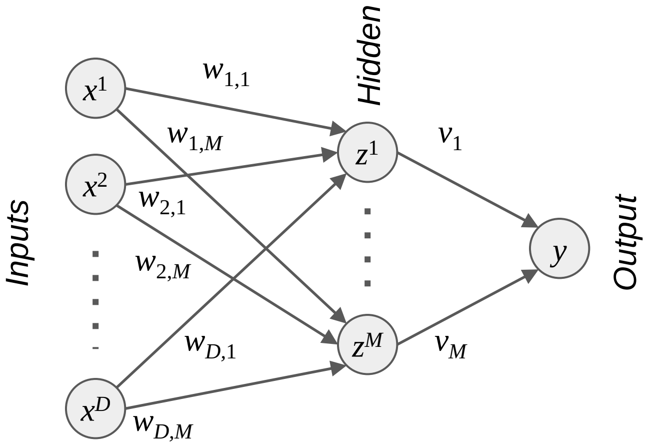
- Inspired by a **biological neuron**, which has axons, dendrites and a nucleus, which fires when a certain threshold is crossed
- Perceptron is very limited, and can only model linear decision boundaries
- More complex nonlinear boundaries are required in practical ML

Single layer Perceptron is very limited, and can only model linear decision boundaries, that is why we need more complex nonlinear boundaries in practical ML

Neural networks: deep learning

#Code

- Extend the perceptron to two or more layers of weighted combinations (**linear layers**) with **nonlinear activations** connecting them



$$z = \max(0, W^T x), \; y = \max(0, v^T z)$$

- Intermediate nodes are known as **hidden neurons**, whose output *z* is fed into the **output layer** which produces the final output *y*
- Modern **deep learning algorithms** usually have multiple hidden neurons, in multiple additional (hidden) layers
- Greatly extends the complexity of decision boundaries (**piecewise linear boundaries**)

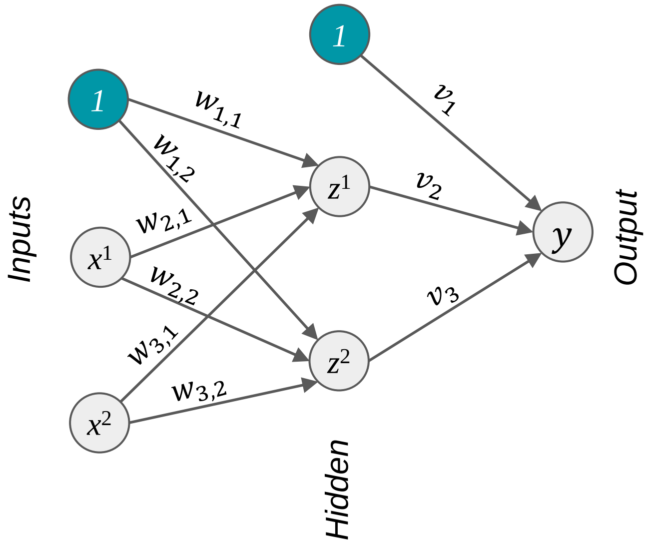
piecewise linear boundaries: 分段线性边界

Neural networks: deep learning

#Code

- Example of a multilayer neural network

What is this ‘1’???



$$z^1 = f\left(w_{1,1}1 + w_{2,1}x^1 + w_{3,1}x^2\right)$$

$$z^2 = f\left(w_{1,2}1 + w_{2,2}x^1 + w_{3,2}x^2\right)$$

$$W^T = \begin{bmatrix} w_{1,1} & w_{2,1} & w_{3,1} \\ w_{1,2} & w_{2,2} & w_{3,2} \end{bmatrix}$$

$$z = f(W^T x)$$

$$y = f(v_1 1 + v_2 z^1 + v_3 z^2)$$

$$y = f(v^T z)$$

#Code

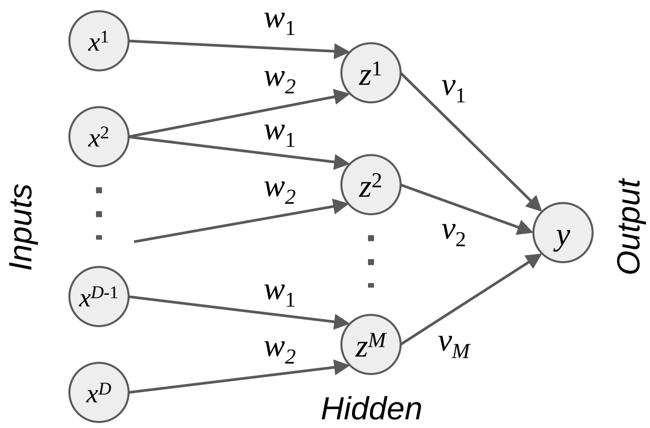
Neural networks: weights consideration

- **Fully connected networks** (every node in each layer connected to every node in the previous layer)

