STRATFUSION: SYNTHESIZING DIVERSE TRADING STRATEGIES IN EVOLVING STOCK MARKETS

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ABSTRACT

The abstract of the "StratFusion: Synthesizing Diverse Trading Strategies in Evolving Stock Markets" report introduces "StratFusion", a framework integrating multiple innovative trading strategies to enhance performance and mitigate risks in the volatile stock market. It covers four main strategies: RegimeEvoVAE, Pairs Trading with Copula Mispricing Index, Advantage Actor Critic (A2C) with LSTM Agent, and GPT-based Sentiment Strategy. Each strategy is designed for different market conditions and aspects. "StratFusion" aims to revolutionize traditional trading methodologies with its adaptive, resilient multi-strategy approach. The report further details the strategies, their integration, and empirical results from testing.

1 INTRODUCTION

The stock market, a complex and ever-evolving landscape, presents a plethora of challenges and opportunities for investors and traders alike. In this dynamic environment, the efficacy of trading strategies is continually tested against the backdrop of market volatility and shifting economic conditions. Our project, "StratFusion: Synthesizing Diverse Trading Strategies in Evolving Stock Markets," seeks to address the critical need for adaptive and robust trading methodologies that can thrive amidst these changing market regimes.

The conventional approach to stock market trading often falls short in its adaptability and scope, primarily focusing on singular or static strategies. This limitation becomes particularly pronounced in the face of market fluctuations and unprecedented economic events. To bridge this gap, "StratFusion" introduces a comprehensive framework that integrates multiple, innovative trading strategies, each uniquely designed to capitalize on different market aspects and conditions. This multifaceted approach aims not only to enhance the performance of trading activities but also to mitigate the risks associated with market volatility.

Our project encompasses four cutting-edge strategies:

- **RegimeEvoVAE:** This strategy leverages Market Regime-Specific Evolving Variational Autoencoders to gain cross-sectional returns. It adapts to various market conditions by evolving its parameters, ensuring optimal performance across different market regimes.
- **Pairs Trading with Copula Mispricing Index:** By identifying and exploiting mispricings in pairs trading, this strategy utilizes the Copula Mispricing Index to uncover profitable opportunities in seemingly correlated assets.
- **Advantage Actor Critic (A2C) with LSTM Agent:** Combining the robustness of A2C algorithms with the predictive capabilities of LSTM neural networks, this strategy offers a sophisticated approach to decision-making in stock trading.

- **GPT-based Sentiment Strategy:** Utilizing advanced natural language processing, this strategy analyzes market sentiment through various data sources, harnessing the predictive power of public sentiment in financial markets.

Crucially, 'StratFusion' excels in the art of strategic integration, expertly weaving together these diverse methodologies to create a harmonious and adaptive trading system. This synergetic combination leverages the distinct advantages of each strategy, ensuring a robust response to the multifaceted nature of today's financial markets. In developing 'StratFusion,' our approach draws inspiration from seminal works in the field of algorithmic trading. Notably, the research on 'Multiple strategies for trading short-term stock index futures based on visual trend bands' Chou & Hung (2021) offers a notable perspective on enhancing day trading strategies by emphasizing the use of visual trend bands, combined with machine learning algorithms like sequential minimal optimization, to improve predictions in short-term stock index futures. Similarly, 'A Multi Strategy Approach to Trading Foreign Exchange Futures' Srivastava et al. (2019) offers valuable insights into the synergistic potential of combining various indicators, such as technical and fx carry indicators, to enhance trading performance in foreign exchange markets. Additionally, 'A Multi-Strategic Approach to Automated Trading' Mahajan et al. (2021) highlights the effectiveness of systematic, multi-strategy approaches in the context of Forex trading, underlining the importance of multiple optimization procedures. These studies collectively underscore the potential of multi-strategy trading systems like 'StratFusion' in navigating the complexities of today's financial markets and form a critical part of the theoretical backdrop against which our project is positioned.

"StratFusion" aims to push the boundaries of traditional trading methodologies, offering a novel, multi-strategy framework that is both adaptive and resilient. The project's goal is to set a new benchmark in trading strategy synthesis, providing a template for future innovations in the realm of stock market trading. The ensuing sections of this report will delve into the technicalities of each strategy, the integration process, and the empirical results from our extensive back-testing and real-world application trials.

2 REGIMEEVOVAE: GAINING CROSS-SECTIONAL RETURNS WITH MARKET REGIME-SPECIFIC EVOLVING VARIATIONAL AUTOENCODERS

2.1 BACKGROUND

Factor Model The Factor Model Fama & French (1992) Fama & French (2020) is a fundamental concept in finance, particularly in the analysis of portfolio returns and risk management. It decomposes the returns on a security into several different factors and an idiosyncratic component. The general form of a Factor Model can be expressed as:

$$R_i = \alpha_i + \sum_{j=1}^{n} \beta_{ij} F_j + \varepsilon_i \tag{1}$$

where:

- R_i is the return of asset i.
- α_i represents asset-specific return not explained by the factors.
- β_{ij} is the sensitivity of the ith asset to the jth factor.
- F_j represents the jth factor affecting the asset returns.
- ε_i is the idiosyncratic return of asset i.

The Factor Model is widely used in various forms, such as the Capital Asset Pricing Model (CAPM) Treynor (1961)Sharpe (1964)Lintner (1975), Arbitrage Pricing Theory (APT) Ross (2013), and multi-factor models in portfolio management and risk assessment.

Market regime Market regimes refer to the different phases or environments that financial markets experience over time. These regimes are characterized by distinct patterns in asset price movements, volatility, and market sentiment. Commonly identified regimes include bullish trends, bearish trends, high volatility periods, and low volatility periods. Understanding market regimes is crucial for investors and financial analysts, as it aids in making informed decisions about asset allocation, risk management, and investment strategy formulation. The identification and analysis of market regimes involve the study of economic indicators, market data, and investor behavior to discern patterns and predict potential shifts in market conditions.

Examples of common Market Regimes include:

- **Bull Market**: A period marked by rising asset prices, investor optimism, and generally favorable economic conditions.
- **Bear Market**: A period characterized by falling asset prices, investor pessimism, and often economic downturns.

Identifying and adapting to different Market Regimes can enhance investment performance and mitigate risk. Strategies effective in a Bull Market might not work well in a Bear Market, and vice versa. Therefore, dynamic and flexible approaches are often required to navigate through varying market conditions.

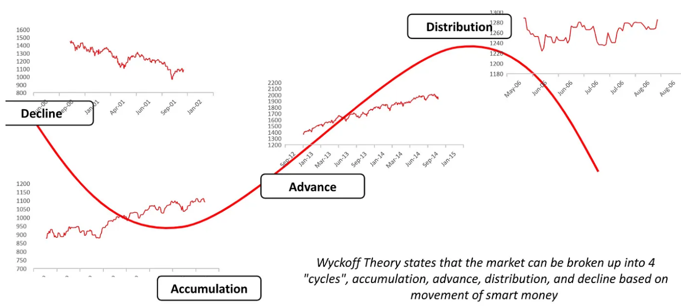

Figure 1: Regime Switch

Regime-Switching Factor Models Regime-Switching Factor Models Wang et al. (2020) are a sophisticated class of financial models that recognize the existence of different regimes in financial markets. These models are an extension of the standard factor models and incorporate the possibility that the relationships between assets and underlying factors can change significantly in different market conditions or regimes.

The core idea behind Regime-Switching Factor Models is that financial markets exhibit different 'states' or 'regimes', each characterized by distinct statistical properties. For example, a market could have a 'bull' regime where prices are generally rising and a 'bear' regime where prices are falling. The model allows the relationships between asset returns and factors to change depending on the prevailing regime. The mathematical representation of a basic Regime-Switching Factor Model is:

$$R_{it} = \alpha_{it}^{(r)} + \sum_{j=1}^{n} \beta_{ijt}^{(r)} F_{jt}^{(r)} + \epsilon_{it}^{(r)} \tag{2}$$

where:

- R_{it} is the return of asset i at time t.
- $\alpha_{it}^{(r)}$, $\beta_{ijt}^{(r)}$, and $\epsilon_{it}^{(r)}$ are the regime-dependent intercept, factor loadings, and idiosyncratic term, respectively.
- $F_{jt}^{(r)}$ is the value of the jth factor at time t in regime r.
- r denotes the current regime.

Variational Autoencoder Variational Autoencoders (VAEs) Kingma & Welling (2013) are a class of generative models that belong to the family of autoencoders. They were introduced as a probabilistic approach to encoding and decoding data, particularly in the field of unsupervised learning. VAEs differ from traditional autoencoders in that they are designed to generate new data points that resemble the input data, thus providing a powerful framework for the modeling of complex data distributions.

The key innovation in VAEs is the introduction of a probabilistic latent space. A VAE consists of two main components: an encoder and a decoder. The encoder maps input data to a latent distribution, typically assumed to be Gaussian, capturing the underlying probability distribution of the data. This mapping involves learning the parameters of the distribution - mean and variance - for each data point. The decoder then samples from this latent space to reconstruct the input data, allowing the model to generate new data points that are similar to the original inputs.

Mathematically, a VAE can be expressed as follows:

$$\text{Encoder: } q_\phi(z|x) = \mathcal{N}(z; \mu_\phi(x), \sigma_\phi^2(x)) \tag{3}$$

$$\text{Decoder: } p_\theta(x|z) = \mathcal{N}(x; \mu_\theta(z), \sigma_\theta^2(z)) \tag{4}$$

where x represents the input data, z is the latent variable, q_ϕ is the encoding distribution with parameters ϕ, and p_θ is the decoding distribution with parameters θ.

VAEs are trained by maximizing the variational lower bound (or evidence lower bound, ELBO) on the marginal likelihood of the observed data. This involves a trade-off between the fidelity of the reconstructed data and the regularization of the latent space.

VAEs have found numerous applications in diverse fields such as image generation, natural language processing, and anomaly detection, due to their ability to learn complex, high-dimensional data distributions and generate new samples from these distributions.

Recently, Variational Autoencoders (VAEs) have seen applications in predicting stock returns. One study, referenced as Duan et al. (2022), employs VAEs to extract nonlinear probabilistic hidden factors from stock features. Another notable work, cited as Wei et al. (2023), introduces a hierarchical encoder with a two-level latent space, reflecting a market-stock hierarchy. This model is coupled with regime-specific decoders that are designed to reconstruct or predict stock returns, adapting to varying market conditions.

2.2 Problem Formulation

According to regime-switching factor models, we have

$$R_{it} = \alpha_{it}^{(r)} + \sum_{j=1}^{n} \beta_{ijt}^{(r)} F_{jt}^{(r)} + \epsilon_{it}^{(r)} \tag{5}$$

The formulation of the task is to learn a regime-switching dynamic factor model with parameter Θ, for predicting future cross-sectional returns from historical data.

$$\hat{y}_s = f(x_s; \Theta) = \alpha^{(r)}(x_s) + \beta^{(r)}(x_s) z^{(r)}(x_s) \tag{6}$$
$$r = \Phi(x_s) \tag{7}$$

where $x_s \in \mathbb{R}^{N_s \times T \times C}$ is the historical stock characteristics (such as volatility, liquidity) of past T time-steps, N_s is the number of stocks in cross-section at time step s (we only consider the stocks that exist in cross-section at all T time steps), C is the number of characteristics, and r is market regime.

We formally define the problem as:

Input: A set of samples $\{(x_s, y_s)\}$, where $x_s \in \mathbb{R}^{N_s \times T \times C}$ is the sequential characteristics of stocks, and $\hat{y}_s \in \mathbb{R}^{N_s}$ is the future returns of cross-sectional stocks.

