Quantitative Risk Management Project 5

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Problem 1

Use the data in problem1.csv. Fit a Normal Distribution and a Generalized T distribution to this data. Calculate the VaR and ES for both fitted distributions.

Overlay the graphs the distribution PDFs, VaR, and ES values. What do you notice? Explain the differences.

Answer

1. Finding:

We use data to construct MLE fitted Normal and T distribution then we calculate the corresponding VaR and Expected Shortfall.

1. For both fitted normal and T, their Expected Shortfall is larger than VaR, which can also be proved by math. See the following.
2. We could find that the VaR of the fitted normal distribution is larger than that of the fitted T distribution but the Expected Shortfall of the fitted T distribution is larger than that of fitted normal distribution.

2. Reasoning:

The reason why this w will happen is that the fitted T distribution has a heavier tail than the fitted normal. Expected Shortfall is the average of the VaRs which are larger than the VaR at tail probability alpha when the underlying distribution is continuous.

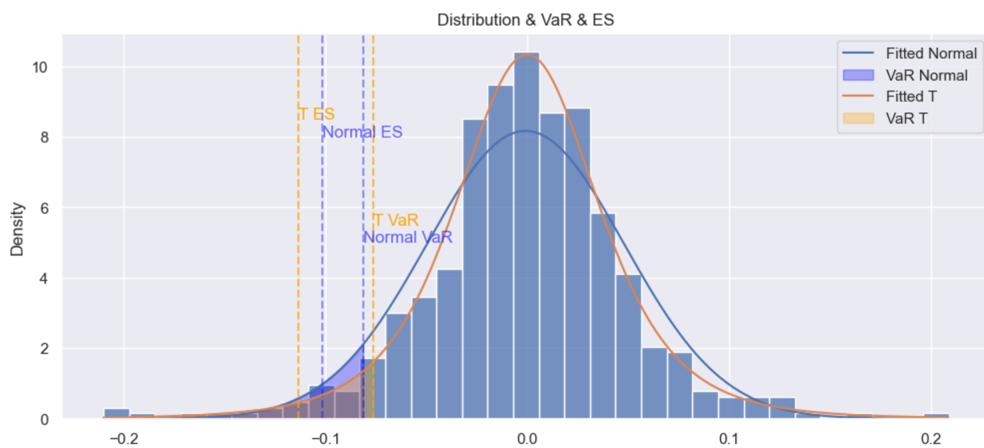
VaR only measures the maximum loss given a certain confidence interval, ignoring the possibility of extreme losses outside of that threshold. That means VaR ignores the tail risk.

Expected Shortfall could tackle such a problem. It considers the whole tail risk and averages them to get an expected potential loss. Even though at some times, the Var is low but the Expected Shortfall may be very high.

Therefore, we could not only use VaR to help us discover the risk level of our portfolio. We should combine VaR and ES to have a better understanding.

Table 1: Time period = 1

Risk Metrics	Distribution	Value
VaR	Normal	0.08125
	T	0.07648
Expected Shortfall	Normal	0.1017
	T	0.1132



3. Mathematical Reasoning:

The finding that Expected Shortfall is bigger than VaR can be proved:

If we assume the distribution is continuous, then we have

$$
\begin{aligned}
ES_\alpha(X) &= -\frac{1}{\alpha}\int_0^\alpha VaR_p(X)dp \\
&= E(-X|X \leq -VaR_\alpha(X)) \\
&= E(-X - VaR_\alpha(X) + VaR_\alpha(X)|X \leq -VaR_\alpha(X)) \\
&= E(-X - VaR_\alpha(X)|X \leq -VaR_\alpha(X)) + E(VaR_\alpha(X)|X \leq -VaR_\alpha(X)) \\
&= VaR_\alpha(X) - E((X + VaR_\alpha(X))^-|X \leq -VaR_\alpha(X)) \\
&\geq VaR_\alpha(X)
\end{aligned}
$$

Problem 2

In your main repository, create a Library for risk management. Create modules, classes, packages, etc as you see fit. Include all the functionality we have discussed so far in class.

Make sure it includes

1. Covariance estimation techniques.
2. Non PSD fixes for correlation matrices
3. Simulation Methods
4. VaR calculation methods (all discussed)
5. ES calculation

Create a test suite and show that each function performs as expected.

