Integer Linear Programming Model of Routing, Core and Frequency Slot Assignment for Time Division Multiplexing, Wavelength Division Multiplexing and Orthogonal Frequency Division Multiplexing over Multi-Core Fiber

Qihan Zhang (lengkudaodi@outlook.com)

July 8, 2021

In [1], the authors developed an Integer Linear Programming (ILP) model to minimize the index of multi-dimensional resources occupied by multiple types of services in the time, frequency, and space domains. To elaborate the model in details for usual understanding, this work formulates the Routing, Core and Frequency Slot Assignment (RSCA) problem for different multiple access such as Time Division Multiplexing (TDM), Wavelength Division Multiplexing (WDM) and Orthogonal Frequency Division Multiplexing (OFDM) over Multi-Core Fibers (MCFs), and also programs the ILP model in C++ edited in Visual Stusio Code (VSCode) [2] by employing IBM ILOG CPLEX Optimization Studio 12.8 (CPLEX) [3] to search for the optimal solutions.

The ILP model of the RSCA problem in [1] derives from the other classic problem-virtual optical network embedding (VONE) problem in [4]. Therefore, some constraints' formulation in this work could be similar to [4]. The difference is that there are only two nodes (source node and destination node) and one link to be embedded for a service request in RSCA problem, thus, node mapping constraint should be replaced by known source and destination.

Firstly, the notations and variables are described as follows:

1. Notations for the RSCA problem:

   - R: The set of service requests;
   - G: The topology of the substrate network;
   - G^r: The service request of the index r, which is abstracted as a virtual network topology with two virtual nodes and one virtual link;
   - C_v^r: The embedded substrate node of the v-th virtual node in the r-th service request;
   - Ω^r: The number of frequency slots requested by the r-th service request, with the value of 1 for WDM while with the value of positive integer for OFDM;
   - T^r: The set of time slots requested by the r-th TDM service request, with the value of positive integer;

- C_{xDM}: The set of assigned cores by a certain xDM service request (xDM indicates TDM, WDM or OFDM. All links in the substrate network should own the same cores set in term of the same kind of service request).

2. Variables for the RSCA problem:

   - $Ms^r_{e,(s,d)}$: An integer decision variable indicates the maximize frequency slot index allocated in the s-to-d substrate link for the e-th virtual link in the r-th service request when allocates frequency slots for WDM and OFDM service;
   - $x^r_{e,(s,d)}$: A binary decision variable indicates whether the e-th virtual link has been mapped in the s-to-d substrate link or not when maps the virtual links of the r-th service request. If the link is mapped, $x^r_{e,(s,d)} = 1$, otherwise, $x^r_{e,(s,d)} = 0$;
   - $y^r_{v,V}$: A binary decision variable indicates whether the v-th virtual node has been embedded in the V-th substrate node or not when embeds the virtual nodes of the r-th service request. If the v-th virtual node is embedded, $y^r_{v,V} = 1$, otherwise, $y^r_{v,V} = 0$;
   - $X^r_{c,(s,d)}$: A binary decision variable indicates whether the c-th core in the s-to-d substrate link has been assigned when assigns the cores in the r-th service request. If the core is assigned, $X^r_{c,(s,d)} = 1$, otherwise, $X^r_{c,(s,d)} = 0$;
   - $f^r_{e,(s,d)}$: An integer decision variable indicates whether the frequency slots in the e-th virtual link has been allocated in the s-to-d substrate link or not when allocates the frequency slots of the r-th service request for WDM and OFDM service. If the frequency slots is allocated, $f^r_{e,(s,d)}$ is a positive integer describing the allocated start frequency slot, otherwise, $f^r_{e,(s,d)} = 0$;
   - $t^{r,slot}_{e,(s,d)}$: An integer decision variable indicates whether the slot-th time slot in the e-th virtual link has been allocated in the s-to-d substrate link or not when allocates the time slots of the r-th service request for TDM service. If the time slot is allocated, $t^{r,slot}_{e,(s,d)}$ is a positive integer describing the allocated time slot, otherwise, $t^{r,slot}_{e,(s,d)} = 0$.

Then, according to the notations and variables, the ILP model of the RSCA problem for TDM, WDM and OFDM service over MCF could be formulated as follows:

1. Objective: The objective function in [1] is to minimize the index of multi-dimensional resources occupied by all types of service requests, which means minimizing the resources of the substrate network. Here, we minimize the summation of the maximize frequency slot index allocated or time slot index allocated and the number of mapping links to formulate the minimal substrate resources occupying.

$$\text{Minimize:} \quad \begin{cases} \text{Max}\left\{Ms^r_{e,(s,d)}\right\} + \sum\limits_{\forall r\in R}\sum\limits_{\forall e\in G^r}\sum\limits_{\forall (s,d)\in G} x^r_{e,(s,d)}, & \text{WDM or OFDM} \\ \text{Max}\left\{t^{r,slot}_{e,(s,d)}\right\} + \sum\limits_{\forall r\in R}\sum\limits_{\forall e\in G^r}\sum\limits_{\forall (s,d)\in G} x^r_{e,(s,d)}, & \text{TDM} \end{cases} \tag{1}$$

