Problem 1

Assume you a call and a put option with the following:

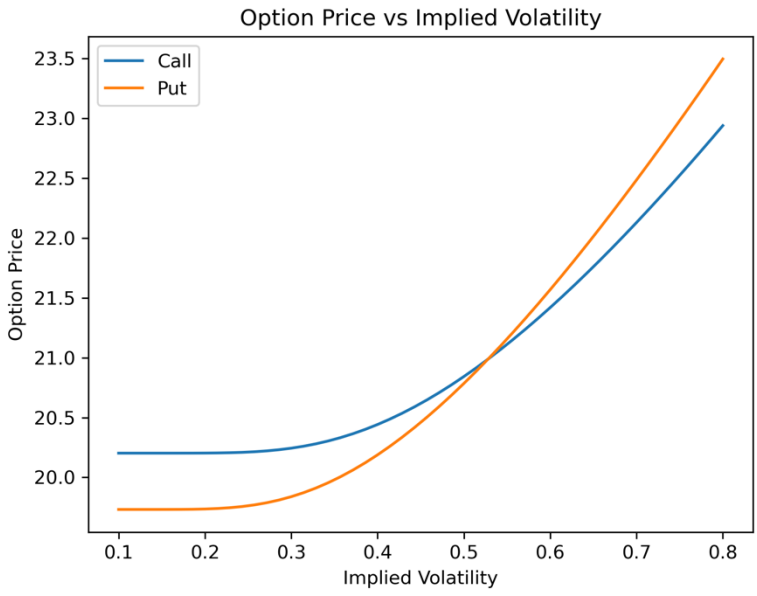
- Current Stock Price $165
- Current Date 03/03/2023
- Options Expiration Date 03/17/2023
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Calculate the time to maturity using calendar days (not trading days).

For a range of implied volatilities between 10% and 80%, plot the value of the call and the put. Discuss these graphs. How does the supply and demand affect the implied volatility?

Answer:

Using the Black-Scholes model to calculate the prices of call options and put options, the results are as follows.



As can be seen from the graph above, both puts and calls are relatively cheap at first, and gradually rise in price as implied volatility increases. Among them, the price of put option is relatively low at the beginning, and then with the increase of implied volatility, the price of put option is higher than that of call option.

This is because the put option gives the holder the right to sell an underlying asset at a predetermined price, which is more valuable when the price of the underlying asset is high. When the implied volatility is low, it indicates that the market expects the price of the underlying asset to be relatively stable. Therefore, the value of the put option is lower than the call option, as the holder of the put option has less chance of making a profit.

However, as the implied volatility increases, both the put and call option prices increase, and the put option price may eventually exceed that of the call option. This is because higher implied volatility means that the market expects the price of the underlying asset to be more volatile and unpredictable, which increases the chance of the put option holder making a profit by selling the underlying asset at a higher price. Therefore, the value of the put option increases, and it can become more expensive than the call option.

About how supply and demand affect implied volatility, it's important to understand that implied volatility is a market-based measure of the expected future volatility of the underlying asset. As demand for options increases, the implied volatility tends to increase as well, as traders are willing to pay higher prices for options that they perceive as having a higher likelihood of providing a profitable payoff. Conversely, when demand for options decreases, implied volatility tends to decrease as well. Similarly, an increase in supply of options can lead to a decrease in implied volatility, as the increased availability of options can make them less valuable to traders. Ultimately, changes in implied volatility are driven by changes in market sentiment and expectations, which can be influenced by a wide range of factors, including economic data releases, geopolitical events, and market trends.