Output: A regime-switching dynamic factor model as Equation 5, which outputs the prediction returns $\hat{y}_s$.

2.3 RegimeEvoVAE Architecture

We take inspiration from Wei et al. (2023) and adopt the structural framework outlined in the same reference. Nevertheless, we plan to employ distinct implementations for each module.

Our model's architectural foundation is built upon a sophisticated VAE encoder-decoder structure. It distinguishes itself from traditional approaches by introducing a hierarchical encoder with a two-tiered latent space, meticulously designed to capture the nuances of the market-stock hierarchy. In tandem with this hierarchical encoder, we employ a series of regime-specific decoders, each specialized in reconstructing or predicting stock returns under distinct market conditions.

To delve into more detail, the market encoder initiates the process by extracting latent factors associated with the market from the market latent features. Subsequently, the stock encoder assumes a conditional role, extracting latent factors specific to individual stocks. This is accomplished by taking into account both the unique stock features and the previously obtained market latent factors. This hierarchical organization enhances the model's ability to discern and comprehend the market's influence on stock latent factors.

At the core of this encoder-decoder architecture resides a pivotal regime-switching module. This module acts as an intermediary, effectively determining and selecting the appropriate regime-specific decoder based on the latent market situation. It empowers the model to adapt dynamically and respond to changing market conditions, thereby amplifying its predictive capabilities.

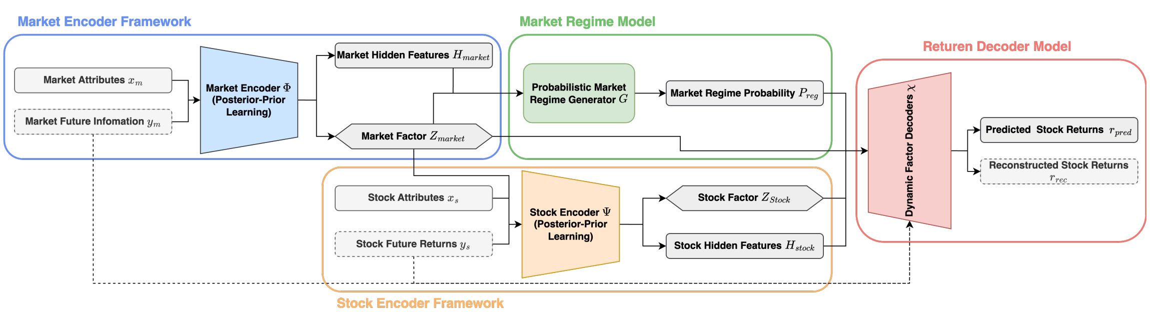

Figure 2: RegimeEvoVAE Structure

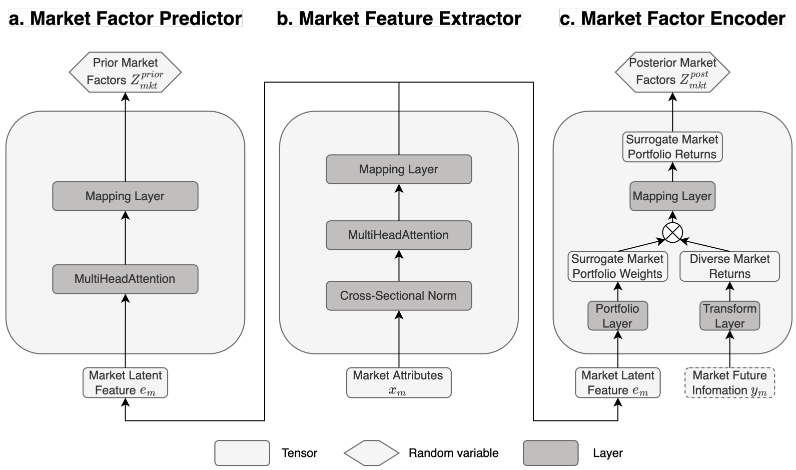

Figure 3: Market Factor

Market Feature Extractor To delve deeper into the hidden intricacies of market dynamics, we've implemented a sophisticated, custom-engineered attention module. This pivotal component of our analysis framework is designed to meticulously sift through each time point in the market data, unveiling a rich tapestry of hidden features.

The cornerstone of this approach lies in the module's advanced attention mechanism. This mechanism is adept at discerning and emphasizing the time points that are most significant, based on the unique market characteristics they embody. It's not just about observing the market; it's about understanding which moments truly matter and why.

Our choice to deploy this method is driven by a desire to dynamically and adaptively concentrate on the most pertinent segments of the market timeline. By assigning greater significance to the time points brimming with relevant insights, our module can unearth and bring to the forefront those elusive market features that are often overlooked but are crucial for a nuanced analysis or accurate forecasting.

This adaptive focus on market dynamics isn't just about gathering data; it's about capturing the essence of market behavior. The result is a more enlightened, comprehensive view of the market, paving the way for more insightful, precise, and actionable outcomes. This enhanced capability to parse through and prioritize market data points ensures that our analysis is not just data-driven, but insight-driven as well, offering a more profound understanding of the complex market landscape.

Market Factor Encoder The market feature encoder is intricately designed to incorporate both historical market data and projected future market insights. This integration significantly augments its ability to develop a sophisticated portrayal of the prevailing market regime. The strategic use of future market information fulfills a twofold objective: it refines the encoder's outputs to more accurately reflect upcoming market conditions and furnishes a detailed reference framework to steer the Predictor module. By anticipating future market dynamics, the encoder acts as a mentor within the predictive system, delineating a blueprint for expected market patterns.

In the prediction phase, the encoder is deliberately omitted to avert the risk of data leakage. The predictive model, therefore, exclusively relies on historical data, which is crucial to preserving the model's veracity and unbiased perspective. The encoder plays an indispensable role during the training phase, laying a solid foundation for the Predictor module by offering a comprehensive perspective of the market and projecting future trajectories. However, it is designed to have no bearing on real-time forecasting, where historical data is the sole input, ensuring the model's predictive capabilities are both reliable and impartial.

To further refine the simulation of market regime shifts, we will construct a series of surrogate portfolios. The returns from these portfolios will serve as market factors, providing a dynamic and representative array of hypothetical market scenarios. This approach enables a deeper exploration of potential market behaviors, improving the model's capacity to navigate and interpret complex market regimes.

Market Factor Predictor For the market factor predictor, an attention mechanism coupled with a linear mapping layer is employed to emulate the functionality of the Market Factor Encoder. This setup approximates the influence that the encoder exerts by focusing on relevant historical data points and mapping them to an appropriate market factor space. Through this process, the predictor can discern the intricate patterns and dependencies within the market data, enabling it to forecast future market factors with greater precision.

Market Regime Extractor To delineate market regimes, we'll harness hidden market features and market factors to form Gaussian clusters representing each regime. These clusters will then be refined through a linear stabilization process, designed to ensure that transitions between regimes occur smoothly. This method not only captures the distinct characteristics of each market regime but also provides a mechanism to gracefully manage the evolution of market conditions over time.

Market Regime Predictor Market factors will be mapped onto a one-dimensional score that reflects a sample drawn from the Gaussian clusters of market regimes. The regime with the highest probability, as indicated by this score, will be selected as the prevailing market regime. This approach effectively translates multifaceted market factors into a quantifiable likelihood, simplifying the identification of the most representative market regime at any given moment.

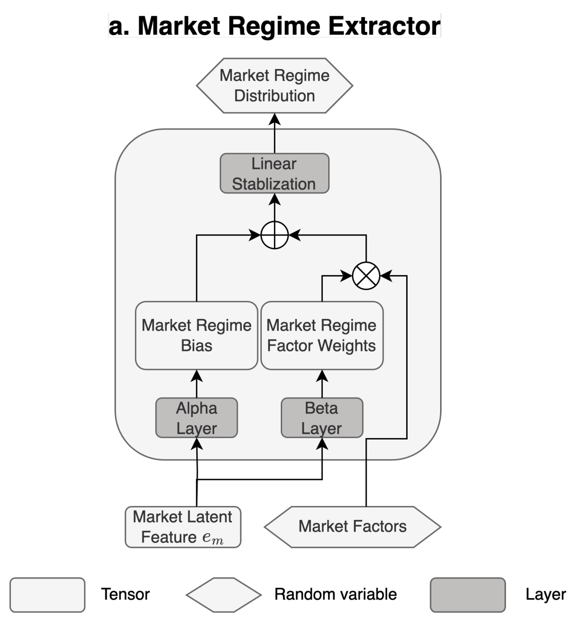

Figure 4: Market Regime Extractor

Stock Feature Extractor The feature extractor module, ϕ_{feat}, is tasked with extracting latent stock features, $\mathbf{e}$, from the historical sequence of stock characteristics, $\mathbf{x}$, and is defined as $\mathbf{e} = \phi_{\text{feat}}(\mathbf{x})$. To capture the temporal dependencies within these sequences, we employ the Gate Recurrent Unit (GRU), a refined variant of recurrent neural networks. At each time step t, the update process for the i-th stock is given by:

$$\begin{aligned} h_{\text{proj}}^{(i,t)} &= \text{LeakyReLU}(w_{\text{proj}}\mathbf{x}^{(i,t)} + b_{\text{proj}}) \\ h_{\text{gru}}^{(i,t)} &= \text{GRU}\left(h_{\text{proj}}^{(i,t)}, h_{\text{gru}}^{(i,t-1)}\right) \end{aligned}$$

where $\mathbf{x}^{(i,t)} \in \mathbb{R}^C$ represents the characteristics of the i-th stock at time step t, $h_{\text{proj}}^{(i,t)}$ and $h_{\text{gru}}^{(i,t)}$ are the hidden states of dimension H, and the LeakyReLU activation function is defined as:

$$\text{LeakyReLU}(x) = \begin{cases} x, & \text{if } x \geq 0 \\ \xi x, & \text{otherwise} \end{cases}$$

where ξ is a small, negative slope.

Finally, the latent features of stocks are encapsulated by the hidden state of the GRU at the final time step T, denoted as $\mathbf{e} = h_{\text{gru}}^{(T)}$.

Stock Factor Encoder The module is tasked with the processing of latent stock features, anticipated stock returns, and market factors. It employs a linear transformation to reconfigure market factors, which are then integrated with the latent stock features to produce an augmented set of features. These enhanced features are utilized in the construction of portfolios, with the generated portfolio returns defining the stock factors. Analogous to its prior role, this module serves as an instructive teacher for the prediction models in the training phase.

Stock Factor Predictor In this module, we will adopt a similar approach to merge market factors with stock features, creating a novel set of features. Subsequently, these features will be subjected to a multi-headed attention mechanism to derive the predicted factors.

