Part (4):

Using the shape and temporal modes that we found in part (3), we now want to find the vectors V_1 and V_2 that represent the first two dimensions of the shape space. V_i can be found by multiplying the ith singular value by the ith temporal mode, or

$$V_i = \sigma_i * \vec{v}_i$$

as the V matrix found in part (3) represents the temporal modes. We then want to plot V_1 against V_2, which gives

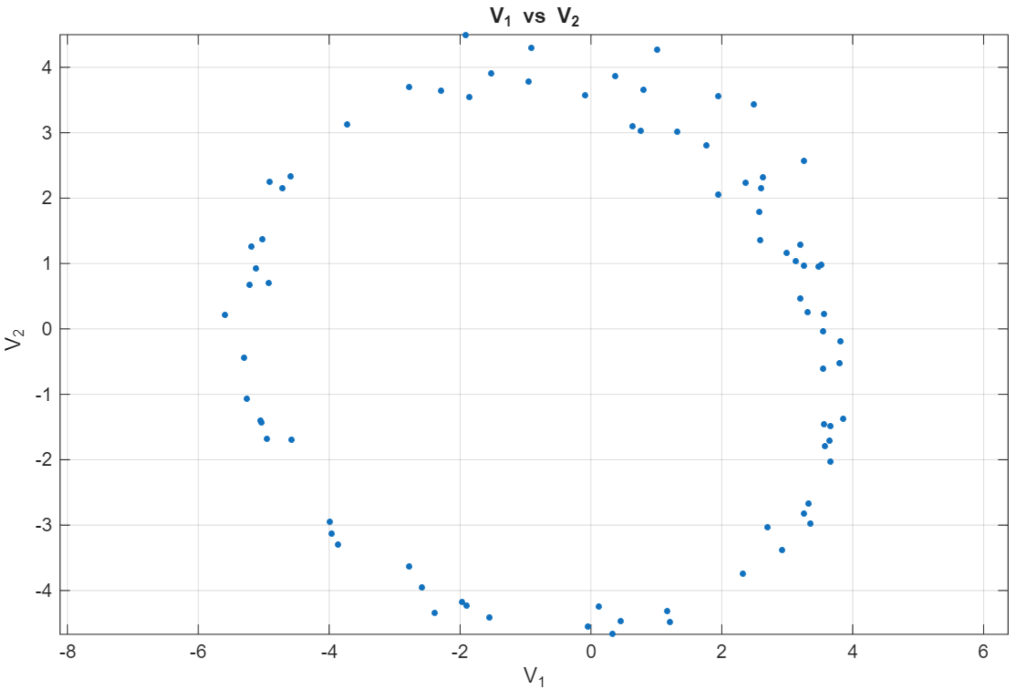

Figure 1: V_1 vs V_2

It can be observed that these points form a closed loop. We want to find the best fitting low dimensional Fourier series $A\cos(\theta) + B\sin(\theta)$ to describe the loop. To do this, we want to find the angle(θ) between the V_1 and V_2. This can be done with the MATLAB code

$$\vec{theta} = \text{unwrap(atan2(V(:,2),V(:,1)))}$$

Now, we can use least squares to fit $V_i = A_i\cos(\theta) + B_i\sin(\theta)$ to find A and B for both V_1 and V_2. To perform this, we want to solve the least squares equation:

$$A^T A\vec{x} = A^T B$$

where $B = V_i$, $A = \begin{bmatrix} \vec{cos(\theta)} & \vec{sin(\theta)} \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} A_i & B_i \end{bmatrix}^T$. Doing these calculations in MATLAB gives

$$\begin{aligned} V_1 &\approx 4.2925\cos(\theta) - 0.1475\sin(\theta) \\ V_2 &\approx -0.1124\cos(\theta) + 4.2011\sin(\theta) \end{aligned}$$

Plotting these best fit series against the data gives

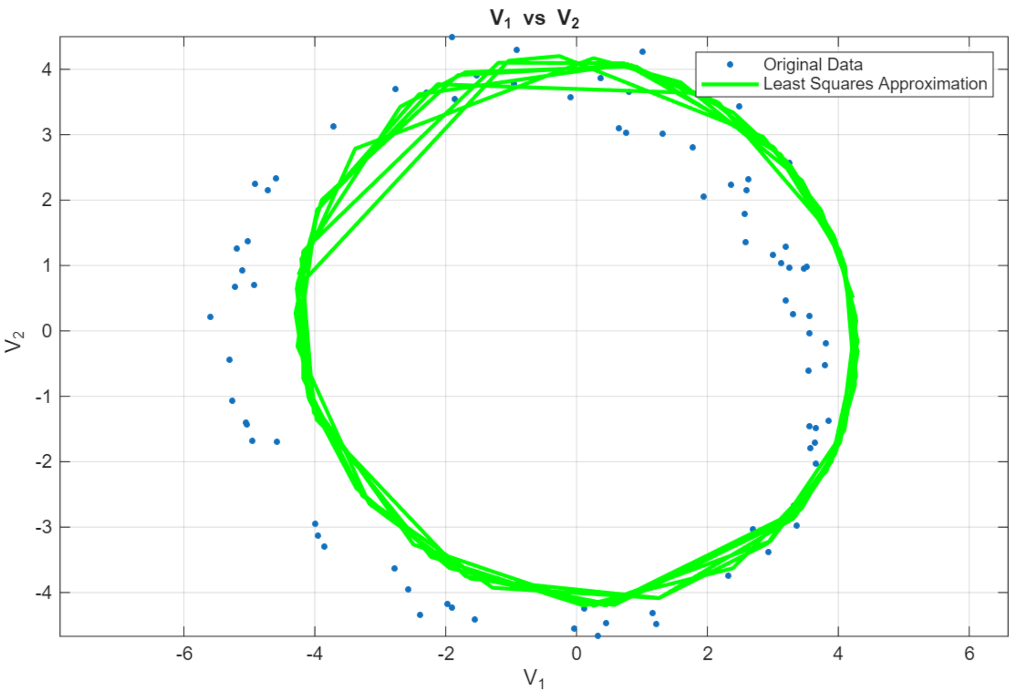

Figure 2: Best Fit Fourier Series