#Code

Neural networks: weights consideration

- **Fully connected networks** (every node in each layer connected to every node in the previous layer) means rapid growth in the number of weights
- **Weight-sharing**, forcing certain connections between nodes to have the same weight, is sensible for certain special applications
- Widely-used example (particularly suited to ordered data: images or time series) is **convolutional** sharing



$$z^1 = \max(0, w^T[x^1\ x^2]^T)$$

$$z^2 = \max(0, w^T[x^2\ x^3]^T)$$

$$\ldots$$

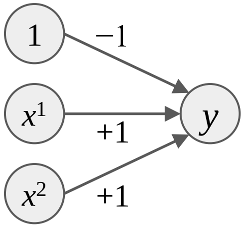
$$z^M = \max(0, w^T[x^{D-1}\ x^D]^T)$$

$$y = \max(0, v^T z)$$

Deep neural logic networks: example

- With sign activation function (similar to step activation), **logical neural networks** have simple weights
- Use these to implement basic logical functions "and", "or" and "not", encoding *True* as $+1$, *False* as -1

$y = x^1 \wedge x^2$ (and)



$$True = +1, False = -1$$

$$f_{\text{and}}(x_1, x_2) = \text{sign}(x_1 + x_2 - 1)$$

#Code

Deep neural logic networks: example

- With sign activation function (similar to step activation), **logical neural networks** have simple weights
- Use these to implement basic logical functions "and", "or" and "not", encoding *True* as $+1$, *False* as -1
- Any complex logical function can be implemented by composing these basic neurons together

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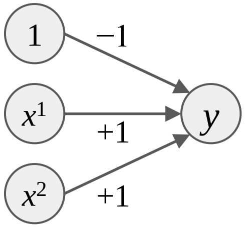

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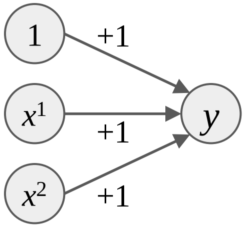
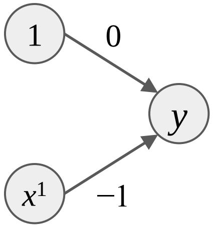


$y = \neg\, x^1$ (not)



$$True = +1,\ False = -1$$

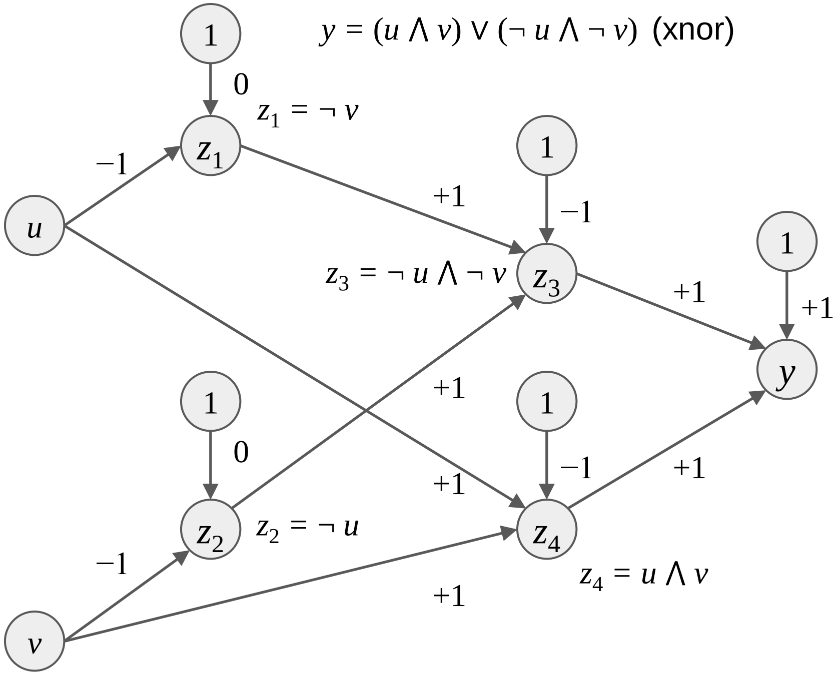
$$f_{\text{and}}(x_1, x_2) = \text{sign}(x_1 + x_2 - 1)$$

