# Axiomatic quantum physics

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# 1. Introduction

This article provides a brief introduction to **axiomatic quantum physics**, which is founded on three core axioms.

First, we recall that a physical system is described by its state—a mathematical entity—and its time evolution, which is typically governed by a differential equation. In classical physics, the state is defined by generalized coordinates $q = (q_1, ..., q_f)$ and their corresponding generalized momenta $p = (p_1, ..., p_f)$. Then the time evolution is determined by **Hamilton function** and **Hamilton's canonical equation**.

We give the definition of the state of a quantum system as following axiom.

**Axiom 1.1**: The **state** of a quantum system is represented by a unit vector (referred to as the **state vector**) in a complex Hilbert space $\mathcal{H}$. For any state vector $\psi \in \mathcal{H}$ and any complex number $\alpha$ with absolute value 1, the vectors $\psi$ and $\alpha\psi$ represent the same physical state.

**Axiom 1.2**: Any observable physical quantity is represented by a self-adjoint operator on $\mathcal{H}$.

**Remark**: Self-adjoint operators are used because their eigenvalues, which correspond to the possible measurement outcomes of a physical quantity, are always real numbers.

An observable quantity $T$ of a state $\psi \in \mathcal{H}$ in quantum physics is stochastic. Then we define a random variable $T_\psi$. Here we add an axiom for $T_\psi$ on a probability space $(\Omega, \mathcal{B}(\mathbb{R}), P)$.

**Axiom 1.3**: Let $B \in \mathcal{B}(\mathbb{R})$, then $P(T_\psi \in B) = \|E_{T(B)}\psi\|^2$, where $E_T$ is the spectral measure of $T$.

Then we obtain the following useful thoerem.

**Theorem 1.1**: The expectation value of the random variable $T_\psi$ is given by $E[T_\psi] = \langle \psi, T\psi \rangle$.

*Proof*: Let $P_{T_\psi}$ the probability distribution of $T_\psi$.

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\begin{aligned}
E[T_\psi] &= \int_{\mathbb{R}} \lambda dP_{T_\psi}(\lambda) \\
&= \int \lambda d\|E_{T(B)}\psi\|^2 \\
&= \int_{\mathbb{R}} \lambda d\langle E_{T(B)}\psi, E_{T(B)}\psi \rangle \\
&= \int_{\mathbb{R}} \lambda d\langle \psi, E_{T(B)}^2\psi \rangle \because \text{adjoint operator} \\
&= \int_{\mathbb{R}} d\langle \psi, \lambda E_{T(B)}\psi \rangle \because \text{projection oprerator} \\
&= \langle \psi, T\psi \rangle \because \text{spectral measure}
\end{aligned}
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