2. Subject to the following constraints which are common to three types of service.

(a) Constraint of node embedding:

$$y_{v,V}^{r}=\begin{cases}1, & V=C_{v}^{r}\\ 0, & \text{otherwise}\end{cases},\quad \forall r\in R,\forall v\in G^{r},V\in G \tag{2}$$

(b) Constraints of link mapping:

- Each link in every service request should be mapped into only one direction from source substrate node to destination substrate node, i.e., no circle for link mapping.

$$x_{e,(s,d)}^{r}+x_{e,(d,s)}^{r}\leq 1,\quad \forall r\in R,\forall e\in G^{r},\forall\left(s,d\right)\in G \tag{3}$$

- For every service request, if the source virtual node is embedded into a substrate node, the number of inflow links should be one more than the number of outflow links in that substrate node. If the destination virtual node is embedded into a substrate node, the number of inflow links should be one less than the number of outflow links in that substrate node. Except for the two cases, the number of inflow links should be the same as the number of outflow links in the substrate node, i.e., the flow conversation.

$$\sum_{\forall(s,d)\in G}\left[x_{e,(s,d)}^{r}-x_{e,(d,s)}^{r}\right]=y_{e_s,s}^{r}-y_{e_d,s},\quad \forall r\in R,\forall e\in G^{r},\forall s\in G \tag{4}$$

where e_s, e_d is the source virtual node and the destination virtual node of the e-th virtual link, respectively.

(c) Constraint of core assigning: The mapped substrate edge should only assign one core for a service request.

$$\sum_{\forall c\in C_{xDM}}X_{c,(s,d)}^{r}=\sum_{\forall e\in G^{r}}x_{e,(s,d)}^{r},\quad \forall r\in R,\forall\left(s,d\right)\in G \tag{5}$$

3. Subject to the following constraints which are specific for a certain type of service.

(a) Constraints for WDM or OFDM service:

- The start frequency slot index of each service request is associated with whether all links of the service request are mapped or not.

$$\begin{cases}\text{If} & x_{e,(s,d)}^{r}=0 & \text{Then} & f_{e,(s,d)}^{r}=0\\ \text{If} & x_{e,(s,d)}^{r}=1 & \text{Then} & f_{e,(s,d)}^{r}\geq 1\end{cases},\forall r\in R,\forall e\in G^{r},\forall\left(s,d\right)\in G \tag{6}$$

- When two service requests are assigned to the same core in the same link, the frequency slots should not be overlapped, i.e. Non-overlapped frequency slots.

$$\begin{aligned}&\text{If}\quad X_{c,(s,d)}^{r}+X_{c,(s,d)}^{\hat{r}}=2\quad\text{Then}\\ &f_{e,(s,d)}^{r}-f_{e,(s,d)}^{\hat{r}}\geq\Omega^{\hat{r}}||f_{e,(s,d)}^{\hat{r}}-f_{e,(s,d)}^{r}\geq\Omega^{r},\\ &\quad\forall\left(r,\hat{r}\right)\in R,\forall c\in C_{WDM}\ or\ C_{OFDM},\forall e\in G^{r},\forall\left(s,d\right)\in G\end{aligned} \tag{7}$$

- The allocating maximize frequency slot index of a service request equals to the summation of the start frequency slot index allocated and the number of frequency slots requested minus one, i.e., frequency slots continuity.

$$f^{r}_{e,(s,d)} + \Omega^{r} - 1 = Ms^{r}_{e,(s,d)}, \quad \forall r \in R, \forall e \in G^{r}, \forall (s,d) \in G \tag{8}$$

- In the transparent optical networks, each substrate node does not have the function of photoelectric conversion. When a virtual link in a service request is mapped into two or more substrate links, the start frequency slot index allocated in each link should be the same, i.e., frequency slots consistency. While in the opaque optical networks, the function of photoelectric conversion in every substrate node invalidates the frequency slots consistency. In this situation, this constraint could be omitted.

$$f^{r}_{e,(s,d)} = f^{r}_{e,\left(\hat{s},\hat{d}\right)}, \quad \forall r \in R, \forall e \in G^{r}, \forall (s,d), \left(\hat{s},\hat{d}\right) \in G \tag{9}$$

(b) Constraints for TDM service:

- The start time slot index of each service request is associated with whether all links of the service request are mapped or not.

