Stat 333: Applied Regression Analysis
Spring 2015
Due Date: Friday, March 13 in class

Relevant text chapters: Ch. 5

Instructions: You may (and are encouraged to) discuss homework problems with other students, but the solutions that you should provide should be your own and not directly copied from another student. Show your work wherever possible, and write out the formulas you used to arrive at your solutions. If you have any questions or difficulties, you are more than welcome to come see me in my office.
Note that datasets for textbook problems can be found on the CD that came with the textbook or can be downloaded from

https://netfiles.umn.edu/users/nacht001/www/nachtsheim/index.html

1. Ch. 5, Problem 5.5 (p. 210).

2. Ch. 5, Problem 5.8 (p. 210)

3. Ch. 5, Problem 5.13 (p. 211)

4. Ch. 5, Problem 5.24 (p. 212)

5. Prove that the hat matrix,

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$$

is idempotent. In addition, prove that the matrix $\mathbf{I} - \mathbf{H}$ is idempotent.

6. For the hat matrix $\mathbf{H}$, it can be shown that

$$\text{tr}(\mathbf{H}) = \sum_{i=1}^{n} h_{ii} = p$$

where p is the number of columns in the n x p design matrix $\mathbf{X}$. Using this fact, prove in matrix notation that when $\mathbf{Y} \sim N(\boldsymbol{\mu}, \sigma^2\mathbf{I_n})$, $E(SSE) = \sigma^2(n-p)$ and hence that $E(MSE) = E\left(\frac{SSE}{n-p}\right) = \sigma^2$.