ECE 523 Ordinary Differential Equations Homework

Consider the following ODE and accompanying initial condition:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 998 & 1998 \\ -999 & -1999 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The analytic solution to these equations is:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 4e^{-t} - 3e^{-1000t} \\ -2e^{-t} + 3e^{-1000t} \end{bmatrix} \quad \text{for } t \geq 0$$

These are stiff equations because the time constants are 1s and 1ms. $e^{-1000t} \approx 0$ after 10 ms so the step size should be able to increased a great deal after this term get very close to zero.

Solve this set of equations over the time span $0 \leq t \leq 1$ using the following techniques:

1. Forward Euler algorithm with (a) $h = 0.01$s, (b) $h = 0.001$s, (c) $h = 0.001$ over $0 \leq t \leq 0.1$ s then $h = 0.01$ over $0.1 < t \leq 1$ s. Can the step size be increased after 0.1s?

2. Repeat (1) above using the Heun algorithm. (Note that this algorithm is a basic Runge-Kutta algorithm.)

3. Repeat (1) above using the Trapezoidal algorithm. Implement and document your own algorithm for solving the implicit equations to find x_{k+1}. For this particular problem, the differential equations are linear so x_{k+1} might be able to be found by solving a set of linear equations.

4. What did you learn from the above?

5. Use MATLAB to compute the solution in each of the following ways.

(a) ode45 with odeset options ‘Refine’ = 1, ‘RelTol’ = 1e-3, ‘AbsTol’ = 1e-6

(b) ode45 with odeset options ‘Refine’ = 1, ‘RelTol’ = 1e-6, ‘AbsTol’ = 1e-9

(c) ode15s with odeset options ‘Refine’ = 1, ‘RelTol’ = 1e-3, ‘AbsTol’ = 1e-6

(d) ode15s with odeset options ‘Refine’ = 1, ‘RelTol’ = 1e-6, ‘AbsTol’ = 1e-9

6. For each of these cases keep track of the number of steps required to complete the results. Plot (i) the step size h versus time, and (ii) the global truncation error versus time.

7. Compare the results of using the non-stiff method ode45 to the stiff method ode15s. Compare the loose tolerance results to the tight tolerance results.

8. What did you learn from the above?