Answer

All the test is passed. Please refer to my Github.

```
test_return_calculate (__main__.TestCalculateReturn) ... ok
test_EWMA_corr (__main__.TestCovarianceEstimation) ... ok
test_EWMA_cov (__main__.TestCovarianceEstimation) ... ok
test_Norm (__main__.TestModelFitter) ... ok
test_T (__main__.TestModelFitter) ... ok
test_Higham_psd (__main__.TestNonPsdFixes) ... ok
test_chol_psd_PD (__main__.TestNonPsdFixes) ... ok
test_chol_psd_PSD (__main__.TestNonPsdFixes) ... ok
test_near_psd (__main__.TestNonPsdFixes) ... ok
test_ES (__main__.TestRiskMetrics) ... ok
test_VaR (__main__.TestRiskMetrics) ... ok
test_DirectSimulation (__main__.TestSimulationMethods) ... ok
test_PCA (__main__.TestSimulationMethods) ... ok
test_PCASimulation (__main__.TestSimulationMethods) ... ok

----------------------------------------------------------------------
Ran 14 tests in 0.362s
```

Problem 3

Use your repository from #2.

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios.

Fit a Generalized T model to each stock and calculate the VaR and ES of each portfolio as well as your total VaR and ES.

Compare the results from this to your VaR form Problem 3 from Week 4.

Answer

Assume the return of all stocks follow generalized T distribution, we use the return data to construct the Gaussian Copula and use Gaussian Copula to simulate the T distributed return.

Gaussian Copula:

$$C_R^{Gauss}(u) = \Phi_R(\Phi^{-1}(u_1), \cdots, \Phi^{-1}(u_n))$$

where Φ^{-1} is the inverse cumulative distribution function of a standard normal and Φ_R is the joint cumulative distribution function of a multivariate normal distribution with mean vector zero and covariance matrix equal to the correlation matrix R.

Finding:

We find the Generalized T model will result in larger VaR and ES than other methods of Normal, like Normal-Delta and Normal Monte Carlo Simulation.

Table 2: VaR of Portfolio

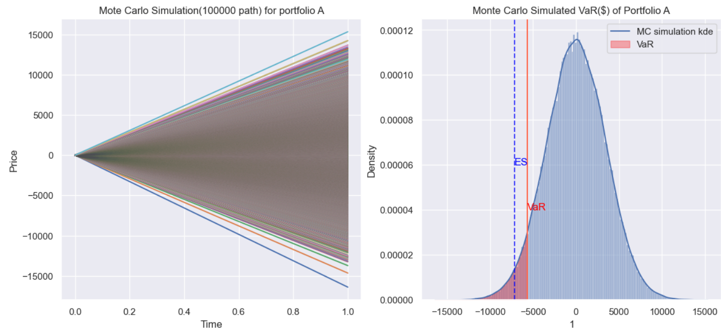
Risk Metrics	Method	portfolio A	portfolio B	portfolio C	Total portfolio
VaR	Normal-Delta	5678.98	4492.44	3775.87	13570.22
	Normal MC	5670.20	4494.59	3786.58	13577.07
	Historical Simulation	9070.10	7351.16	5802.652	21275.01
	T + Gaussian Copula	7866.03	6908.44	5742.72	20288.10
ES	Normal MC	7155.39	5635.36	4759.14	7031.40
	Historical Simulation	10640.56	9183.29	7568.59	26579.18
	T + Gaussian Copula	10573.47	9221.20	7755.23	26418.80

As we can see from the chart that for VaR, under Normal assumption, the portfolio has the samllest Value at Risk. Under the T assumption, the portfolio has almost same VaR as Historical Simulation except fot portfolio C.

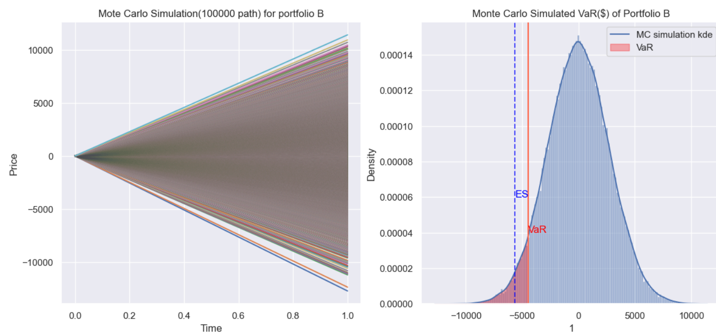
Therefore, we can tell that Generalized T model could better catch the Value at Risk even though there is some difference between Historical Simulation and Generalized T model. For the Normal Model, it underestimate the potential loss.