$$f_{\text{or}}(x_1, x_2) = \text{sign}(x_1 + x_2 + 1)$$

$$f_{\text{not}}(x) = \text{sign}(-x)$$

Deep neural logic networks: XNOR

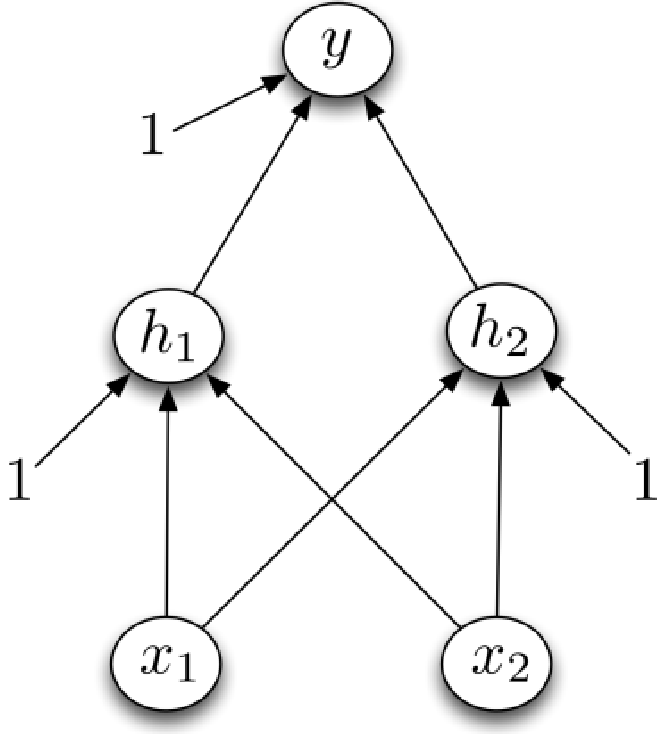
#Code



- **Exclusive not-or** "xnor" function constructed using the basic logical neural networks
- In this implementation, need **two hidden layers** z_1, z_2 and z_3, z_4 to compute intermediate terms in the expression
- Example simple function which cannot be computed using a single layer linear neural network

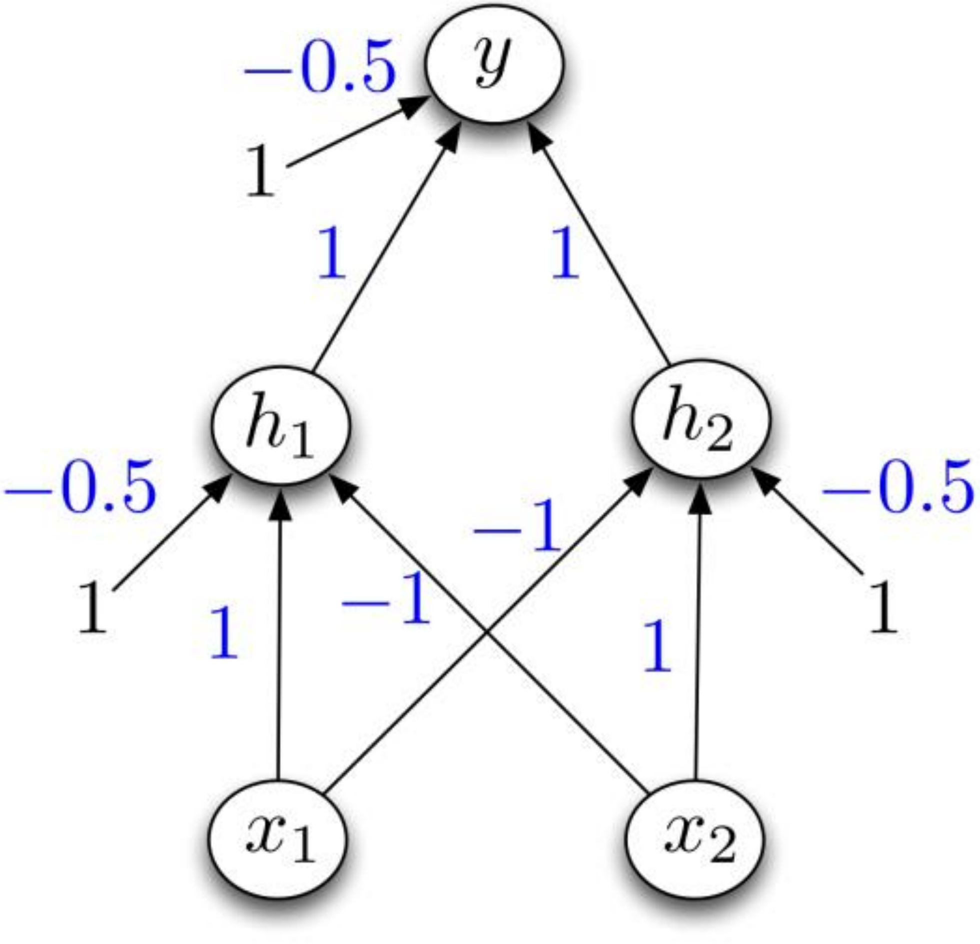
Designing a network to compute XOR:

Assume hard threshold activation function



XOR

#Code



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To recap

- We learned the basic concepts of a **neural network**
  - Extended the concept to multiple (hidden) layers: **deep learning**
  - Problem of the number of weights & weight sharing: **convolutional neural network**
- **Next**: How to optimize the weights of a neural network
  - Pre-Reading: **Lecture Notes**, Section 14

Further Reading

- **PRML**, Section 5.1
- **H&T**, Section 11.3