Quantitative Risk Management Project 7

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Problem 1

- Current Stock Price $165
- Strike Price $165
- Current Date 03/13/2022
- Options Expiration Date 04/15/2022
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%
- implied volatility 20%

Implement the closed form greeks for GBSM. Implement a finite difference derivative calculation.

Compare the values between the two methods for both a call and a put.

Implement the binomial tree valuation for American options with and without discrete dividends.

Assume the stock above:

- Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put. Calculate the Greeks of each.

Answer

As to first problem, I compare the effect of the greeks closed form of Gengeralized Black Scholes Model and finite difference methods for European options. The results are following:

Table 1: Greeks: Closed Form Vs. Finite Differnece

		Detla	**Gamma**	**Vega**	**Theta**	**Rho**	**Carry Rho**
Closed Form	Call	0.534009	0.040038	19.71018	-24.89852	7.583586	7.966246
	Put	-0.465512	0.040038	19.71018	-18.787	-7.277011	-6.944416
Finite Difference	Call	0.534009	0.040038	19.7101	-24.93221	7.583554	7.966254
	Put	-0.465512	0.040038	19.7101	-18.82068	-7.277045	-6.944409

As we could see, the greeks of European options from cloesd form and finite difference is very similar. The difference between the two is very small.

Use the binomial tree methods to calculate the value and greeks of call and put American options(consider whether there is continuously compounding coupon or not):

Table 2: Greeks: American Option with Continuously Compounding Coupon

	Without Discrete Dividend		With Discrete Dividend	
	Call	Put	Call	Put
Valuation	4.227506	3.714324	4.074976	4.147062
Detla	0.533966	-0.473668	0.530507	-0.495856
Gamma	0.280568	0.246837	0.031063	0.002387
Vega	19.685473	19.655211	19.576769	19.815527
Theta	-24.904878	-19.299822	-24.464277	-18.948359
Rho	7.583365	-5.980132	6.797344	-7.248928
Carry Rho	7.965619	-5.738138	7.123061	-6.913934
Sensitivity to Dividend Amount	NaN	NaN	-0.115203	0.515291

We vary the changing amount of all variable to better make our sensitivity reasonable. Finally, we set the amount the price change is \$0.2 and all the other changing amount is 0.01.

Table 3: Greeks: American Option without Continuously Compounding Coupon

	Without Discrete Dividend		With Discrete Dividend	
	Call	Put	Call	Put
	Call	Put	Call	Put
Valuation	4.269859	3.684138	4.112836	4.110535
Detla	0.537384	-0.471987	0.532904	-0.493280
Gamma	0.280501	0.242952	0.021277	0.003252
Vega	19.680769	19.642167	19.574566	19.824035
Theta	-25.388799	-18.988711	-24.897100	-18.573133
Rho	7.630524	-5.896065	6.834995	-7.202673
Carry Rho	8.016606	-5.659202	7.163720	-6.870814
Sensitivity to Dividend Amount	NaN	NaN	-0.115497	0.512470

Problem 2

Using the options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming :

- American Options
- Current Date 03/03/2023
- Current AAPL price is 151.03
- Risk Free Rate of 4.25%
- Dividend Payment of $1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns

- assume 0 mean return.

Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.

Calculate VaR and ES using Delta-Normal.

Present all VaR and ES values a $ loss, not percentages.

Compare these results to last week's results.

Answer

Use Normal distribution to simulate the returns 10 days ahead, apply those returns to the current AAPL price and we get the following:

Table 4: Normal Simulated Portfolio Value 10 Days Later For American Option

	Mean(Portfolio Value)	Mean(Change)	VaR	ES
Straddle	13.573306	2.034698	1.175029	1.221071
SynLong	1.756892	-0.163474	17.931696	22.04644
CallSpread	4.503748	0.034049	3.760683	4.071643
PutSpread	3.426409	0.508251	2.670079	2.794192
Stock	151.392764	0.362764	16.740359	20.588025
Call	7.665099	0.935612	5.97523	6.309565
Put	5.908207	1.099086	4.495994	4.656884
CoveredCall	146.359195	-0.595879	12.87181	16.616972
ProtectedPut	155.276754	1.22444	7.485625	7.832187

Table 5: AR1 Simulated Portfolio Value 10 Days Later For European Option

Portfolio	**Mean(Portfolio Value)**	**Mean(Variation)**	**VaR**	**ES**
Straddle	13.236627	1.586627	1.378317	1.387136
SynLong	2.050838	0.100838	16.24976	19.975054
CallSpread	4.519231	-0.070769	3.89146	4.181353
PutSpread	3.292349	0.282349	2.6562	2.811006
Stock	151.334622	0.304622	16.007244	19.710478
Call	7.643733	0.843733	6.039037	6.362422
Put	5.592895	0.742895	4.401389	4.601255
CoveredCall	146.323396	-0.656604	12.191148	15.780425
ProtectedPut	154.976053	0.936053	8.074428	8.681969

Here, we could find that almost all the portfolio of American Options values more than European Options. The Sensitivity between the two is very small.

Problem 3

Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the following stocks.

Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.

Based on the past 10 years of factor returns, find the expected annual return of each stock.

Construct an annual covariance matrix for the 10 stocks.

AAPL	FB	UNH	MA
MSFT	NVDA	HD	PFE
AMZN	BRK-B	PG	XOM
TSLA	JPM	V	DIS
GOOGL	JNJ	BAC	CSCO

Assume the risk free rate is 0.0425. Find the super efficient portfolio.

Answer

Use Fama French 3 factor and Carhart Carhart Momentum to fit the 4 factor model and based on the past 10 years of factor returns, we get the expected annual return of each stock like following:

AAPL	META	UNH	MA
23.06%	7.65%	20.39%	19.70%
MSFT	NVDA	HD	PFE
22.39%	22.36%	16.92%	15.63%
AMZN	BRK-B	PG	XOM
33.89%	14.39%	17.60%	8.61%
TSLA	JPM	V	DIS
11.82%	13.41%	18.70%	11.94%
GOOGL	JNJ	BAC	CSCO
29.18%	11.21%	13.59%	21.78%

Then we optimize the portfolio sharpe ratio to find the super efficient portfolio.

The best sharpe ratio is 0.832215456951802

Here is the weight of each asset:

	AAPL	**META**	**UNH**	**MA**
Weight	0	0	37%	0
	MSFT	**NVDA**	**HD**	**PFE**
Weight	0	0	0	0
	AMZN	**BRK-B**	**PG**	**XOM**
Weight	10%	0	28%	0
	TSLA	**JPM**	**V**	**DIS**
Weight	0	0	0	0
	GOOGL	**JNJ**	**BAC**	**CSCO**
Weight	17%	0	0	8%