Problem 1

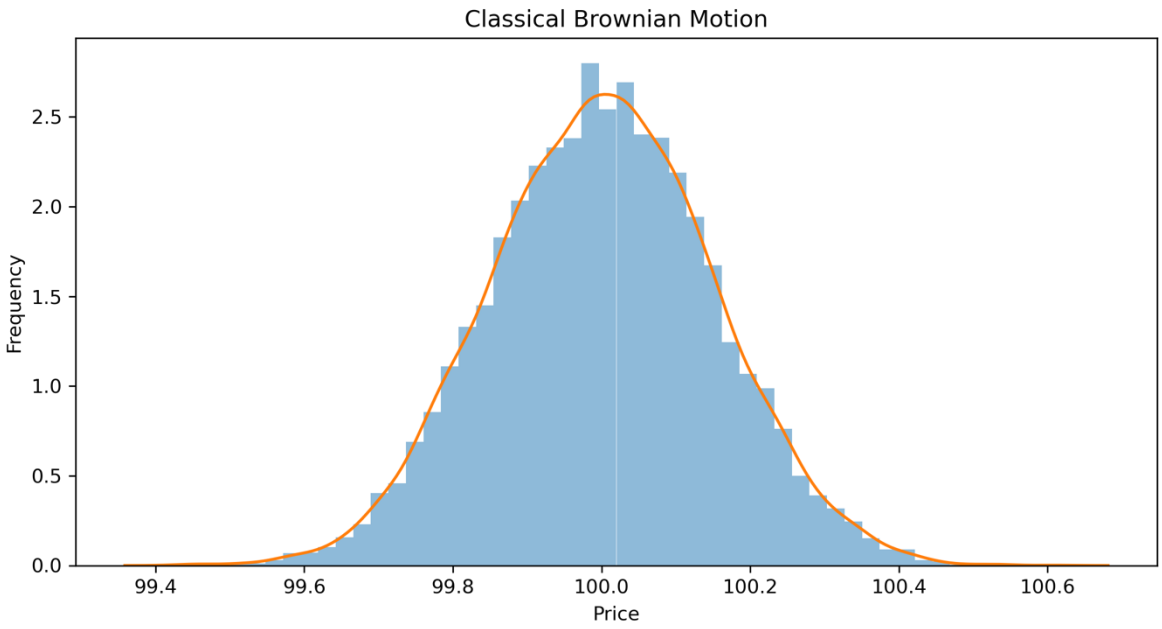
Calculate and compare the expected value and standard deviation of price at time t (P_t), given each of the 3 types of price returns, assuming $r_t \sim N(0, \sigma^2)$. Simulate each return equation using $r_t \sim N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

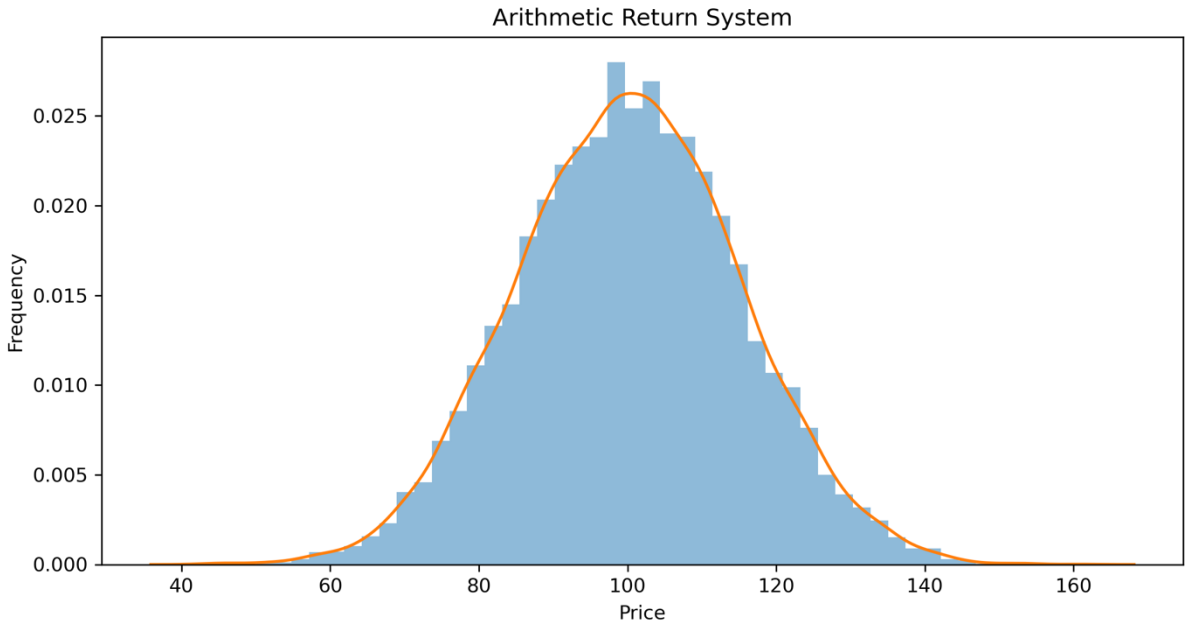
Answer:

