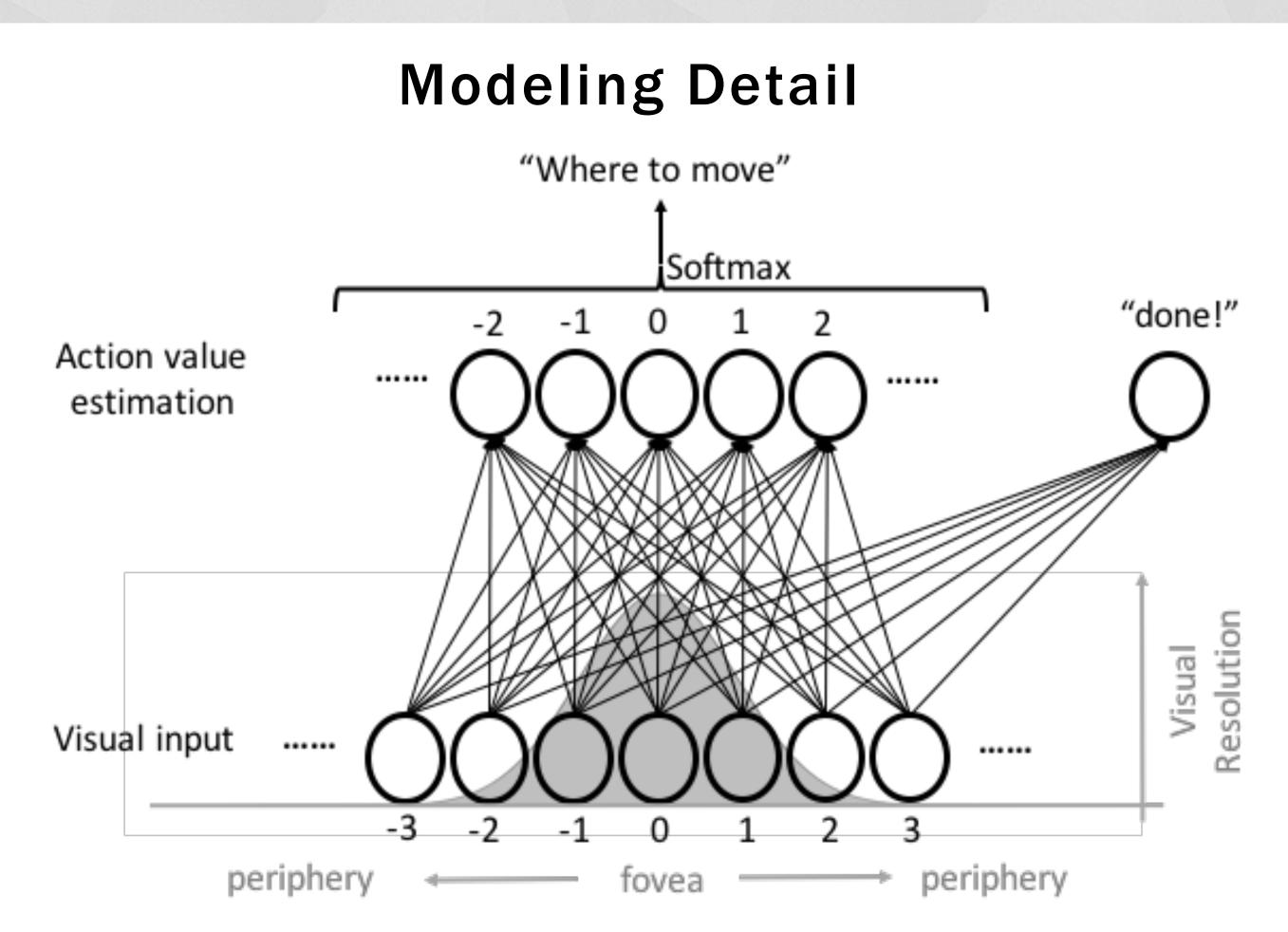
Background

Counting skill is a foundation for more sophisticated math concepts. The current project tries to capture a sub-task of counting: given an array of objects, touch every object exactly once, which is related to the one-to-one principle [1].