$$\begin{cases} \text{If} \quad x^{r}_{e,(s,d)} = 0 \quad \text{Then} \quad t^{r,slot}_{e,(s,d)} = 0 \\ \text{If} \quad x^{r}_{e,(s,d)} = 1 \quad \text{Then} \quad t^{r,slot}_{e,(s,d)} \geq 1 \end{cases}, \forall r \in R, \forall slot \in T^{r}, \forall e \in G^{r}, \forall (s,d) \in G \tag{10}$$

- When two service requests are assigned to the same core in the same link, the time slots should not be overlapped, i.e. Non-overlapped time slots.

$$\begin{aligned} &\text{If} \quad X^{r}_{c,(s,d)} + X^{\hat{r}}_{c,(s,d)} = 2 \quad \text{Then} \\ &\quad t^{r,slot}_{e,(s,d)} - t^{\hat{r},\hat{slot}}_{e,(s,d)} \geq \Omega^{\hat{r}} || t^{\hat{r},\hat{slot}}_{e,(s,d)} - t^{r,slot}_{e,(s,d)} \geq \Omega^{r}, \\ &\qquad \forall (r,\hat{r}) \in R, \forall \left(slot, \hat{slot}\right) \in T^{r}, \forall c \in C_{TDM}, \forall e \in G^{r}, \forall (s,d) \in G \end{aligned} \tag{11}$$

- No matter in transparent or opaque optical networks, when a link in a service request is mapped into two or more substrate links, the time slot index allocated in each link should be the same, i.e., time slots consistency.

$$t^{r,slot}_{e,(s,d)} = t^{r,slot}_{e,\left(\hat{s},\hat{d}\right)}, \quad \forall r \in R, \forall slot \in T^{r}, \forall e \in G^{r}, \forall (s,d), \left(\hat{s},\hat{d}\right) \in G \tag{12}$$

- Although time slots continuity for a service request does not need to meet, the order of the time slots index should be maintained, i.e., time slots order maintaining.

$$t^{r,slot}_{e,(s,d)} \leq t^{r,slot+1}_{e,(s,d)}, \quad \forall r \in R, \forall (slot, slot+1) \in T^{r}, \forall e \in G^{r}, \forall (s,d) \in G \tag{13}$$

We have implemented the ILP model above in C++ language and make it freely available. However, CPLEX is a commercial software with the copyright of IBM. The free edition could be tried based on [3]. Therefore, although the open source code we provide includes some classes or functions from CPLEX, we only give the usage of them and the compile and link method rather than any head files or libraries of CPLEX. For more details information about CPLEX, please visit their official site in [3].

We have built an executable version named “TDMWDMOFDMonMCF.exe” for 64-bit Windows 10 platform. Followers can execute it directly in the Windows ”cmd” or ”PowerShell” terminal. The usage of the arguments for the executable file could be obtained by run the command:

TDMWDMOFDMonMCF.exe -h

or

TDMWDMOFDMonMCF.exe -help

in the terminal.

The running of the executable file will need two input files. One is the topology of the substrate network. It is a txt file with the format as follows:

Link ID	Source	Destination	S Capacity	D Capacity	Frequency/Time slots
1	1	2	0	0	320

The txt file“xDMN6S8.txt” or“xDMNSFNET.txt” could be taken as the examples.

The other one is the traffic of different service requests. It is also a txt file, with the format as follows:

Source	Destination	Type	Frequency/Time slots requested
1	3	2	21

The txt file “tdm1000.txt”, “wdm5000.txt” or “ofdm5000.txt” could be taken as the examples. Note that the examples only give 1000 or 5000 service requests, hence, the number of service requests should not exceed this limit in case of the file input mistake.

After the execution, the result files including the model “*.lp” file called “xDMonMCF⟨The number of service requests⟩.lp” and the solution “*.txt” file called “xDMonMCFILP⟨The number of service requests⟩.txt” will be saved in the same folder of the executable file. In addition, solving ILP is very time consuming, it may take a long time to wait for the big number of the service requests, even encounter the out-of-memory error.

Finally, when rebuild the open source code in VSCode, the CPLEX inlude paths or library paths in “tasks.json” file may be different because of the different options during the software installation. The file gives the default CPLEX installation paths and they should be carefully checked and replaced by the right paths when rebuild.

References

[1] X. Zhang, C. Feng, X. Gong, Q. Zhang, Y. Zong, W. Hou, and L. Guo, “On throughput optimization in software-defined multi-dimensional space division multiplexing optical networks,” *Journal of Lightwave Technology*, vol. 39, no. 9, pp. 2635–2651, 2021.

[2] Microsoft, “Visual Studio Code,” 2021. [Online]. Available: https://code.visualstudio.com/

[3] IBM, “IBM ILOG CPLEX Optimization Studio,” 2021. [Online]. Available: https://www.ibm.com/products/ilog-cplex-optimization-studio

[4] L. Gong and Z. Zhu, “Virtual optical network embedding (vone) over elastic optical networks,” *Journal of Lightwave Technology*, vol. 32, no. 3, pp. 450–460, 2014.