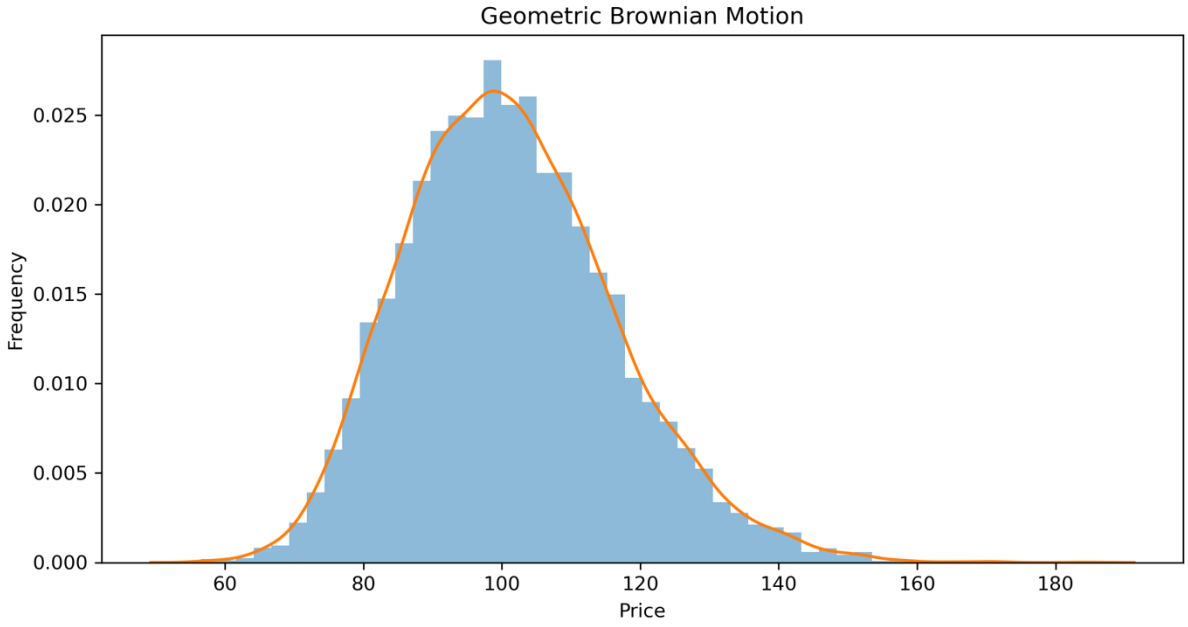
For the three methods of calculating return, simulations are carried out, and the results obtained are as follows. Assuming $\sigma = 0.15$ $P_t = 100$.