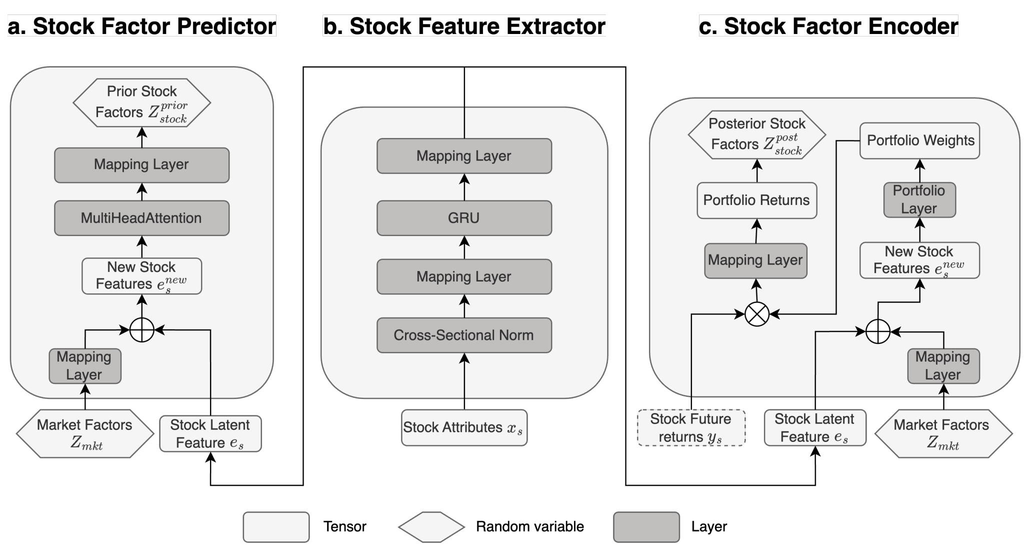

Figure 5: Stock Factor

Single Factor Decoders These decoders take both market and stock factors as input. Leveraging the latent stock features, they extract the alpha and beta coefficients. These coefficients are crucial as they allow us to compute the predicted stock returns within the context of the identified market regime, effectively translating the diverse market signals into actionable financial insights.

Regime-based Factor Decoders Within our system, each market regime is associated with a specialized decoder tailored to its unique characteristics. Upon predicting the current regime, we select the corresponding decoder to meticulously interpret the market factors, stock factors, and stock features. This targeted decoding ensures that the nuances of each regime are accurately reflected in the analysis, allowing for a more precise understanding of the interplay between these elements within the specified market context.

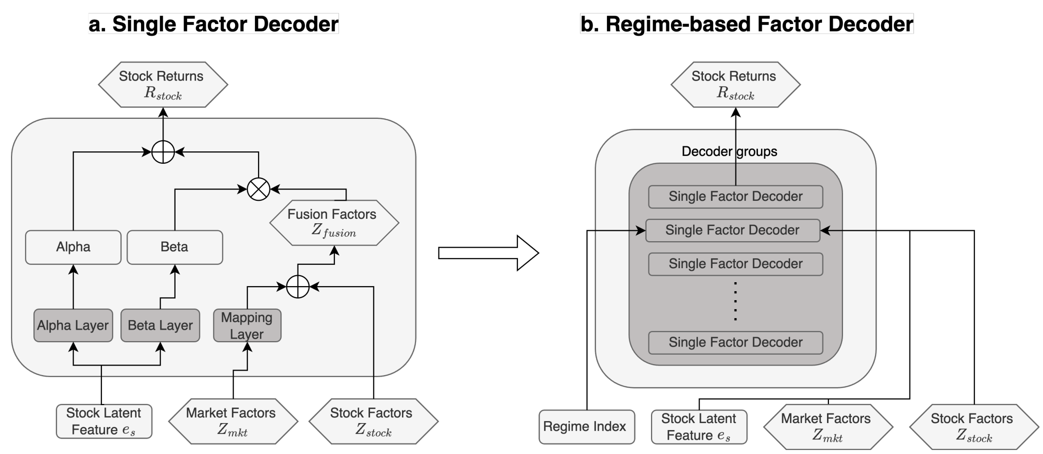

Figure 6: Regime Decoder

2.4 Strategy

2.4.1 Enhanced TopkDropoutStrategy in Quantitative Trading

In the dynamic realm of quantitative trading, the **Enhanced TopkDropoutStrategy** stands as a sophisticated approach for optimizing portfolio composition. This strategy is characterized by its pivotal parameters: *Topk* and *Drop*, which collectively govern the portfolio's structure and its periodic adjustment.

Operational Dynamics Initially, the portfolio is initialized without any assets. At each designated trading interval, this strategy conducts a meticulous evaluation of both the existing portfolio and the broader stock universe, ranking each asset based on its anticipated performance metrics.

- *Selection Criterion*: The *Topk* parameter sets the fixed number of assets to be consistently maintained in the portfolio.
- *Rebalancing Mechanism*: The *Drop* parameter, on the other hand, dictates the number of assets to be liquidated during each trading cycle. Specifically, the least promising stocks, as per their predictive performance, equivalent in number to *Drop*, are divested.
- *Portfolio Renewal*: Concurrently, an equivalent number of new stocks, showcasing the most promising predictive performance and not currently held, are acquired, ensuring the portfolio remains populated with *Topk* assets at all times.

This strategic cyclical renewal ensures the portfolio not only retains a consistent size but also dynamically aligns with evolving market trends. The turnover rate of the strategy is quantitatively defined by the ratio $\frac{2\times\text{Drop}}{\text{Topk}}$, facilitating an equilibrium between stability and responsiveness to market fluctuations.

Asset Weighting Approaches Incorporating a suitable asset weighting methodology is crucial for the efficacy of the TopkDropoutStrategy. Two prevalent methods are:

1. **Equal Weighted Approach**: Here, every asset in the portfolio is allocated an identical weight, advocating for a balanced exposure across all holdings.

2. **Inverse-Variance Weighting**: In this method, each asset's weight is inversely proportional to its variance (σ), symbolically represented as $\frac{1}{\sigma}$. This approach favors assets with lower risk profiles, aiming to minimize overall portfolio volatility.

By integrating these weighting strategies, the Enhanced TopkDropoutStrategy adeptly balances between risk management and capitalizing on market opportunities, making it a robust tool in the arsenal of quantitative trading methodologies.

Below is a visual elucidation of the Topk-Drop maneuver:

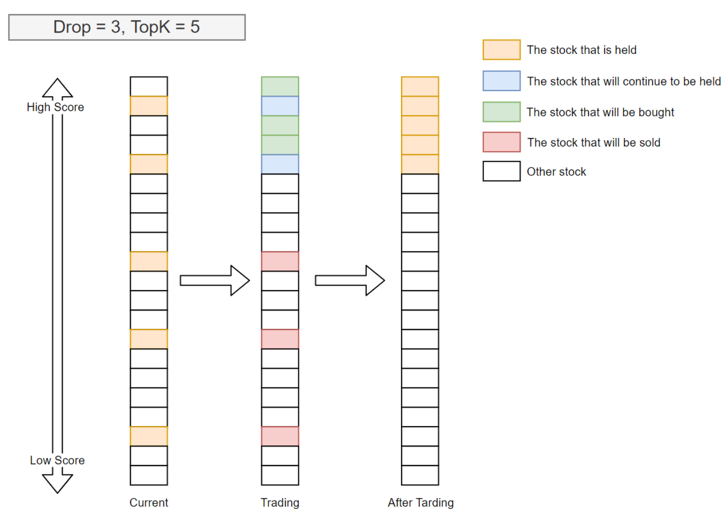

Figure 7: An illustration of the Topk-Drop strategy in action.

2.5 Backtest Results

2.5.1 Equal Weighted TopkDropoutStrategy

In-Sample (Training Period) Period: 1/1/2017 – 1/1/2021

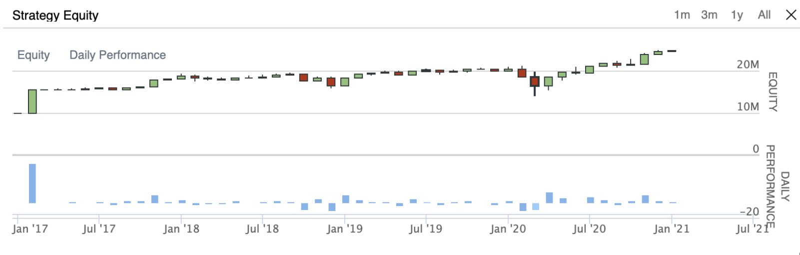

Figure 8: 1/1/2017 – 1/1/2021

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Out-of-Sample Period A: 1/1/2022 – 11/1/2022

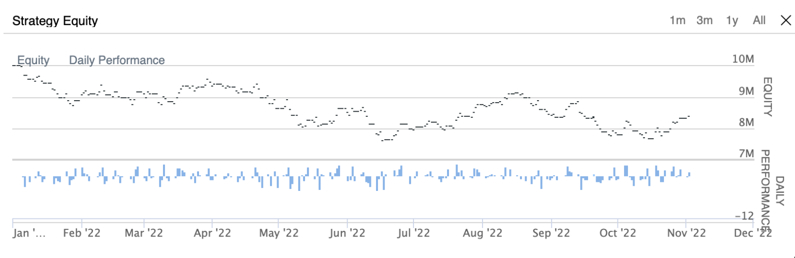

Figure 9: 1/1/2022 – 11/1/2022

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Out-of-Sample Period B: 1/1/2016 – 1/1/2017

Figure 10: 1/1/2016 – 1/1/2017

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Out-of-Sample Period C: 3/10/2023 - 10/10/2023

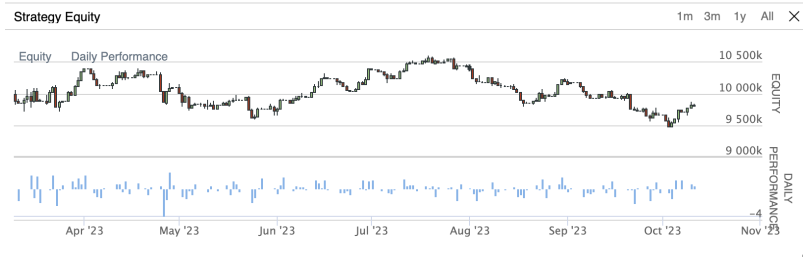

Figure 11: 3/10/2023 - 10/10/2023

- Backtesting link

	Sharpe Ratio	Max Drawdown	Compounding Annual Return
IS 2017.1.1-2021.1.1	0.789	34.20%	25.70%
OS 2016.1.1-2017.1.1	0.465	12.90%	8.94%
OS 2022.1.1-2022.11.1	-0.658	23.70%	-18.75%
OS 2023.3.10-2023.10.10	-0.448	10.30%	-3.17%

Table 1: EQUAL WEIGHTED TopkDropoutStrategy

2.5.2 Inverse-Variance Weighted TopkDropoutStrategy

In-Sample (Training Period) Period: 1/1/2017 – 1/1/2021

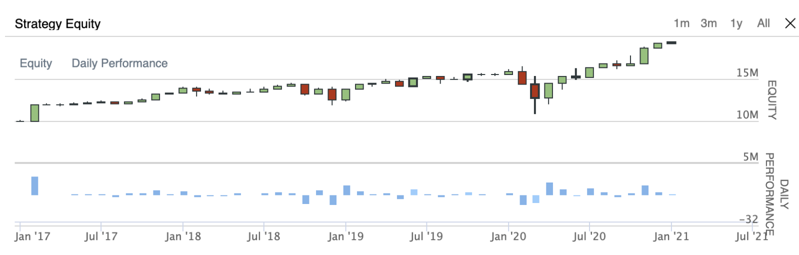

Figure 12: 1/1/2017 – 1/1/2021

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Out-of-Sample Period A: 1/1/2022 – 11/1/2022

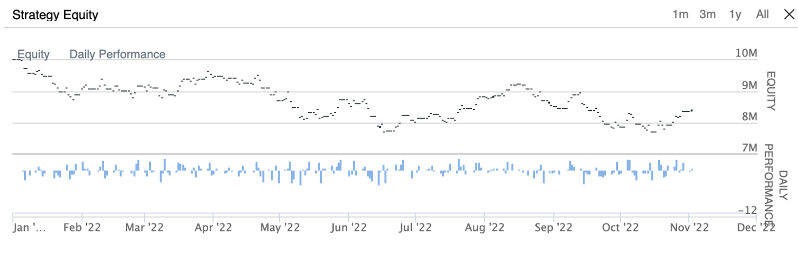

Figure 13: 1/1/2022 – 11/1/2022

- Backtesting link

Out-of-Sample Period B: 1/1/2016 – 1/1/2017

Figure 14: 1/1/2016 – 1/1/2017

- Backtesting link

Out-of-Sample Period C: 3/10/2023 - 10/10/2023

Figure 15: 3/10/2023 - 10/10/2023

- Backtesting link

	Sharpe Ratio	Max Drawdown	Compounding Annual Return
IS 2017.1.1-2021.1.1	0.693	34.20%	18.07%
OS 2016.1.1-2017.1.1	0.576	10.500%	10.297%
OS 2022.1.1-2022.11.1	-0.663	23.00%	-18.89%
OS 2023.3.10-2023.10.10	-0.344	10.70%	-1.44%

Table 2: Inverse-Variance Weighted TopkDropoutStrategy

2.5.3 Discussion

The inverse-variance strategy shows a higher Sharpe Ratio in the in-sample (IS) period from 2017 to 2021, indicating a better risk-adjusted return compared to the equal weight strategy. Although the Drawdown is the same for the out-of-sample (OS) period from 2016 to 2017 for both strategies, the inverse-variance strategy again presents a higher Sharpe Ratio and a significantly higher Compounding Annual Return, suggesting it was able to better capitalize on the available investment opportunities while maintaining the same level of maximum loss from peak to trough.