As to Expected Shortfall, the order of three Model is very clear. THe ES of Generalized T model is the largest and the ES of Normal is the samllest. The Historical ES is in the middel of those two.

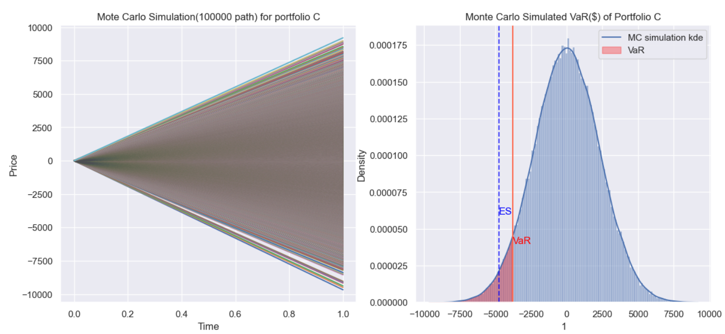
We could also see there are more outliers in the Generalized T model. It's not surprise this will happen. The T Model capture more information of the tail. It considers more tail risk than normal.



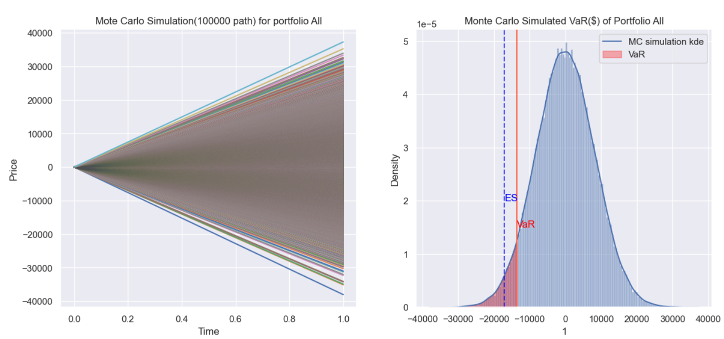
(a) Portfolio A



(b) Portfolio B

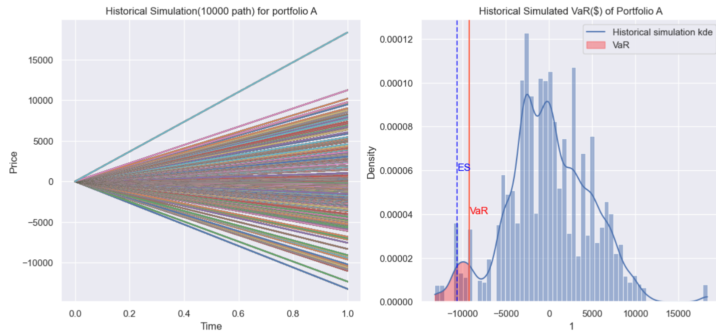


(c) Portfolio C