Method	Formula	Mean		Standard Deviation	
		Expectation	Result	Expectation	Result
Classical	$P_{t+1} \sim N(P_t, \sigma^2)$	P_t	100.00	σ	0.1497
Arithmetic	$P_{t+1} \sim N(P_t, P_t^2\sigma^2)$	P_t	100.15	$P_t \cdot \sigma$	14.97
Log	$P_{t+1} \sim LN(\ln(P_t), \sigma^2)$	$P_t \cdot e^{\frac{\sigma^2}{2}}$	101.27	$P_t \cdot e^{\frac{\sigma^2}{2}} \cdot \sqrt{e^{\sigma^2} - 1}$	15.25

Draw the price distributions simulated by the three methods as follows.




Arithmetic Return System
Frequency
0.025
0.020
0.015
0.010
0.005
0.000
40
60
80
100
120
140
160
Price


Geometric Brownian Motion
Frequency
0.025
0.020
0.015
0.010
0.005
0.000
60
80
100
120
140
160
180
Price

Problem 2

Implement a function similar to the “return_calculate()” in this week’s code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

Remove the mean from the series so that the mean(META)=0

Calculate VaR

1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a fitted AR(1) model.
5. Using a Historic Simulation.

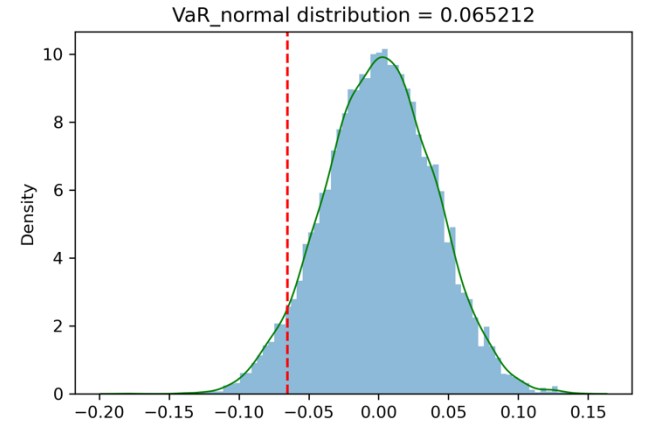
Compare the 5 values.

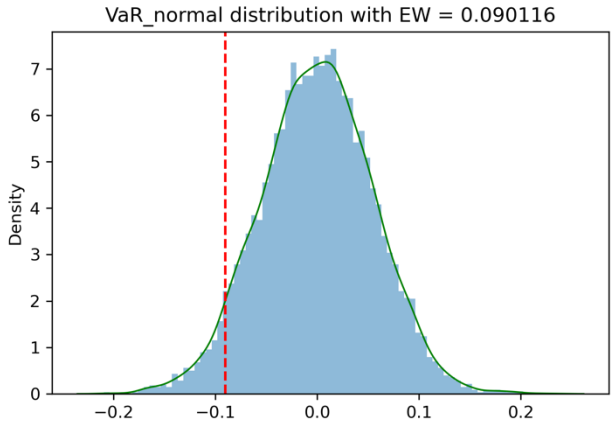
Answer:

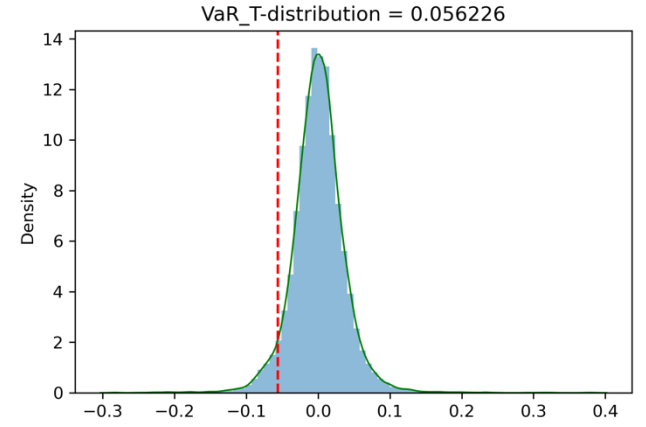
Using the stock price data of META and normalizing the return, the results of simulating and calculating VaR are as follows.