For the OS period from 2022, both strategies performed poorly, as indicated by negative Sharpe Ratios and Compounding Annual Returns. However, the inverse-variance strategy had a slightly less negative Sharpe Ratio and a smaller absolute value of negative Compounding Annual Return, which could be interpreted as it being less adversely affected by whatever market conditions prevailed at the time.

Finally, for the most recent period in 2023, although both strategies experienced a negative performance, the inverse-variance strategy had a smaller Drawdown and a less negative Compounding Annual Return than the equal weight strategy.

Considering these metrics, the inverse-variance strategy consistently shows either superior or less negative performance figures compared to the equal weight strategy across all timeframes, making it the preferable choice.

3 Pairs Trading with Copula Mispricing Index

3.1 Introduction

Our pairs trading strategy incorporates the copula methods and by doing so, delves deeper into the dependency structure between financial instruments. This approach corrects a theoretical misunderstanding of assuming joint stocks returns to be bivariate Gaussian distribution. However, in reality, pairs stocks return often shows high correlation when markets are very negative or positive. This is where copula comes to work.

Copulas is a mathematical functions used to describe the joint distribution of multiple random variables, provide a nuanced way to capture and model the relationship between the price movements of different assets. In pairs trading, this typically involves identifying two securities whose prices are statistically co-integrated, indicating a historical tendency to move in tandem. In particular, the copula in our pairs trading strategy identifies the joint distribution of quantiles of stock prices and depicts the dependencies of stock movements, especially tail dependencies. Copulas capture the level of mispricing as well as the strength of the mean reversion. The level of mispricing is captured by determining the probability associated with a market-observed value with respect to another market variable to which it was historically highly correlated

By applying copula models, we can more accurately discern strength of the dependency between stocks. This refined understanding allows for the identification of instances when the relationship between the pair deviates from the historical norm, signaling potential trading opportunities. We then exploit these deviations by taking opposing positions on the pair (long on the undervalued asset and short on the overvalued one), betting on the reversion of their price relationship to the historical mean.

3.2 Coupla and Stock Price

Sklar's Theorem Let $X_1, \ldots, X_d$ be random variables with distribution functions $F_1, \ldots, F_d$, respectively. Then, there exists an d-copula C such that:

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)),$$

for all $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$. If $F_1, \ldots, F_d$ are all continuous, then the function C is unique; otherwise C is determined only on $\mathrm{Im}F_1 \times \ldots \times \mathrm{Im}F_d$.

A two dimensional copula fitting into pairs trading strategy, is a function $C : [0, 1]^2 \to [0, 1]$, and is defined as the joint cumulative density on quantile of each marginal random variable:

$$C(u_1, u_2) = \mathbb{P}(U_1 \leq u_1, U_2 \leq u_2).$$

Proposition. Let U1 and U2 be two random variable with distribution U(0,1). Then,

$$\mathbb{P}(U_1 \leq u_1 | U_2 = u_2) = \frac{\partial C(u_1, u_2)}{\partial u_2},$$
$$\mathbb{P}(U_2 \leq u_2 | U_1 = u_1) = \frac{\partial C(u_1, u_2)}{\partial u_1}.$$

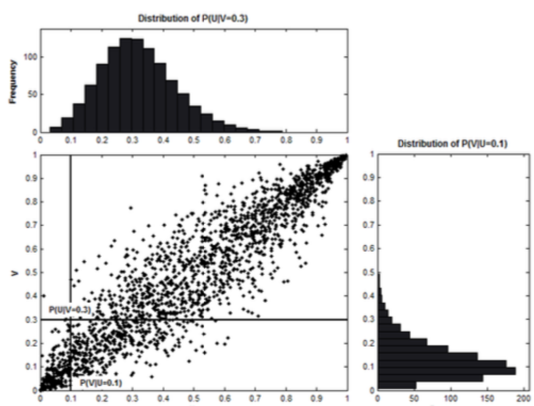

Figure 16: An illustration of the conditional distribution function of V for a given value of U.

According to the graph Johannes Stübinger & Krauss (2018), we can find that when the U and V have strong tail dependence. When at the tail, there is a point away from the majority, we can regarded it as a misplaced point. For the next point, it will back to the majority cluster due to the tail dependence.

Because of the dependence idea, we select Archimedean copulas as our choice which not only gets rid of the Gaussian Distribution Assumption, but also can have a non parametric estimation for its parameter. It will be helpful for our trading stability Goda (2010). For each trading pairs, we will select the optimal copula for it by minimizing the Akaike information criterion.

Name of copula	**Bivariate copula** $C_\theta(u,v)$	**Tail Dependence**
Clayton	$\left[\max\{u^{-\theta}+v^{-\theta}-1;0\}\right]^{-1/\theta}$	Lower Tail Dependence
Frank	$-\frac{1}{\theta}\log\left[1+\frac{(\exp(-\theta u)-1)(\exp(-\theta v)-1)}{\exp(-\theta)-1}\right]$	No tail dependence but stronger in the center
Gumbel	$\exp\left[-\left((-\log(u))^{\theta}+(-\log(v))^{\theta}\right)^{1/\theta}\right]$	Upper Tail Dependence

Table 3: Selected Archimedean copulas functions.

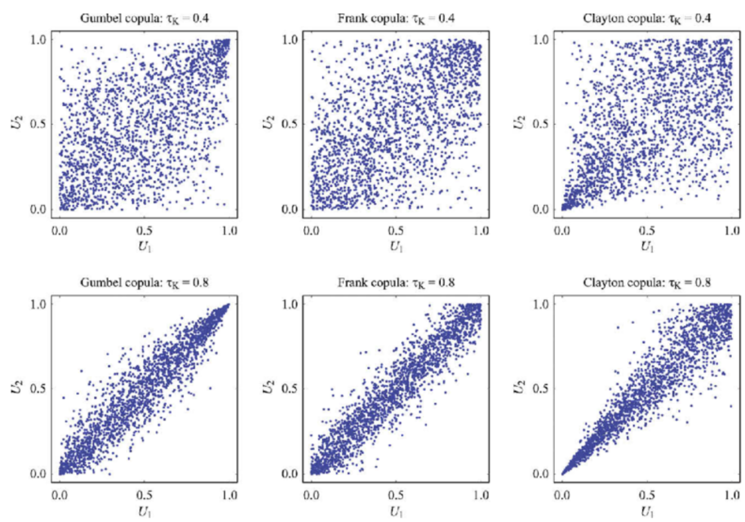

Figure 17: Simulation from Gumbel, Clayton and Frank Copulas

3.3 Mispricing Index Signal

The marginal distribution selected in this strategy is the empirical distribution. We use the empirical cumulative density function to transfer the stock returns. Then, denote:

$$U1 = ECDF(stockXreturn)$$
$$U2 = ECDF(stockYreturn)$$

U1 and U2 are two random variable with distribution U(0,1). Then,

$$\mathbb{P}(U_1 \leq u_1 | U_2 = u_2) = \frac{\partial C(u_1,u_2)}{\partial u_2},$$
$$\mathbb{P}(U_2 \leq u_2 | U_1 = u_1) = \frac{\partial C(u_1,u_2)}{\partial u_1}.$$

By using the fact that the partial derivative of the copula function gives the conditional distribution function, define a measure to denote the degree of mispricing:

Definition - Mispricing Index Signal: Let U_t^X and U_t^Y represent the quantile of random daily returns of stocks X and Y at time t, and r_t^X and r_t^Y represent the realizations of the those returns at time t. Then define:

$$\mathbb{MI}^{X|Y} = \frac{\partial C(u_1,u_2)}{\partial u_2} = \mathbb{P}(U_t^X \leq u_t^X | U_t^Y = u_t^Y)$$
$$\mathbb{MI}^{Y|X} = \frac{\partial C(u_1,u_2)}{\partial u_1} = \mathbb{P}(U_t^Y \leq u_t^Y | U_t^X = u_t^X)$$

Therefore, the conditional probabilities $MI^{X|Y}$ and $MI^{Y|X}$ indicate whether the return of X is considered high or low at time t, given the information on the return of Y at time t and the historical relation between the two stocks' returns, and vice-versa. When we observe a value of $MI^{X|Y}$ equal to 0.5, r_t^X is neither too high nor too low given r_t^Y and their historical relation, i.e., we can say that this reflects no mispricing. In other words, the historical data indicates that, on average, there is an equal number of observations of the return of X being larger or smaller than r_t^X if the return of stock Y is equal to r_t^Y and therefore, a conditional value of 0.5 means that the stock X is fairly priced priced relative to stock Y at day t.

Note that the conditional probabilities, $MI_t^{X|Y}$ and $MI_t^{Y|X}$, only measure the degrees of relative mispricing for a single day. We determine an overall degree of relative mispricing. Initially, let $m_{1,t}$ and $m_{2,t}$ be the cumulative mispricing indexes of stocks X and Y, defined by ($MI_t^{X|Y}$ - 0.5) and ($MI_t^{Y|X}$ - 0.5), respectively. Then, at the beginning of each trading period two cumulative mispricing indexes $CMI_{1,t}$ and $CMI_{2,t}$ are set to zero and evolve for each day via

$$CMI_{1,t} = CMI_{1,t-1} + m_{1,t} = \sum_{i=0}^{t} m_{1,i}$$
$$CMI_{2,t} = CMI_{2,t-1} + m_{2,t} = \sum_{i=0}^{t} m_{2,i}$$

This approach reflects the mispricings over multiple periods, thus reflecting how farther away the prices are out from their equilibrium. In contrast to the mispricing indices, it can lead to optimal trading strategies, since it results in a more stable strategy. Positive (negative) $CMI_{1,t}$ and negative (positive) $CMI_{2,t}$ are interpreted as stock 1 (stock 2) being overvalued relative to stock 2 (stock 1) **?**.

3.4 Stock Selection

The security selection range are ETF and Stocks. The ETFs are divided into Energy, Metals, Technology, Treasures, Volatility and SP500 ETF Sectors. The stock selection from sectors are using MorningStar Sector Division, which are including:

MorningStar Sectors	
Basic Materials	ConsumerCyclical
FinancialServices	RealEstate
RealEstate	ConsumerDefensive
Healthcare	Utilities
CommunicationSerivces	Energy
Industrials	Technology

For each sectors, stocks are selected from their belonging sectors. We plan to select multiple pairs and trade them together. To reduce the interaction influence among pairs, we finally selected Basic Materials, Financial Services, Healthcare, Utilities, Energy and Technology to minimize the industry correlation.

