

# My Research Report

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*This article provides a brief introduction to axiomatic quantum physics.*

## 1. Introduction

First, we recall that a physical system is described by its state—a mathematical entity—and its time evolution, which is typically governed by a differential equation. In classical physics, the state is defined by generalized coordinates  $q = (q_1, \dots, q_f)$  and their corresponding generalized momenta  $p = (p_1, \dots, p_f)$ . Then the time evolution is determined by **Hamilton function** and *Hamilton's canonical equation*. We give the definition of the state of a quantum system as following axiom.

### Axiom 1.1.

The **state** of a quantum system is represented by a unit vector (referred to as the **state vector**) in a complex Hilbert space  $\mathcal{H}$ . For any state vector  $\psi \in \mathcal{H}$  and any complex number  $\alpha$  with absolute value 1, the vectors  $\psi$  and  $\alpha\psi$  represent the same physical state. Moreover, Any observable physical quantity is represented by a self-adjoint operator on  $\mathcal{H}$ .

### Remark

Self-adjoint operators are used because their eigenvalues, which correspond to the possible measurement outcomes of a physical quantity, are always real numbers.

An observable quantity  $T$  of a state  $\psi \in \mathcal{H}$  in quantum physics is stochastic. Then we define a random variable  $T_\psi$ . Here we add an axiom for  $T_\psi$  on a probability space  $(\Omega, \mathcal{B}(\mathbb{R}), P)$ .

### Axiom 1.2.

Let  $B \in \mathcal{B}(\mathbb{R})$ , then  $P(T_\psi \in B) = \|E_{T(B)}\psi\|^2$ , where  $E_T$  is the spectral measure of  $T$ .

### Theorem 1.3. (From [Som22])

The expectation value of the random variable  $T_\psi$  is given by  $E[T_\psi] = \langle \psi, T\psi \rangle$ .

*Proof.* Let  $P_{T_\psi}$  the probability distribution of  $T_\psi$ .

$$\begin{aligned}
 E[T_\psi] &= \int_{\mathbb{R}} \lambda dP_{T_\psi}(\lambda) = \int_{\mathbb{R}} \lambda d\|E_{T(B)}\psi\|^2 = \int_{\mathbb{R}} \lambda d\langle E_{T(B)}\psi, E_{T(B)}\psi \rangle \\
 &= \int_{\mathbb{R}} \lambda d\langle \psi, E_{T(B)}^2\psi \rangle \because E_{T(B)} \text{ is self adjoint operator} \\
 &= \int_{\mathbb{R}} d\langle \psi, \lambda E_{T(B)}\psi \rangle \because E_{T(B)} \text{ is projection operator} \\
 &= \langle \psi, T\psi \rangle \because E_{T(B)} \text{ is spectral measure}
 \end{aligned} \tag{1}$$

□

## Bibliography

[Som22] Someone, "A article," *arXiv?*, 2022.