Problem 2

Use the options found in AAPL_Options.csv

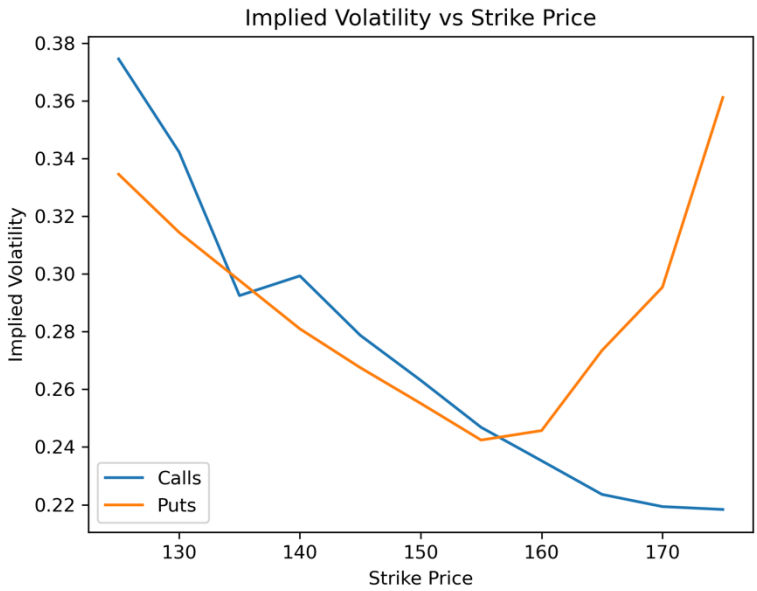
- Current AAPL price is 151.03
- Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

Calculate the implied volatility for each option.

Plot the implied volatility vs the strike price for Puts and Calls. Discuss the shape of these graphs. What market dynamics could make these graphs?

Answer:

Using the Black-Scholes model, calculate the implied volatility and draw the results as follows.



When I use the Black-Scholes model to draw the relationship between implied volatility and strike price, the implied volatility of call options continues to fluctuate and decrease with the increase of strike price. When the strike price is lower than the current underlying asset price, the implied volatility of the put option gradually decreases, and when it is higher than the underlying asset price, the implied volatility gradually increases.

The shape of these graphs can tell us a lot about market dynamics. In general, implied volatility tends to be higher for options that are farther out of the money (i.e., have a strike price far away from the current stock price) and for options that are closer to expiration. This is because these options are riskier and have a higher chance of expiring out of the money.

The shape of the implied volatility curve can also provide information about market sentiment. If the curve is relatively flat, this suggests that investors have a relatively neutral outlook on the stock price and are not expecting any major moves up or down. If the curve is skewed to the left (i.e., implied volatility is higher for out of the money puts than for out of the money calls), this suggests that investors are more bearish on the stock and are expecting it to decline. Conversely, if the curve is skewed to the right (i.e., implied volatility is higher for out of the money calls than for out of the money puts), this suggests that investors are more bullish on the stock and are expecting it to rise.

In a typical volatility smile, the implied volatility of call options tends to increase as the strike price decreases and the implied volatility of put options tends to increase as the strike price increases, creating a smile shape.

However, there may be specific circumstances or market conditions that could cause the implied volatility to behave differently. For example, if there is a particular event or news that affects the underlying asset, it could cause the market to perceive different risks at different strike prices.

One possible explanation for the patten above is that the market perceives a greater risk of a downward movement in the underlying asset price than an upward movement. This could cause the implied volatility of call options to decrease as the strike price increases, as traders may feel less need to protect against a potential upward movement in the underlying asset price. On the other hand, the implied volatility of put options may first decrease as the strike price increases, as traders may feel less need to protect against a

downward movement in the underlying asset price, but then rise again at higher strike prices as the risk of a downward movement increases.

It's worth noting that the Black-Scholes model assumes a constant volatility, but in reality, volatility can vary based on market conditions and events. As such, the behavior of implied volatility in the options market can be complex and may not always fit a standard volatility smile pattern.

Problem 3

Use the portfolios found in problem3.csv

- Current AAPL price is 151.03
- Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes.