SECTOR CORRELATIONS Source: Bloomberg, as of 10/19/17

	Consumer Discretionary	Consumer Staples	Energy	Financials	Healthcare	Industrials	Information Technology	Materials	Telecom	Utilities	Real Estate
Consumer Discretionary	1.00	0.52	0.45	0.78	0.51	0.85	0.72	0.74	0.54	0.26	0.70
Consumer Staples	0.52	1.00	0.34	0.58	0.65	0.57	0.27	0.47	0.39	0.43	0.55
Energy	0.45	0.34	1.00	0.49	0.35	0.60	0.37	0.67	0.31	0.43	0.37
Financials	0.78	0.58	0.49	1.00	0.60	0.81	0.51	0.69	0.42	0.33	0.72
Healthcare	0.51	0.65	0.35	0.60	1.00	0.56	0.39	0.43	0.41	0.37	0.51
Industrials	0.85	0.57	0.60	0.81	0.56	1.00	0.66	0.83	0.49	0.37	0.69
Information Technology	0.72	0.27	0.37	0.51	0.39	0.66	1.00	0.54	0.51	0.16	0.53
Materials	0.74	0.47	0.67	0.69	0.43	0.83	0.54	1.00	0.39	0.30	0.62
Telecom	0.54	0.39	0.31	0.42	0.41	0.49	0.51	0.39	1.00	0.30	0.34
Utilities	0.26	0.43	0.43	0.33	0.37	0.37	0.16	0.30	0.30	1.00	0.44
Real Estate	0.74	0.55	0.37	0.72	0.51	0.69	0.53	0.62	0.34	0.44	1.00

Past performance is not a guarantee of future results.

Figure 18: Industry Correlation Map

We divided the stock selection into fundamental analysis and statistics tests.

3.4.1 Fundamental Analysis

Holding the goal to find stocks with mispricing probabilities, we want to reduce possibilities that stock prices will finally divergent. Stocks selected are top 10 Market Capitalization in each industry sections on Jan. 1st, 2017, because we holds the belief that their valuations have been confirmed by the market. Among each sectors, High Market Cap Stocks will have a stable long-term correlation. Moreover, for each pairs construction, we form a stock pair based on their underlying business scope to stabilize their long-term stock price pairs correlation. The selected ETF and stock symbols have been attached in the Appendix.A.1.

3.4.2 Statistical Tests

Stocks and ETF filtered through fundamental analysis should pass our statistical tests. Then it will be regarded as a candidate for our pairs trading strategies. We use in-sample stock log returns datasets between 2017-01-01 and 2018-01-01 to perform our statistical testing.

2.1 Augmented Dickey-Fuller Test
The fundamental objective of the ADF test is to determine the presence or absence of a unit root in a time series, a factor that indicates non-stationarity. The presence of a unit root implies that the time series is susceptible to shocks or changes that can have lasting effects, thereby rendering its statistical properties, such as mean and variance, inconsistent over time.

Consider a time series Y_t. The ADF test models Y_t as an autoregressive process:

$$Y_t = \rho Y_{t-1} + \sum_{i=1}^{p} \phi_i \Delta Y_{t-i} + \epsilon_t,$$

where $\Delta Y_t = Y_t - Y_{t-1}$ represents the first difference of the series, ρ is the coefficient of the lagged level of the series, ϕ_i are the coefficients of the lagged differences, and ϵ_t is the error term. The number of lags p is chosen such that ϵ_t is a white noise process.

In the ADF test, the hypothesis testing is:

$$H_0 : \rho = 1 \text{: Non-Stationarity}$$
$$H_1 : \rho < 1 \text{: Stationarity}$$

For a equity pair P and Q, we assume a regression slope β (also called hedge ratio), regression residual ε, the standard deviation of ε and initial capital for a certain pair C. The linear regression equation relates to the log prices is as follows:

$$\log P_t = \beta \log Q_t + \alpha + \varepsilon_t$$

Then, we run the Augmented Dickey-Fuller Test on ε_t to confirm the co-integration relationship between Equity P and Equity Q.

2.2 Mangold's Test

Mangold's Test Johannes Stübinger & Krauss (2018) introduced a multivariate linear rank test of independence based on the Nelsen copula, and was used in as a method to select tradable stocks. It is a statistic that tests for variable independence (and therefore dependence). It registers extremal co-moves at the tail of distributions well because of the involvement of copula, therefore is a great candidate for copula-based trading methods with mean-reversion bets. The Nelsen copula is parametric. The 2D formula is the following:

$$C_\theta(u, v) = C_{A_1,A_2,B_1,B_2}(u, v) =$$
$$= uv\left[1 + (1-u)(1-v)\left(\frac{u^{-B_2} + v^{-B_1} - 1}{B_1 + B_2}\right) + (1-u)(1-v)A_1 + (1-u)(1-v)A_2\right]$$

Keep in mind that the coefficients are not trivially bounded. The property we utilize is when $\theta = (A_1, A_2, B_1, B_2) = 0$, the Nelsen copula becomes the independent copula:

$$C_0 = uv \quad \text{iff} \quad \theta = (A_1, A_2, B_1, B_2) = \theta_0$$

which serves as its null hypothesis. The test statistic asymptotically has a 1-D Gaussian distribution, and we can therefore use a χ^2 test with $DOF = q = 2d$, and d is the number of stocks.

In the Mangold's test, the hypothesis testing is:

$$H_0 : \theta = 0 \text{: No Tail Dependency}$$
$$H_1 : \theta \neq 0 \text{: Tail Dependency}$$

2.3 Quantile-Quantile Distance Verification

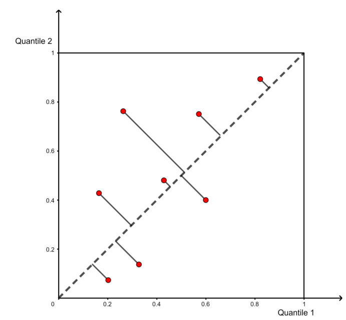

Figure 19: Q-Q plot's averaged diagonal distance

A Quantile-Quantile (Q-Q) plot's averaged diagonal distance can serve as a measure of association between two distributions. The smaller the averaged diagonal distance, the more "related" the distributions are considered to be (in one direction). This relationship suggests a higher degree of similarity between the distributions, as points in the Q-Q plot would lie closer to the 45-degree line that represents theoretical quantile correspondence. Consequently, this measure can be interpreted as a statistic for the empirical copula, which captures the dependency structure between the marginal distributions of multivariate data. In this context, the empirical copula's Q-Q plot provides a visual and numerical method to assess the strength and nature of the dependency, which is crucial for understanding the joint behavior of the variables under study. We use the average quantile to quantile plot's averaged diagonal distance to compare between pairs. The smaller the averaged diagonal distance is, the higher correlation between pairs.

3. Stock Selection Results

Our selected pairs:

Equity 1	**Equity 2**
AAPL	META
USB	WFC
PPG	CRHCY
ABBV	BMY

- Apple Inc. (AAPL) is a behemoth in the technology sector, renowned for its innovative consumer electronics like the iPhone, iPad, and Mac computers, as well as its proprietary software and services. Meta Platforms, Inc. (META), dominates the social media landscape with platforms such as Facebook, Instagram, and WhatsApp, and is expanding into virtual reality and other technologies.
- Wells Fargo Company (WFC) and U.S. Bancorp (USB) are key players in the financial services industry, offering banking, mortgage, and investment services.
- PPG Industries, Inc. (PPG) specializes chemicals in consumer products, industrial, and transportation markets. Croda International Plc (CRHCY) develops and markets specialty chemicals for a wide array of industries, including personal care and health care.
- AbbVie Inc. (ABBV) is a biopharmaceutical giant that researches and develops a broad portfolio of drugs and therapies, focusing on areas such as immunology, oncology, and virology. Bristol Myers Squibb Company (BMY) is another global biopharmaceutical firm, with a focus on discovering and developing treatments for serious diseases including cancer, heart disease, and immune disorders.

3.4.3 TRADING STRATEGY EXECUTION

Our trading strategy execution is implemented through QuantConnect. The general strategy excution are Fitting the Coupla, Calculating Cumulative Mispricing Index(CMI) and Order Execution.

- **Fitting the Coupla - Events happens at the First Day of Each Month:**
  We use the historical 125 days returns data fit an empirical marginal density distribution. Meanwhile, we will use the same historical datasets to select an optimal copula for each trading pairs.

- **Calculating Mispricing Index - Events happens on Everyday Market Open:**
  The mispricing index for Day_t is calculated using close data Day_{t-1}. Then the mispricing index at Day_t will be added to Cumulative Mispricing Index at Day_{t-1}.

- **Order Execution - Events happens on Everyday Market Open:** Order will be submitted when it meet the following condition:
  - When $CMI_{1,t} \geq$ EnterPoint or $CMI_{2,t} \leq$ -EnterPoint, Short $W_{t,1}$ * Stock 1 and Long $W_{t,2}$ * Stock 2;
  - When $CMI_{1,t} \leq$ -1 * EnterPoint or $CMI_{2,t} \geq$ EnterPoint, Long $W_{t,1}$ * Stock 1 and Short $W_{t,2}$ * Stock 2;
  - When Absolute Value of $CMI_{1,t}$ or $CMI_{2,t} \geq$ Exit Point, Liquidate All the Position and Set both $CMI_{1,t}$ and $CMI_{2,t}$ to 0.

1. Enter or Exit Threshold Selection
The Optimization Method is Grid Search. We run the Enter and Exit Threshold Optimization on the in-sample data from 2017-01-01 to 2021-01-01.

Parameter	Min	Max	Step Size
enter	0	2	0.5
exit	2	4	0.1

Table 4: Parameter optimization ranges.

The final choice are:

- Enter Threshold: 0.5
- Exit Threshold: 2

The threshold is reasonable for the real world understanding. For example, for enter threshold, it means that when you at least have 2 excessive days with probability higher than 0.5, you need to enter the position.

2. Order Weights
Since our selected pairs have passed Augmented Dickey-Fuller Test (i.e. Our selected pairs have a stationary spread relationship). We make the assumption that when there is a tail dependence, it should be the linear relationship. So, we can have:

$$W_{t,1} = \frac{1}{1+\beta} \text{ for } Stock1_t$$

$$W_{t,2} = \frac{\beta}{1+\beta} \text{ for } Stock2_t$$

When we execute order, stock 1 and stock 2 will match with weights assigned to them. The weights will re-calculate every month. Strategies' Backtest Performances are attached at the Section: Backtesting and Results.