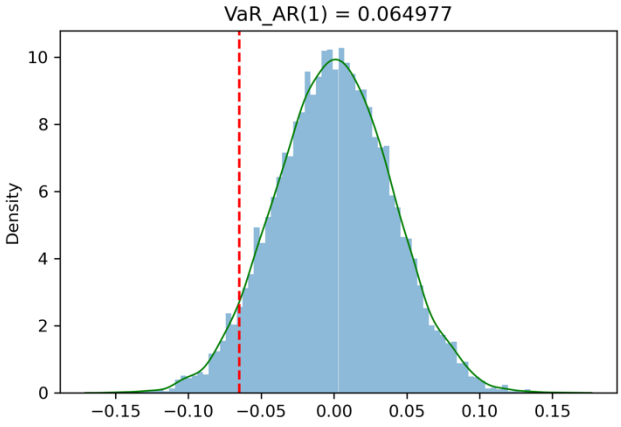
Simulation	VaR	Dollar Loss
Normal Distribution	0.065490	11.6549
Normal Distribution(EW)	0.090170	16.0460
T-Distribution	0.057023	10.1485
AR(1)	0.065466	11.6507
Historic	0.054620	9.7209

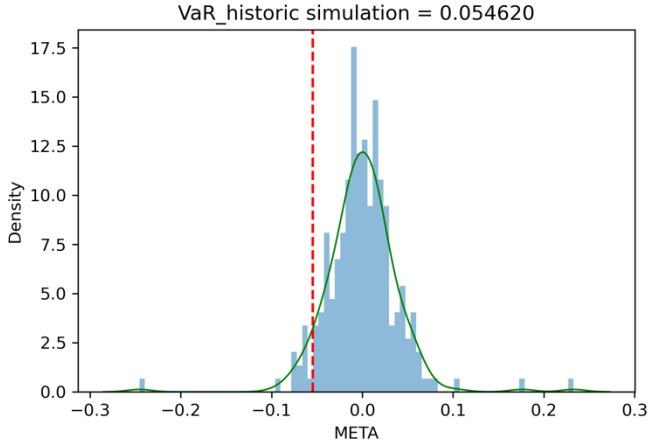
The simulation results of return are distributed as follows.











It can be seen from the above results that the simulation results of the T distribution are closer to the past historical data. Therefore, the results given by the T distribution and Historic simulation are relatively close.

The value of alpha calculated in the AR(1) simulation is about 10, which means that its simulation result is almost a normal distribution, and has little to do with the return of the previous day, which also explains the two The VaR calculated by the method is close to the reason.

The Normal distribution with an Exponentially Weighted variance method assigns a higher weight to the recent data, so the results given are more deviated from the other methods. This may be because the return of the recent META is in a relatively volatile, a higher risk situation.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.

Discuss your methods and your results.

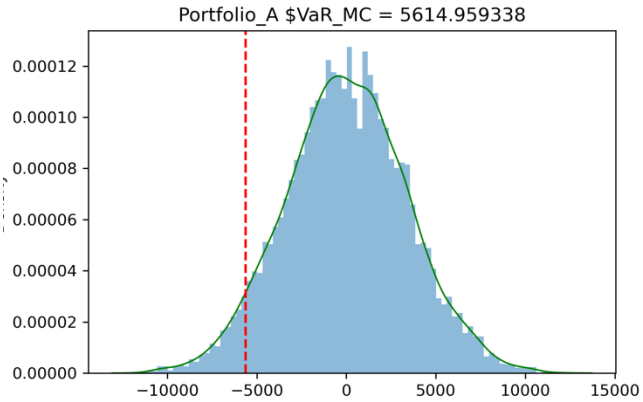
Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

