Problem 1

Assume you a call and a put option with the following:

- Current Stock Price $151.03
- Strike Price $165
- Current Date 03/13/2022
- Options Expiration Date 04/15/2022
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Implement the closed form greeks for GBSM. Implement a finite difference derivative calculation. Compare the values between the two methods for both a call and a put.

Implement the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:

- Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put. Calculate the Greeks of each. What is the sensitivity of the put and call to a change in the dividend amount?

Answer:

Using the Black-Scholes model to calculate the greeks of call options and put options, the results are as follows.

	Call		Put	
	Closed form	**Finite difference derivative**	**Closed form**	**Finite difference derivative**
Delta	0.534	0.471	-0.466	-0.525
Gamma	0.040	0.058	0.040	0.058
Vega	19.710	13.488	19.710	13.488
Theta	-24.899	-25.103	-18.787	-39.089
Rho	7.584	3.203	-7.277	-3.808
Carry Rho	7.966	6.509	-6.944	-7.488

It can be seen from the above results that the results calculated by the two methods are roughly the same. But it is worth noting, because the calculation of rho in the closed form is for the case of b=r, that is, the case of no dividends. Therefore, the results calculated by this method are not accurate, and the calculation of finite difference derivative is more accurate.

Using the binomial tree valuation for American options with and without discrete dividends methods to calculate the greeks and price of call options and put options, the results are as follows.

	Call	**Put**
Price with no Div	4.27	3.68
Price with Div	4.12	4.11
Delta	0.539	-0.493
Gamma	0.000	0.000
Vega	19.515	19.824
Theta	-24.795	-18.533
Rho	6.830	-7.220
Senstivity to Div	-0.094	0.513

Problem 2

Using the options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming :

- American Options
- Current Date 03/03/2023
- Current AAPL price is 151.03
- Risk Free Rate of 4.25%
- Dividend Payment of $1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.

Calculate VaR and ES using Delta-Normal.

Present all VaR and ES values a $ loss, not percentages. Compare these results to last week's results.

Answer:

Use normal distribution to simulate returns for the next ten days, and calculate the portfolio value after ten days. Subtracting the corresponding current value from the portfolios value after ten days, the results obtained by calculating Mean, VaR and ES are as follows.

	Mean			**VaR**			**ES**		
	GBSM	American	Hedged	GBSM	American	Hedged	GBSM	American	Hedged
Stock	0.71	0.71	0.00	15.59	15.60	0.00	19.65	20.22	0.00
SynLong	0.50	-0.20	-0.94	15.83	16.93	1.66	19.92	21.75	1.82
Straddle	1.68	1.64	1.56	1.38	1.65	1.64	1.39	1.68	1.67
Call	1.09	0.72	0.31	5.98	6.10	1.22	6.34	6.47	1.23
CallSpread	0.04	-0.18	-0.39	3.84	3.94	0.42	4.16	4.28	2.46
CoveredCall	-0.44	-0.18	-0.62	11.80	11.76	4.98	15.73	16.26	7.54
Put	0.58	0.92	1.25	4.43	4.55	0.42	4.63	4.70	0.44
PutSpread	0.19	1.45	0.57	2.68	2.77	0.02	2.83	2.89	0.05
ProtectedPut	1.23	1.45	0.96	7.98	7.29	0.47	8.64	7.81	0.47

From the above results, it can be seen that the risk of a portfolio that does not use the delta neutral strategy to hedge risks, that is, the options are purely American options with dividends, is roughly the same as the result calculated last week. Using the delta neutral strategy to hedge the risk of the portfolio, the risk is significantly reduced.

It is worth noting that the risk of the SynLong portfolio has not changed significantly after hedging. This is because the portfolio is made up of puts and calls with approximately the same absolute value of delta. In other words, this portfolio itself is already the result of effective risk hedging.

Problem 3

Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the following stocks.

AAPL	META	UNH	MA
MSFT	NVDA	HD	PFE
AMZN	BRK-B	PG	XOM
TSLA	JPM	V	DIS
GOOGL	JNJ	BAC	CSCO

Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.

Based on the past 10 years of factor returns, find the expected annual return of each stock. Construct an annual covariance matrix for the 20 stocks. Assume the risk free rate is 0.0425. Find the super efficient portfolio.

Answer:

Using the Fama-French four-factor model and the data of the past 10 years, the expected annual returns of these 20 stocks are calculated as follows.

Symbol	Return	Symbol	Return	Symbol	Return	Symbol	Return
AAPL	0.157	**META**	0.018	**UNH**	0.254	**MA**	0.223
MSFT	0.156	**NVDA**	0.280	**HD**	0.121	**PFE**	0.077
AMZN	-0.043	**BRK-B**	0.130	**PG**	0.082	**XOM**	0.522
TSLA	-0.033	**JPM**	0.098	**V**	0.241	**DIS**	-0.155
GOOGL	-0.017	**JNJ**	0.124	**BAC**	-0.112	**CSCO**	0.148

The weights of each stock in the found super efficient portfolio are as follows.

Symbol	Weight %	Symbol	Weight %	Symbol	Weight %	Symbol	Weight %
AAPL	0.00	**META**	0.00	**UNH**	22.57	**MA**	0.00
MSFT	0.00	**NVDA**	0.00	**HD**	0.00	**PFE**	0.00
AMZN	0.00	**BRK-B**	0.00	**PG**	0.00	**XOM**	57.44
TSLA	0.00	**JPM**	0.00	**V**	12.93	**DIS**	0.00
GOOGL	0.00	**JNJ**	7.05	**BAC**	0.00	**CSCO**	0.00

The Portfolio's Sharpe Ratio is 1.47.