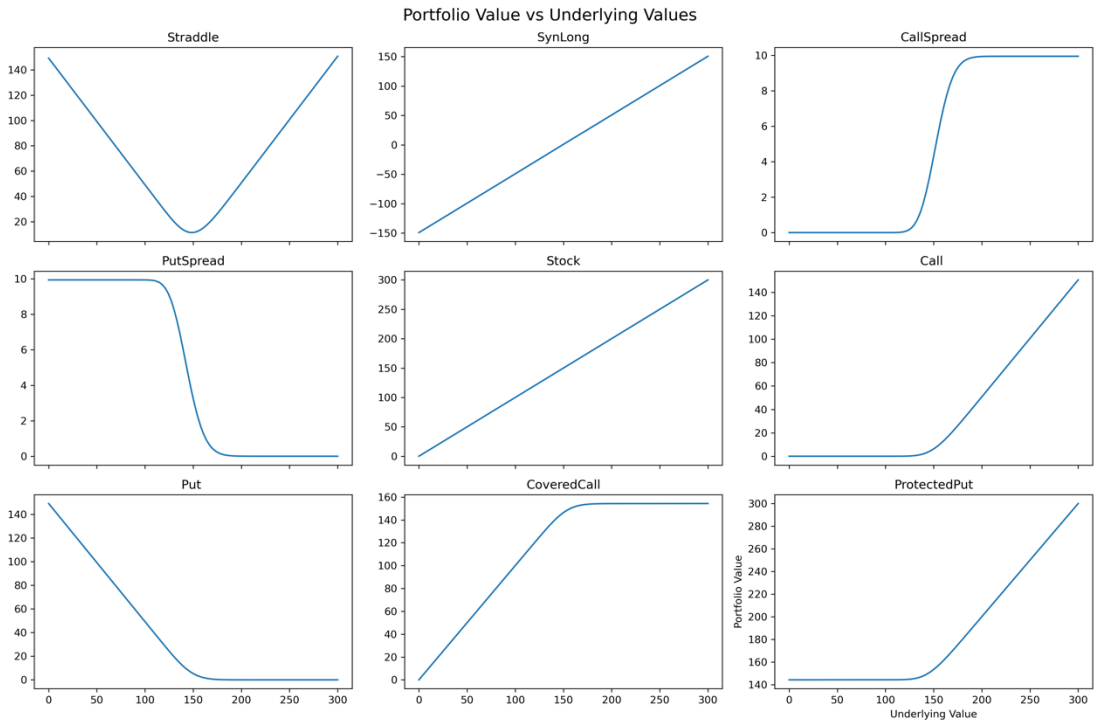
Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

Hints:

- you will need to calculate the implied volatility - might not be the same as #2.
- you need to take into account the change in dates for option valuations. You are simulating forward in time and options valuations are a function of time.
- Calculate the PL from the current portfolio value using Current Date.

Answer:

The value of various portfolios drawn using the Black-Scholes model changes with the price of the underlying asset as follows.



Use the AR(1) model to simulate returns for the next ten days, and calculate the portfolio value after ten days. Subtracting the corresponding current value from the portfolios value after ten days, the results obtained by calculating Mean, VaR and ES are as follows.

Portfolio	Mean	VaR	ES
Stock	0.711966	15.593002	19.653563
SynLong	0.651813	15.724610	19.836270
Straddle	2.673267	0.136114	0.143813
Call	1.662540	5.594155	6.067186
CallSpread	0.173343	3.536593	3.933718
CoveredCall	- 0.952991	11.994509	15.851745
Put	1.010727	4.212505	4.488632
PutSpread	0.243591	2.546705	2.742993
ProtectedPut	1.647564	7.554841	8.321570

From the plots and data above:

- For Call and Put: The basic options. Makes money as the stock price goes up or down. The risks are moderate.
- For CallSpread and PutSpread: They add a limit the the basic Call and Put. Lowering the risk by sacrificing returns.
- For CoveredCall and ProtectedPut: Basically a mix of stocks and options. The patterns are similar as Call or Put. The risks are also between stocks and options.
- For SynLong and Stock: SynLong has the same straightforward pattern as Stock, but with double returns. Both have high risks.
- Straddle: The most special portfolio here, since it makes money as long as the price moves regardless of the direction. The risk is low among all portfolios.