3.5 Backtesting Results

3.5.1 In of Sample Period: 2017.1.1 - 2021.1.1

Figure 20: 2017.1.1 - 2021.1.1

- Backtesting link

3.5.2 Out of Sample Period 1: 2022.1.1 - 2022.11.1

Figure 21: 2022.1.1 - 2022.11.1

- Backtesting link

3.5.3 Out of Sample Period 2: 2016.1.1 - 2017.1.1

Figure 22: 2016.1.1 - 2017.1.1

- Backtesting link

3.5.4 Out of Sample Period 3: 2023.3.10 - 2023.10.10

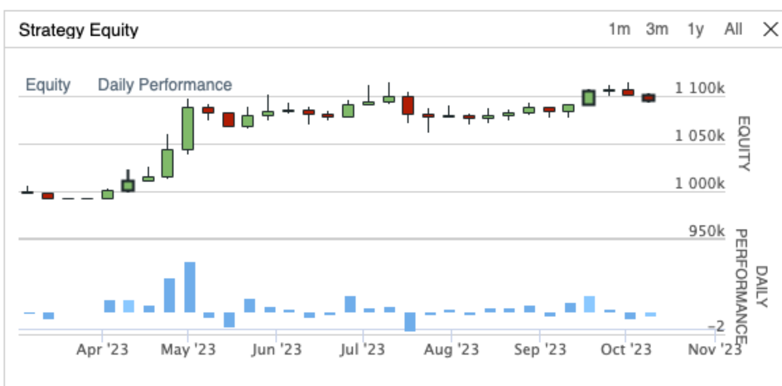

Figure 23: 2023.3.10 - 2023.10.10

- Backtesting link

3.5.5 Stress Testing: 2020.3

Figure 24: Stress Testing

- Backtesting link

3.5.6 Performance Conclusion

	Sharpe Ratio	Max Drawdown	Compounding Annual Return
IS 2017.1.1-2021.1.1	0.532	13.800%	10.220%
OS 2016.1.1-2017.1.1	0.323	12.200%	5.098%
OS 2022.1.1-2022.11.1	2.166	13.200%	59.643%
OS 2023.3.10-2023.10.10	0.831	4.5%	16.894%
ST 2020.3	1.567	7.800%	55.267%

Table 5: Pairs Trading Statistics

3.5.7 Live Trading

We deployed live trading on QuantConnect from Dec 11th, but no order has been submitted, which means from Dec 11th to Dec 12th, there is no mispricing opportunities between selected pairs.

4 Advantage Actor Critic(A2C) with LSTM agent

In the ever-evolving and intricate landscape of financial markets, the quest for innovative and efficient trading strategies is unending. Recently, machine learning (ML) and deep learning (DL) have emerged as some of the most favored techniques in this realm, primarily due to the rapid advancements in the machine learning community. Traditional statistical learning algorithms often fall short in addressing the non-stationary and nonlinear characteristics inherent in stock markets Dang (2019).

Deep reinforcement learning (DRL), a burgeoning subset of machine learning, has recently garnered attention for its potential in automating stock trading. Research indicates that DRL can outperform other methodologies in trading environments. Among various DRL approaches, the Advantage Actor-Critic (A2C) method has demonstrated its capability to yield considerable rewards while mitigating maximum downturns Asodekar et al. (2022).

Current literature on Reinforcement Learning (RL) in trading is primarily divided into three approaches: critic-only, actor-only, and actor-critic. The critic-only method, particularly Deep Q-Networks (DQN), is widely researched. It uses a state-action value function, Q, to assess actions in specific states, usually with discrete action spaces. However, this method risks significant losses during market volatility due to its tendency for fully invested positions, lacking flexibility in adjusting to market conditions.

On the other hand, the actor-only approach directly optimizes objectives like profits without calculating action outcomes, suitable for continuous action spaces. Though it can be optimized using gradient ascent methods, the approach suffers from slow learning and the need for extensive data to refine policies. The actor-critic method, less explored in financial applications, seeks to solve these issues by simultaneously updating two models: the actor, determining the agent's actions, and the critic, evaluating these actions. Our project aims to further investigate the actor-critic approach, offering a more dynamic and responsive framework for trading strategies. For comprehensive insights into these methodologies and their applications in finance, readers are directed to the relevant survey in the field Zhang et al. (2020).

In our project, we have integrated the A2C algorithm with Long Short-Term Memory (LSTM) agents to enhance stock trading strategies. LSTMs are a type of recurrent neural network renowned for their proficiency in processing and learning from sequential data, which is crucial for analyzing time-series financial data AbdelKawy et al. (2021). This synergy is designed to capture complex temporal dependencies and patterns in stock price movements, providing a sophisticated framework for informed trading decisions. The primary objective of this strategy extends beyond mere price prediction; it focuses on learning a trading policy (encompassing buy, sell, and hold actions) that maximizes investment returns (profit and loss, PnL) over a set investment horizon, while carefully considering market risks and uncertainties.

4.0.1 Long Short Term Memory Network (LSTM)

Long Short-Term Memory (LSTM) networks are designed to learn long-term dependencies and avoid the long-term dependency problem. An LSTM unit consists of a cell state c_t and three gates: an input gate i_t, an output gate o_t, and a forget gate f_t, which regulate the information flow.

The LSTM update equations are:

- Forget gate: $f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$
- Input gate: $i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$
- Cell candidate: $\tilde{c}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$
- Update cell: $c_t = f_t * c_{t-1} + i_t * \tilde{c}_t$
- Output gate: $o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$
- Output: $h_t = o_t * \tanh(c_t)$

The optimization problem for training an LSTM involves minimizing a loss function L that measures the discrepancy between the predicted and true outputs:

$$\min_{W,b} \sum_t L(y_t, \hat{y}_t)$$

where y_t is the true output and $\hat{y}_t$ is the predicted output by the LSTM at time t. This optimization is typically performed using algorithms like Stochastic Gradient Descent (SGD) or its adaptive variants.

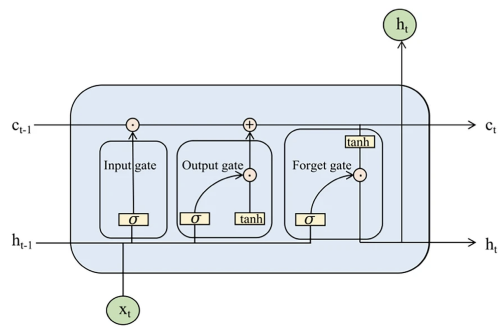

Figure 25: LSTM structure

4.0.2 Restrictions of LSTM

During our experiments with Long Short-Term Memory (LSTM) networks for stock performance prediction, we identified key limitations when using LSTM as a standalone tool. These include a propensity for overfitting amidst the stock market's volatility and noise, and challenges in adapting to rapid market shifts and external influences. These drawbacks underscored the necessity for a more dynamic approach, leading us to integrate the Advantage Actor-Critic (A2C) model. A2C, a deep reinforcement learning algorithm, complements LSTM by not only predicting stock prices but also learning optimal trading actions in a fluctuating market environment. This combination enhances our strategy's adaptability and overall effectiveness in navigating the complexities of financial trading.

4.0.3 Advantage Actor Critic(A2C) algorithm

A2C is a synchronous and deterministic version of the actor-critic algorithm, which utilizes an advantage function to decrease the variability in the policy gradient. This enhancement bolsters the model's robustness. The algorithm synchronizes its updates by accumulating and averaging the experiences of all actors once each has finished its respective experience segment Mnih et al. (2016).

4.0.4 Stock Trading Problem Setting

A Markov Decision Process is used to formalize the stock trading process:

- State $\boldsymbol{s}_t = [\boldsymbol{b}, \boldsymbol{f}]$the information vector at time t, including the features for prediction and current balance.
- Action $\boldsymbol{a}_t = [a_1, a_2, \cdots, a_n] \in \mathbb{R}^n, a_i \in [-1, 1]$ for $i = 1, 2, \cdots, n$ represent the action at time t for the n stock.The element a_i refers the proportion of the cash used to buy/sell/hold the stock(positive numbers refers buying action, nagative numbers refer selling action and 0 refers holding the position.
- Reward: the reward is defined as $PnL_t = P_t - P_{t-1}$, where P_t is the total portfolio value at time t and the reward PnL is the profit and loss between time t and $t-1$.
- Policy function $\boldsymbol{\pi}(\boldsymbol{a}|\boldsymbol{s};\boldsymbol{\theta})$ is responsible for producing the probability distribution over action $\boldsymbol{a}$ given he current state $\boldsymbol{s}$.

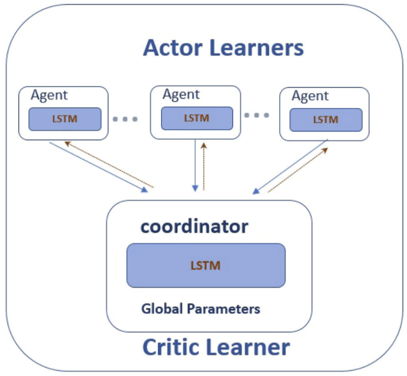

Figure 26: A2C architecture

- Action-value function $Q(\boldsymbol{s}, \boldsymbol{a})$ represents the expected return after taking an action $\boldsymbol{a}$ in state $\boldsymbol{s}$
- State function $V(\boldsymbol{s}; \boldsymbol{\phi})$ is parameterized by ϕ and estimates the value of being in state $\boldsymbol{s}$.
- The advantage function $A(\boldsymbol{s}, \boldsymbol{a}) = Q(\boldsymbol{s}, \boldsymbol{a}) - V(\boldsymbol{s}; \boldsymbol{\phi})$ indicates how much better an action a is compared to the average action at state $\boldsymbol{s}$ The loss function includes the actor loss and critic loss which are fomulized as:

$$\begin{aligned} L^{\text{actor}}(\boldsymbol{\theta}) &= -\log(\pi(\boldsymbol{a}|s; \boldsymbol{\theta}))A(\boldsymbol{s}, \boldsymbol{a}) \\ L^{\text{critic}}(\boldsymbol{\phi}) &= (R_t - V(\boldsymbol{s}; \boldsymbol{\phi}))^2 \end{aligned} \tag{8}$$

  where R_t is the rewards of the following state $\boldsymbol{s}$ at time t
- Margine requirements: the margin requirements is set to 25% in the training. A 'margin call' occurs if the equity (total portfolio value minus borrowed amount) falls below this margin requirement.

4.0.5 Dataset

Stock Selection: In this trading strategy, we have selected the five stocks with the largest market capitalizations from three key sectors: financial services, consumer services, and technology. This approach allows us to focus on leading companies in each sector, providing a diverse yet targeted investment portfolio. By choosing the top players in these sectors, we aim to capitalize on the stability and growth potential of established companies while navigating different market dynamics. The financial services sector offers insights into the broader economic landscape, consumer services reflect changing consumer trends and demands, and the technology sector represents innovation and future growth potential. This strategic selection of stocks from varied but influential sectors underpins our approach to achieving a balanced and potentially lucrative trading outcome. The final selection of the stocks are:

- **JPM (JPMorgan Chase Co.)**: A leading global financial institution, JPM represents a cornerstone in banking and financial services.
- **BRK.B (Berkshire Hathaway Inc.)**: A diversified conglomerate with significant holdings in finance and insurance, BRK.B is synonymous with long-term value investing.
- **AAPL (Apple Inc.)**: An iconic technology company, AAPL is at the forefront of consumer electronics, software, and digital services innovation.

- **AMZN (Amazon.com Inc.)**: A dominant player in online retail and cloud services, AMZN also has expanding interests in media and AI, making it a key stock in consumer and tech sectors.
- **MSFT (Microsoft Corporation)**: A powerhouse in software, cloud computing, and technology solutions, MSFT is integral to the global tech landscape

Indicators Selection: We utilize a range of technical indicators for the strategy to analyze market trends and inform our trading decisions. Each indicator offers unique insights:

- **Relative Strength Index (RSI)**: Measures the magnitude and speed of recent price changes to evaluate overbought or oversold conditions in the stock.
- **Autoregressive Integrated Moving Average (ARIMA)**: A statistical analysis model that uses time series data to better understand and predict future trends.
- **Volatility**: Indicates the degree of variation of a trading price series over time, which helps in assessing risk.
- **Relative Daily Volume(RDV)**: Compares current trading volume to historical averages, useful in identifying unusual trading activity.
- **Bandwidth**: Part of the Bollinger Bands methodology, this measures the difference between the upper and lower bands, indicating market volatility.
- **Percent B (%B)**: Quantifies a stock's price relative to the upper and lower Bollinger Bands, useful for identifying overbought and oversold conditions.
- **Momentum(MOM)**: Measures the rate of change in stock prices, helping to identify the strength of current market trends.