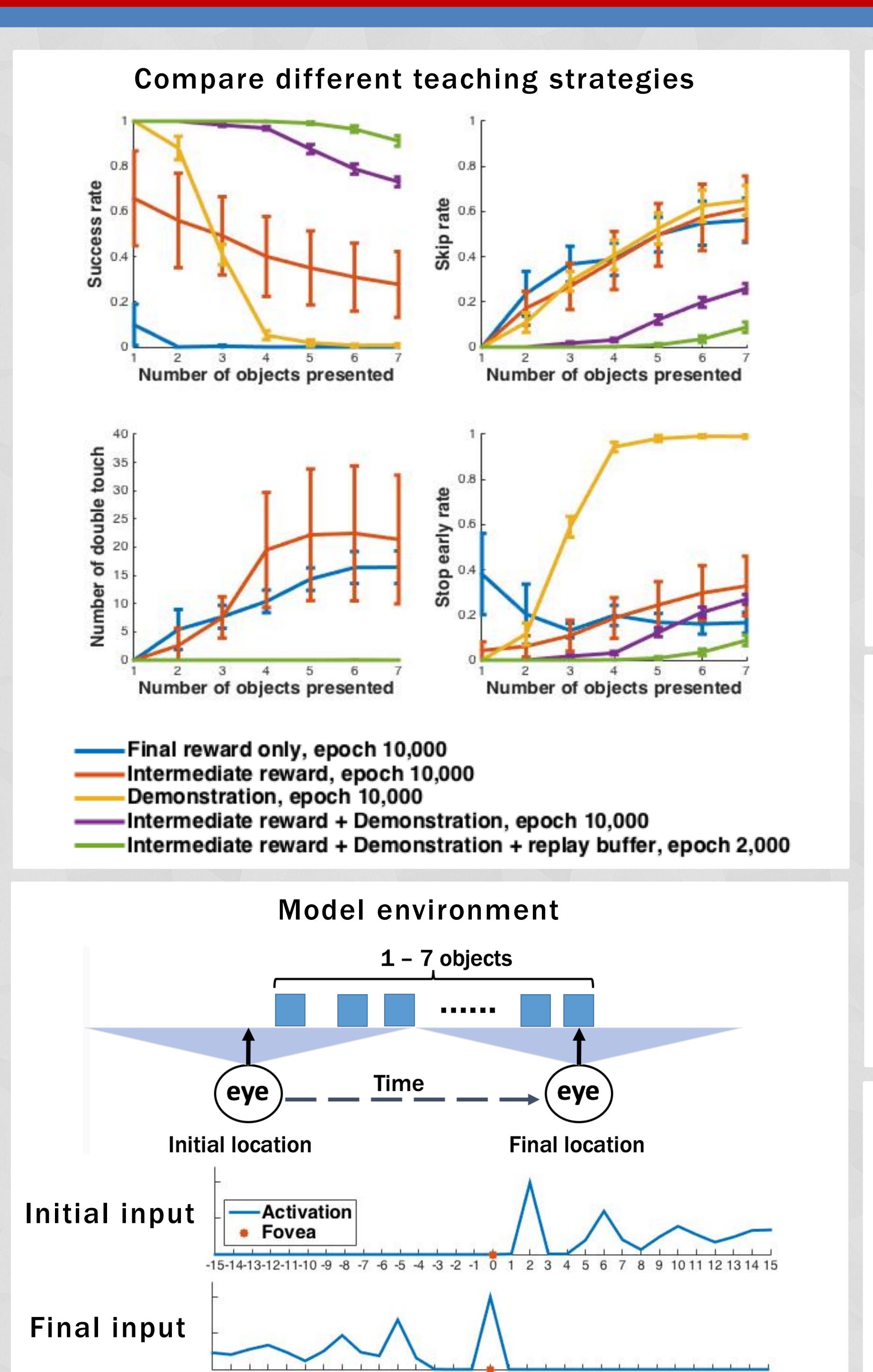
Although some have considered aspects of counting principles as innate, we examine how these skills might be learned from social support, such as enriched feedback and teacher demonstration.



$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \right]$$

In every trial, a one-dimensional object array is randomly initialized on the right-hand side of the model. The goal is to touch every object exactly once and say "done" to end the task. The model has a limited visual field. At each time step, the model can make a movement, which determines its visual input at the next time step. The input vector encodes where things are, represented by some Gaussian bumps (see model environment).

The model approximates the action values with a one-layered neural network with softmax non-linearity, using the Q-learning rule [2], listed above. The model can be augmented with an experience replay buffer [4] and a target network [5] (also see [3]).



Reward policy & Teaching strategies

Errors:

- 1: Stop early say "done" when there exists an untouched object
- 2: Double touch touching the same object twice in a trial
- 3: Skip touching the (k+1)th object before touching the kth object Note: Besides future reward discounting, these errors have no immediate consequence, and touching empty space is also fine.

Teaching strategies:

All models receive a final reward when completing the task. We had two additional feedback signals, motivated by how children learn:

- 1. Intermediate reward: Reward the model for touching the correct next object. The reward magnitude is one-half of the final reward.
- 2. Demonstration: Force the model to execute the maximally efficient action sequence. In this condition, we alternate demonstration trial and the regular self-exploration trial.

Summary

Social scaffolding made learning easier, because:

- Intermediate feedback makes the task more supervised.
- Demonstration forces exposure to the optimal solution.

These results provide insights about how social scaffoldings support learning from a computational perspective. Further research will extend these explorations to multi-layer recurrent architectures and more complex task settings.

References

- [1] Gelman, R., Gallistel, C. R. (1978). The child's understanding of number. Harvard U. press.
- [2] Sutton, R. S., & Barto, A. G. (1998). Reinforcement learning: An introduction. MIT press.
- [3] Mnih, V., et al. (2015). Human-level control through deep reinforcement learning. *Nature*, 518(7540), 529–533.
- [4] Lin, L.-J. (1993). Reinforcement learning for robots using neural networks. Technical Report, DTIC Document.
- [5] Van Hasselt, H., Guez, A., Silver, D. (2015). Deep Reinforcement Learning with Double Q-learning. arXiv.

Simulation source code:

https://github.com/QihongL/mathCognition_PDP_RL