UNIVERSITY OF BIRMINGHAM

School of Computer Science

Data Structures, Algorithms and Databases

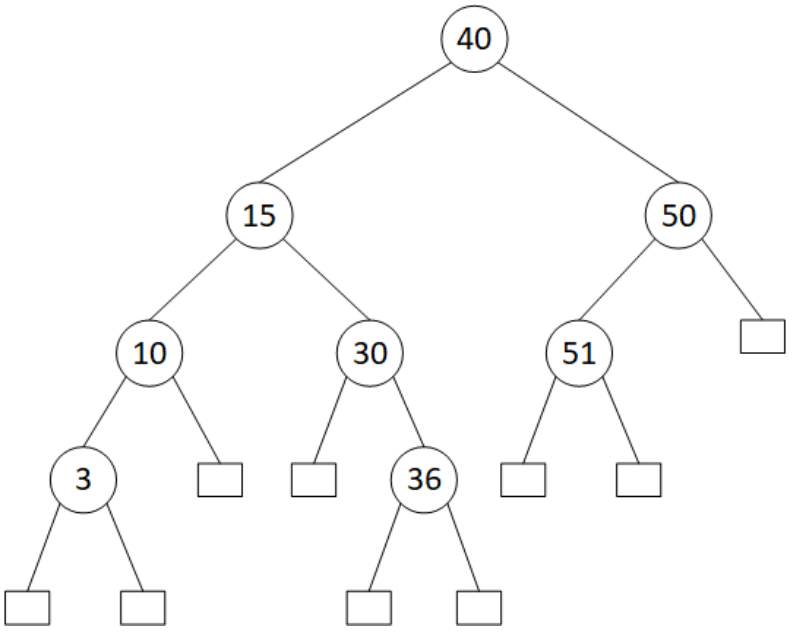
Class Test #1 [2023-24]

7th Feb, 2024

Data Structures, Algorithms and Databases

Question 1 Formula Representation

Consider the following tree structure:



(a) Write the formula representation of the tree using $EMPTY$ and $Fork$. **[2 marks]**

(b) Does this structure qualify as an AVL tree? If it is, justify your answer. Otherwise, suggest a correction to change it into an AVL tree without modifying the shape of the tree. **[2 marks]**

(c) Consider the following operations:

1. $T1 = Fork(10, Fork(5, EMPTY, EMPTY), Fork(20, EMPTY, EMPTY))$
2. $T2 = Fork(30, Fork(25, EMPTY, EMPTY), Fork(35, EMPTY, EMPTY))$
3. $T = Fork(22, T1, T2)$
4. $T = Fork(largest(T2), T1, removeLargest(T2))$

Is T a valid binary search tree? If so, draw the corresponding diagram for T. Otherwise, explain why it is not. **[1 mark]**

Question 2 BST

Part 1 Consider the following pseudocode:

```
function arrayToBST(_array)
  tree = □;
  for element in _array
    tree = insert(element, tree);
  return tree;
```

where *_array* is an input array, *insert*() is a function to add an element to a binary search tree accordingly (without rebalancing).

Assuming the length of *_array* equals to 8 and $\_array = [25, 36, 6, 6, 18, 10, 15, 35]$.

(a) After running this function with *_array* as the input, write down the in-order traversal and the pre-order traversal of the tree. **[3 marks]**

(b) What is the height of the tree after insert another element 19 into it? **[1 mark]**

(c) Without increasing its height, what is the max number of node this tree can have? **[1 mark]**

(d) Is it possible to construct an input with the same length (length = 8) so that running the above algorithm produces a perfectly balanced tree? If so, provide an example array; If not, explain why. **[4 marks]**

Hint: Similar to the in order traversal, pre-order traversal is a way to visit all the nodes in a binary tree by following a specific order: visit the root node first, then recursively perform a pre-order traversal of the left subtree, followed by a recursive pre-order traversal of the right subtree. This process is applied to each node in the tree, ensuring that each node is visited exactly once.

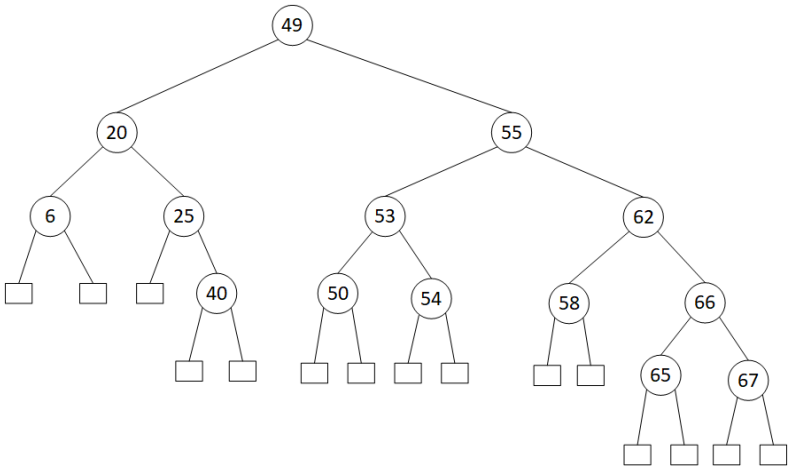
Question 3 AVL

(a) Suppose an AVL tree with 6764 nodes: After a deletion operation, what are the **minimum** and **maximum** possible heights of the resulting tree? Be specific and justify your answer. **[3 marks]**

Hint: Here some Fibonacci number values:

$F(17) = 1597$, $F(18) = 2584$, $F(19) = 4181$, $2^{12} = 4096$, $2^{13} = 8192$

(b) Consider the following AVL tree:



Remove node 6 from the above tree and execute the necessary rotation(s). Illustrate both the intermediate stages and the final outcome. **[3 marks]**