These indicators, collectively, provide a comprehensive view of the market's technical aspects, guiding us in making informed trading decisions based on underlying trends, volatility, and momentum.

In this trading strategy, transactions are executed every 10 trading days, striking a balance between responsiveness to market changes and over-trading. This periodic approach allows for the assimilation of market trends and indicator signals over a meaningful timeframe, enabling more informed trading decisions.

For model training, historical stock price data and historical data of the selected indicators (Relative Strength Index, Autoregressive Integrated Moving Average, Volatility, Relative Daily Volume, Bandwidth, Percent B, Momentum) from January 1, 2017, to January 1, 2021, are utilized. This four-year period provides a substantial dataset that captures a variety of market conditions, including typical market fluctuations and exceptional events. Training the model on this data enables it to learn and adapt to different market scenarios, enhancing its predictive accuracy and robustness for future trading. This historical approach, combined with a disciplined transaction schedule, forms the backbone of our data-driven, algorithmic trading strategy.

4.1 Backtesting and Results

The transactions are made every 10 trade days in all backtesting,

4.1.1 IN SAMPLE PERIOD: 2017.1.1 - 2021.1.1

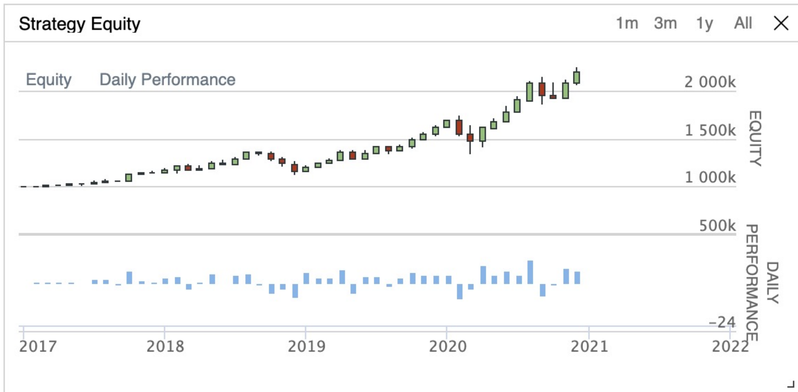

Figure 27: 2016.1.1-2017.1.1 Strategy Equity

- Backtesting link

4.1.2 OUT OF SAMPLE PERIOD 2: 2016.1.1 - 2017.1.1

Figure 28: 2016.1.1-2017.1.1 Strategy Equity

- Backtesting link

4.1.3 Out of Sample Period 1: 2022.1.1 - 2022.11.1

Figure 29: 2022.1.1-2022.11.1 Strategy Equity

- Backtesting link

4.1.4 Out of Sample Period 3: 2023.3.10 - 2023.10.10

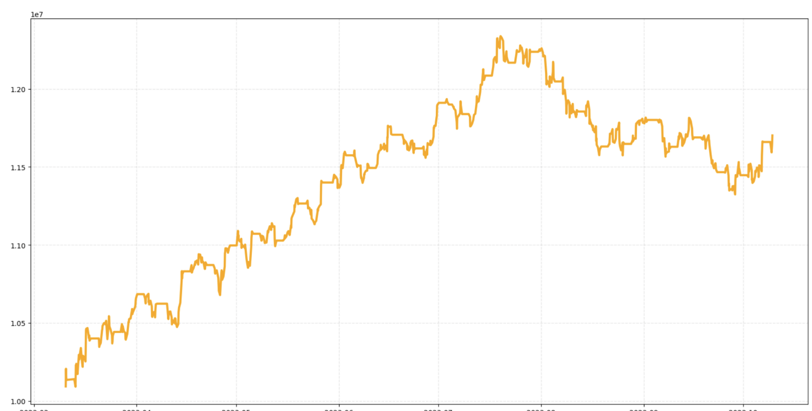

Figure 30: 2023.3.10-2023.10.10 Strategy Equity

- Backtesting link

4.1.5 STRESS TESING PERIOD: 2020.3.1-2020.4.1

Figure 31: 2023.3.10-2023.10.10 Strategy Equity

- Backtesting link

4.1.6 LIVE TRADING2023.12.7-2023.12.12

Figure 32: 2023.3.10-2023.10.10 Strategy Equity

	Sharpe Ratio	Max Drawdown	Compounding Annual Return
IS 2017.1.1-2021.1.1	0.997	21.500%	21.800%
OS 2016.1.1-2017.1.1	1.817	6.200%	17.497%
OS 2022.1.1-2022.11.1	-0.346	14.700%	-4.851%
OS 2023.3.10-2023.10.10	0.610	6.200%	4.788%
Stress Testing 2020.3	-1.438	1.000%	-6.759%

Table 6: A2C Trading Statistics

The above charts and table show that the A2C (Advantage Actor-Critic) trading strategy we implemented demonstrates robust overall performance, particularly in capitalizing on profit opportunities during favorable market trends. However, a notable aspect of this strategy is its relatively high maximum drawdown during periods of market volatility. This indicates that while the strategy is adept at identifying and leveraging profit-making opportunities in trending markets, it is somewhat vulnerable to significant losses during downturns. The drawdown experienced in such scenarios can negatively impact the overall performance of the portfolio. This characteristic underscores the need for further refinement of the strategy, possibly by integrating risk management techniques or adjusting the model parameters, to mitigate losses during adverse market conditions and enhance the strategy's resilience.

5 GPT-ENHANCED SENTIMENT STRATEGY

5.1 BACKGROUND

Sentiment analysis, a subfield of natural language processing (NLP), plays a crucial role in financial markets. It involves analyzing subjective information in text data to determine the prevailing sentiment towards a particular stock or the market as a whole. In stock trading, sentiment analysis is used to gauge investor sentiment, which can be a powerful indicator of market trends. Positive sentiment typically correlates with rising stock prices, while negative sentiment often precedes a decline in stock values. This relationship stems from the behavioral economics principle that investor sentiment can significantly influence market movements. By effectively analyzing market sentiment, traders can gain insights into potential market shifts, allowing for more informed trading decisions.

The advent of artificial intelligence (AI), particularly through machine learning and natural language processing, has significantly transformed sentiment analysis in stock trading. This technology allows for the rapid processing and analysis of extensive textual data, crucial in fast-paced financial markets. ChatGPT, a product of this AI evolution and a variant of the Generative Pre-trained Transformer model by OpenAI, exemplifies this transformation with its adeptness in context comprehension, nuanced interpretation, and efficient sentiment analysis, making it a valuable tool for extracting insights from financial news and reports.

The use of sentiment analysis in stock trading is not new Zhang & Skiena (2010), but the integration of advanced models like ChatGPT is a relatively recent development. Previous approaches primarily relied on simpler text analysis techniques, such as keyword counting and basic natural language processing methods. These earlier models often struggled with the complexities of human language, such as sarcasm, idioms, and contextual meaning.

Recent studies and applications have demonstrated the effectiveness of more advanced AI models in sentiment analysis for finance Kazemian et al. (2016). For instance, research has shown that AI-driven sentiment analysis can predict stock market movements with a higher degree of accuracy compared to traditional methods. The integration of AI tools like ChatGPT in this domain is a promising development, offering a more nuanced and sophisticated analysis of market sentiment.

This project aims to leverage the capabilities of ChatGPT to build a stock trading strategy informed by AI-driven sentiment analysis. By doing so, it seeks to contribute to the growing body of work at the intersection of AI, sentiment analysis, and financial trading.

5.2 DATA COLLECTION

The primary data source for this project is a collection of news articles related to Apple Inc., obtained from Tiingo, a financial data platform. This data was accessed and compiled using Quantconnect, an algorithmic trading platform that provides an interface for data retrieval and strategy implementation. The choice of Apple as the focus of this study stems from its status as a major player in the technology sector and its significant impact on market trends.

The process involved setting up a data pipeline in Quantconnect to regularly query and retrieve news articles from Tiingo that mentioned Apple. This setup ensured a consistent flow of up-to-date information, which is crucial for real-time sentiment analysis. The data collected included various attributes of each news article, such as the publication date, title, and full text content. Emphasis was placed on ensuring the comprehensiveness and reliability of the data, as the accuracy of sentiment analysis heavily depends on the quality of the input data.

5.3 ChatGPT Usage

For the sentiment analysis part of the project, ChatGPT, accessed through the OpenAI API, was utilized. The integration of ChatGPT involved developing a series of prompts to efficiently process the news data and output a sentiment classification. Each news article was fed into ChatGPT, which was then tasked with analyzing the text and categorizing the sentiment as a sentiment score between 1 and 10.

The process began by refining the prompts to ensure that ChatGPT could understand and interpret the context of the financial news accurately. This refinement involved testing different phrasing and structures to find the most effective way to elicit a clear and accurate sentiment analysis from the AI. Once the optimal prompt structure was established, it was used consistently across all data to maintain uniformity in the analysis process.

Incorporating ChatGPT for financial news sentiment analysis in trading offers several distinct advantages, aligning with the core requirements of modern financial analytics. Here's an explanation highlighting the four key merits:

- **Training-Free Application:** Utilizing ChatGPT for financial news sentiment analysis offers significant cost benefits. Its training-free feature is particularly advantageous, as it removes the need for extensive resource allocation towards data collection, annotation, and model training. The pre-trained nature of ChatGPT means that it's ready to be deployed without incurring the substantial costs typically associated with developing a sentiment analysis model from scratch.
- **High Accuracy:** ChatGPT, with its deep learning foundations, demonstrates high accuracy in understanding and interpreting the nuances of financial news. Its ability to process complex language structures and context makes it particularly adept at deciphering the subtleties of financial news sentiment. This high level of accuracy is crucial in trading, where understanding the sentiment behind financial news can lead to more informed and profitable trading decisions.
- **Customization:** ChatGPT's flexibility shines in its ability to be customized via prompt engineering. This involves crafting specific prompts or queries that guide the AI to analyze financial news in a way that aligns with particular trading strategies or market focuses. Prompt engineering allows traders to tailor the model's output to their specific needs without altering the underlying model architecture. This method of customization is not only practical but also efficient, enabling users to extract tailored insights from the AI without the need for complex technical adjustments or retraining of the model.
- **Interpretability:** One of the significant challenges in using advanced AI models for financial analysis is the 'black box' nature of these systems. ChatGPT, however, offers a level of interpretability that is often lacking in other models. It can provide explanations and reasoning behind its analysis, offering traders insights into how it arrived at a particular sentiment assessment. This interpretability is invaluable for traders who need to understand the rationale behind the AI's analysis to make informed trading decisions.

5.4 Prompt Engineering

The prompt designed for ChatGPT in this project reflects several key techniques in prompt engineering, tailored to optimize the AI's performance in analyzing the sentiment of financial news specifically about Apple Inc Zhou et al. (2022) Liu & Chilton (2022) White et al. (2023). The prompt is as follows:

"You will work as a Sentiment Analysis Expert for Financial News for Apple, focusing on financial indicators such as earnings, market trends, and investor opinions. Your answer will include 2 lines. In the first line, you will answer 1 sentence to analyze why the news is good or bad for Apple. Then, in the second line, you will answer with an integer between 1 and 10, with 1 being most BEARISH, 10 being most BULLISH."