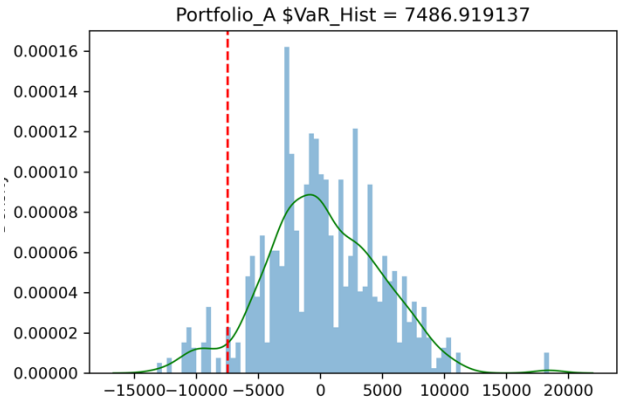
Answer:

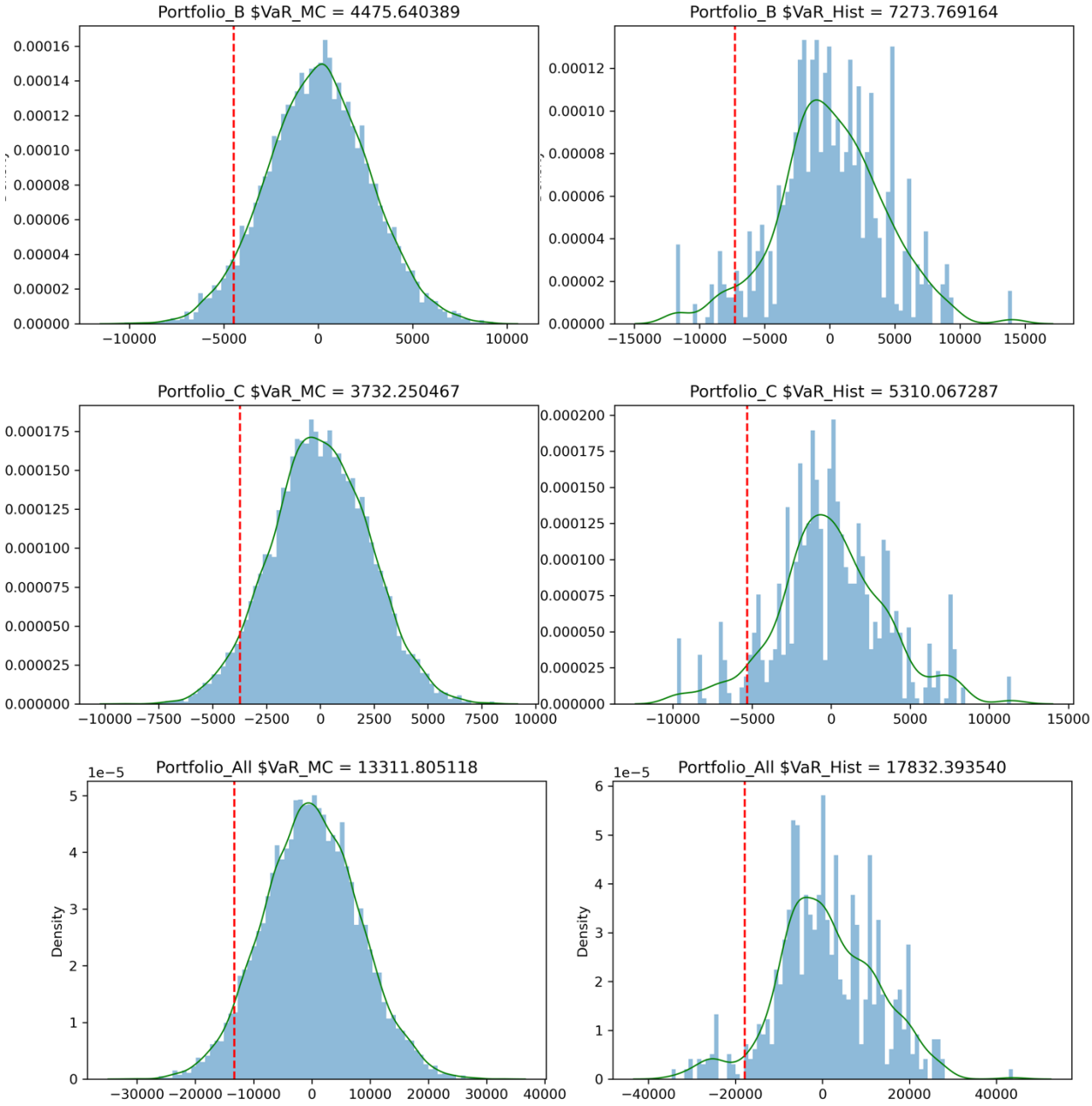
The VaR calculated by Delta Normal, Monte Carlo simulation, and Historic methods are as follows.