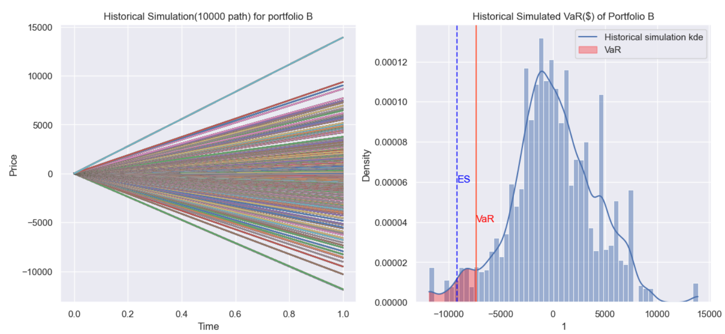


(d) Portfolio ALL

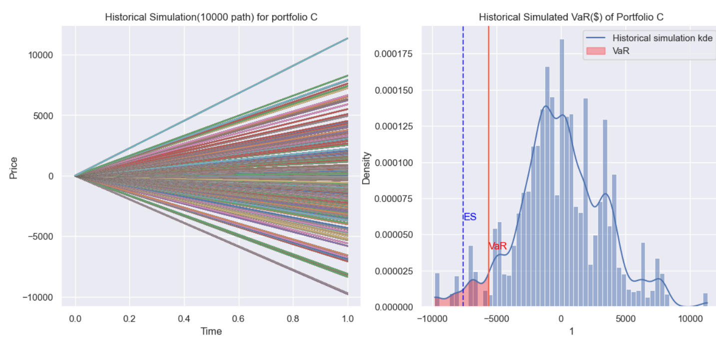
Figure 1: Normal Model Carlo Simulation



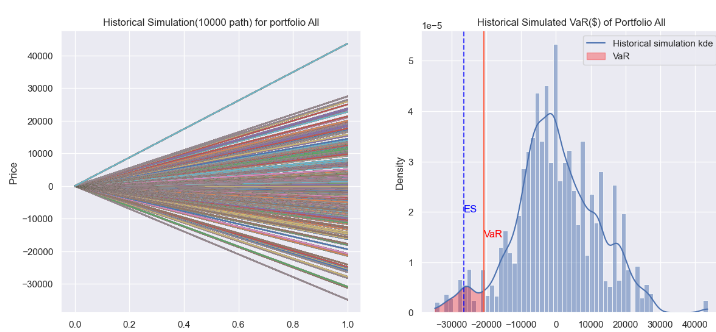
(a) Portfolio A



(b) Portfolio B

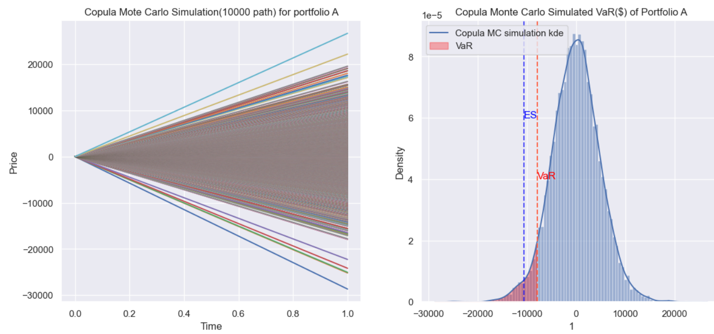


(c) Portfolio C

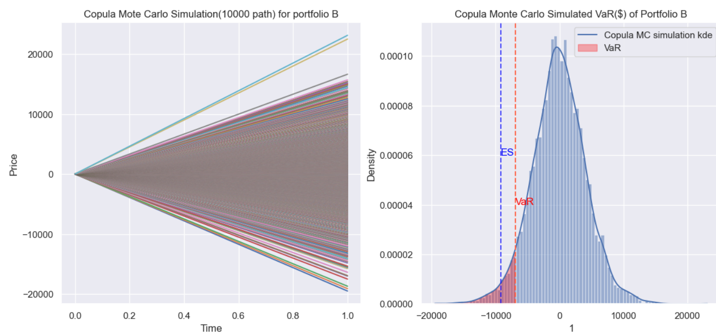


(d) Portfolio ALL

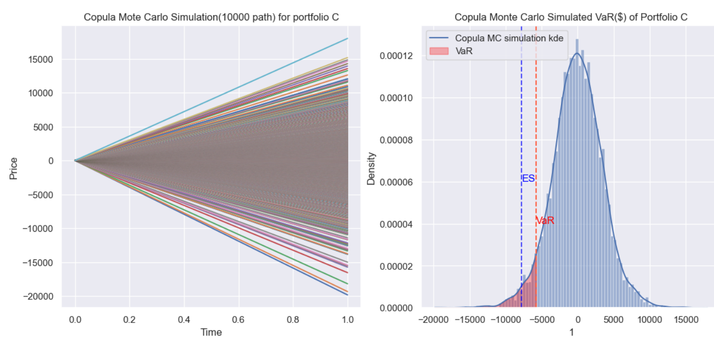
Figure 2: Historical Simulation



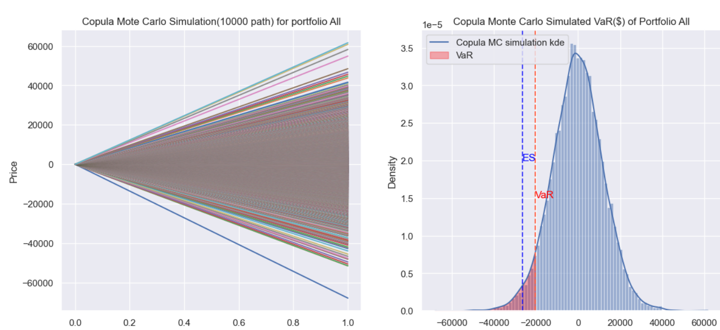
(a) Portfolio A



(b) Portfolio B



(c) Portfolio C



(d) Portfolio ALL

Figure 3: Generalized T model