Portfolio	Current Value	Delta Normal	Monte Carlo	Historic
A	299950.06	5670.20	5614.96	7486.92
B	294385.59	4494.60	4475.64	7273.77
C	270042.83	3786.59	3732.25	5310.07
Total	864378.48	13577.08	13311.81	17832.39

The data obtained by Monte Carlo simulation and Historic simulation are as follows.



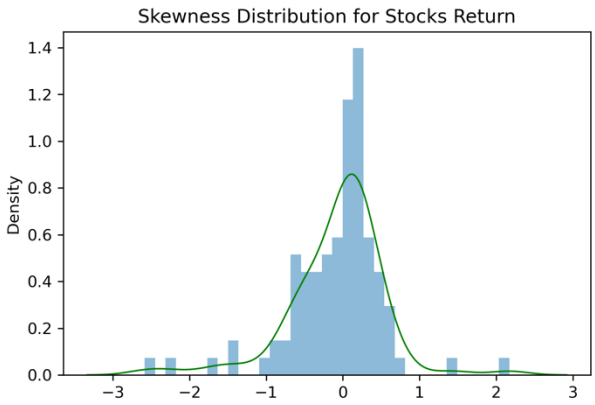


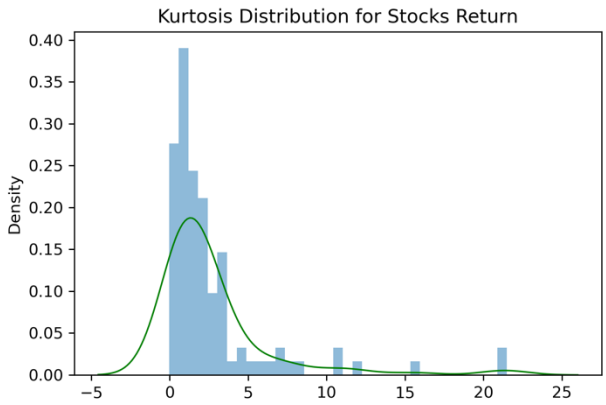


Among them, Monte Carlo simulation and Delta Normal are both based on the assumption that the returns of assets are normally distributed. Compared with Monte Carlo simulation, Delta Normal increases the assumption of linearity. Therefore, although the results given by the two methods are relatively close, the results given by Delta Normal are relatively larger, which will overestimate the risk of Portfolio.

Secondly, because the method of exponentially weighted covariance with lambda = 0.94 is used to generate the covariance matrix, it also gives higher weight to the recent data.

However, considering the assumption of normal distribution, I calculated the skewness and kurtosis of the returns of assets.





It can be seen from the calculated distribution that almost none of the stock data conforms to the normal distribution. At the same time, it can be seen from the simulated results that applying the assumption of normal distribution will underestimate the possibility of extreme situations, so the calculated VaR will be relatively small.

So I chose Historic method to calculate VaR. Compared with the previous two methods, its results are more in line with the past performance of stocks. At the same time, because the probability of extreme situations is not underestimated, the calculated result is larger than the first two. This should also be more in line with the real situation.