- **Role Assignment:** The prompt begins by assigning a specific role to ChatGPT – that of a "Sentiment Analysis Expert for Financial News for Apple." This technique, known as role-playing, helps in guiding the AI to adopt a specific perspective or expertise, in this case, focusing on financial sentiment analysis.
- **Focus on Relevant Factors:** The prompt explicitly directs ChatGPT to focus on "financial indicators such as earnings, market trends, and investor opinions." This specificity ensures that the AI concentrates on the most relevant aspects of the news that are likely to impact sentiment, leading to more accurate and focused analysis.
- **Structured Response Format:** By instructing ChatGPT to provide its response in two distinct lines, with the first line dedicated to an explanatory sentence and the second to a one-word sentiment classification, the prompt ensures clarity and conciseness in the AI's responses. This structured approach aids in the efficient extraction and subsequent analysis of the sentiment data.
- **Analytical Reasoning:** The requirement for ChatGPT to include a rationale in its first line ("analyze why the news is good or bad for Apple") pushes the AI to not only classify the sentiment but also to provide a brief explanation for its assessment. This encourages a deeper level of processing and understanding, yielding more insightful and justified sentiment analysis.
- **Clear Sentiment Labels:** The use of distinct, unambiguous sentiment labels (1-10) in the second line of the response helps in standardizing the output, making it easier to categorize and use in subsequent trading strategy algorithms.

This carefully engineered prompt plays a crucial role in the success of the sentiment analysis process. By providing clear instructions and a structured format, it enables ChatGPT to efficiently process complex financial news and output sentiment data that is both insightful and consistent. This approach aligns with best practices in prompt engineering, leveraging the capabilities of ChatGPT for specialized tasks in a domain as nuanced as financial sentiment analysis.

5.5 EDA on Sentiment Analysis

Correlation Analysis between News Sentiment and Stock Returns The core of the EDA involved a detailed correlation analysis to uncover relationships between the sentiment of Apple news (as classified by ChatGPT) and the subsequent performance of Apple's stock in the short term. This analysis focused on 1-hour return following the publication of a news article. The aim was to identify whether and how sentiment indicators could predict stock price movements within these timeframes.

Methodology To conduct this analysis, each news article's sentiment rating (1-10) was paired with the stock's return data at the specified intervals after the news was published. Stock returns were calculated as the percentage change in the stock price from the time of the news release to the end of each interval. This data was then used to compute correlation coefficients, providing a statistical measure of the relationship between news sentiment and stock movement.

Key Findings One of the most intriguing patterns emerged with '2' and "6 sentiment signals. Contrary to initial expectations, it was observed that when the sentiment analysis classified news as '6,' there was a tendency for the stock price to increase in the following 60 minutes. On the other hand, when the sentiment analysis classified news as '2,' there was a tendency for the stock price to decrease in the following 60 minutes. This pattern suggests a potential behavior of the market in response to neutral and bad news, possibly indicating that investors may interpret neutrality as a positive sign in certain contexts.

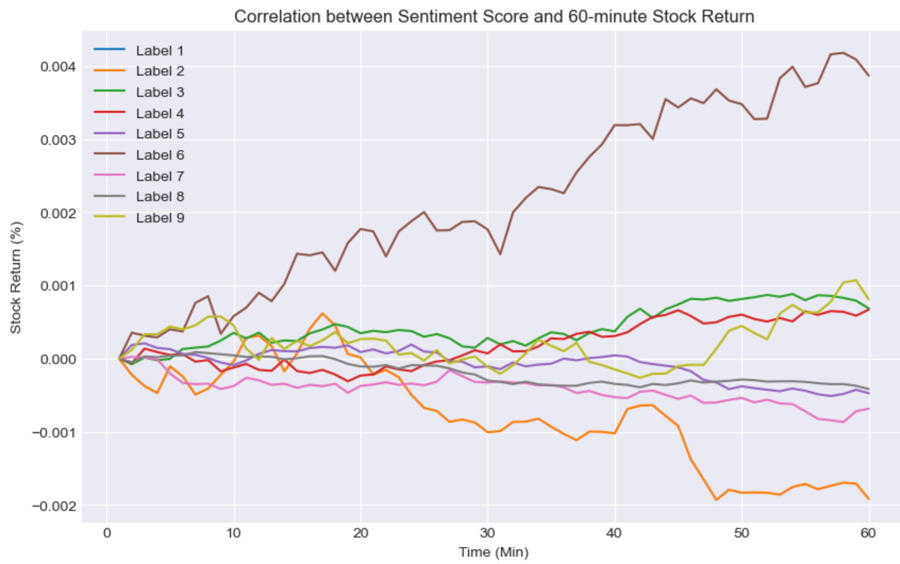

Figure 33: Average Return in 60 minutes for sentiment score

Interpretation and Implications The discovery of the neutral sentiment correlation over a 60-minute period offers a potentially valuable insight into investor behavior and market dynamics. It raises questions about the market's interpretation of 'neutral' news, suggesting that in the absence of strongly positive or negative cues, investors may lean towards optimism or consider other factors not captured by sentiment analysis.

The mixed results in shorter timeframes highlight the challenges of using sentiment analysis for immediate trading decisions. They also underscore the importance of considering multiple factors in trading strategies, as sentiment alone may not be a reliable predictor of short-term stock movements.

5.6 Strategy Formulation

The trading strategy for this project was formulated based on the key patterns and correlations identified in the exploratory data analysis (EDA) phase. The strategy leverages the sentiment analysis of Apple news articles to make informed trading decisions, with specific actions tied to the sentiment classification (positive, negative, or neutral) provided by ChatGPT.

Strong Positive Signals (6): When a news article about Apple is classified as having a positive (6) sentiment, the strategy involves taking a long position in Apple stock for 60 minutes. This counterintuitive approach is informed by the EDA findings, where slightly positive sentiment correlate with immediate raise in stock price, suggesting a delayed market reaction or other compensating factors.

Strong Negative Signals (2): Conversely, when a news article is classified as negative (2), the strategy is to long the stock for 60 minutes. This decision is based on the observed tendency for the stock price to react negatively in a short time frame following negative news.

Slight Positive Signals(3, 4): For news articles classified as slightly negative, the strategy calls for longing the stock for 60 minutes. This decision is based on the observed pattern where slightly negative sentiment often preceded an increase in the stock price after 60 minutes. Since the trend is not as significant as 6, we only apply a 40% weight on these signals. We add these 2 kinds of signals due to the preference to longing a stock.

5.7 Backtest Results and Addressing Look-ahead Bias

5.7.1 Addressing Look-ahead Bias

A critical consideration in this project was the avoidance of look-ahead bias, a common pitfall in financial modeling where future information is inadvertently used in making past decisions. To address this, a specific version of ChatGPT, trained only on data available up to September 2021, was employed for sentiment analysis. This precaution ensured that the AI model did not have access to any information or trends that emerged after September 2021, thereby eliminating the risk of look-ahead bias.

The backtesting of the trading strategy was conducted on data from January 2022 onwards. This approach created a clear temporal separation between the training data of ChatGPT and the period over which the trading strategy was tested. By doing so, the project adhered to a rigorous standard of temporal integrity, ensuring that the sentiment analysis and subsequent trading strategy were based solely on information that would have been available to investors at that time. This methodological rigor enhances the validity and reliability of the backtesting results, providing a more accurate representation of the strategy's potential performance in real-world trading scenarios.

5.7.2 GPT strategy In Sample: 2022.1-2022.12

Figure 34: GPT strategy In Sample

- Backtesting link

5.7.3 GPT strategy Out of Sample: 2023.1-2023.11

Figure 35: GPT strategy Out of Sample

- Backtesting link

5.7.4 GPT strategy result conclusion

	Sharpe Ratio	Max Drawdown	Compounding Annual Return
GPT IS 2022.1.1-2022.12.31	0.378	4.900%	5.929%
GPT OS 2023.1.1-2023.12.1	0.728	4.200%	11.467%

Table 7: Pairs Trading Statistics

6 Market Making Strategy

6.1 Avellaneda Stoikov's market-making strategy Avellaneda & Stoikov (2008)

Avellaneda and Stoikov introduced a quantitative strategy for market making based on stochastic control theory. This approach optimizes the placement of buy and sell orders in a limit order book, considering factors such as inventory risk and adverse selection. In this section, we want to replicate it and see whehter it works.

Price Dynamics The mid-price of an asset is modeled as a Brownian motion:

$$dp_t = \sigma dW_t \tag{9}$$

where p_t is the mid-price at time t, σ represents the volatility, and dW_t is the Wiener process.

Inventory Risk The market maker's inventory risk is modeled using a quadratic cost function:

$$C(q_t) = \gamma q_t^2 \tag{10}$$

where $C(q_t)$ is the cost function, γ is the risk aversion parameter, and q_t is the inventory at time t.

Optimal Bid and Ask Prices The optimal bid and ask prices are determined by:

$$b_t = p_t - \delta(q_t) \tag{11}$$

$$a_t = p_t + \delta(q_t) \tag{12}$$

where b_t and a_t are the bid and ask prices, respectively, and $\delta(q_t)$ is a function of the inventory level.

Spread Function The spread function $\delta(q_t)$ is derived from solving the Hamilton-Jacobi-Bellman equation, indicating the trade-off between earning the spread and controlling inventory risk. We can get the reference price and best spread by solving that euqation. For reservation price:

$$r(s, q, t) = s - \gamma q \sigma^2 (T - t)$$

For the optimal bid and ask spread:

$$\delta^a + \delta^b = \gamma \sigma^2 (T - t) + \frac{2}{\gamma} \ln\left(1 + \frac{\gamma}{k}\right)$$

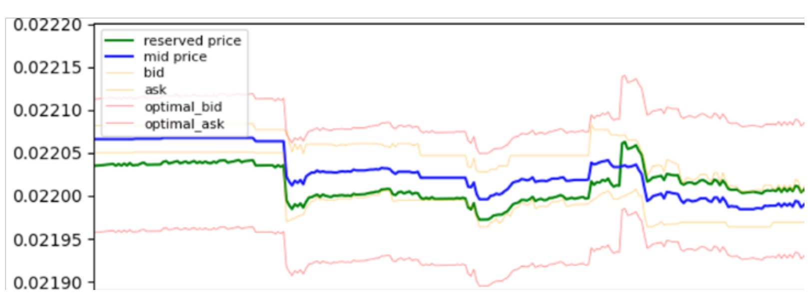

Figure 36: AS Model Demo

6.1.1 BACKTEST RESULTS

6.1.2 WITHOUT TRANSACTION COST

Figure 37: AS Model without Transaction Fee

6.1.3 WITH TRANSACTION COST

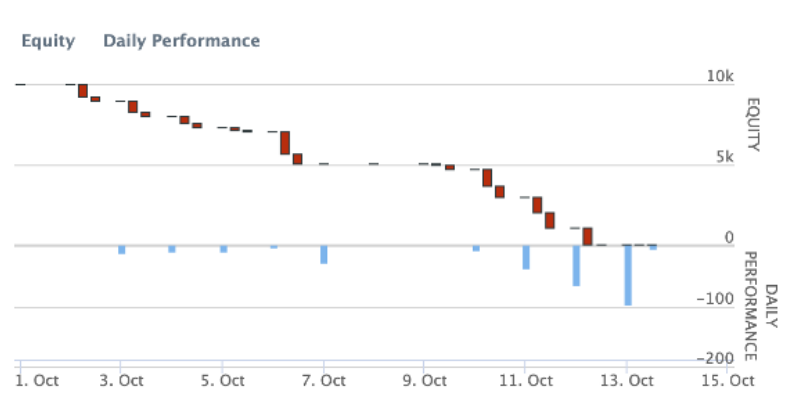

Figure 38: AS Model with Transaction Fee

6.2 AVELLANEDA-STOIKOV-QI MODEL

Guéant et al. (2013) The ASQ model, or the Avellaneda-Stoikov-Qi model, is an extension of the original Avellaneda-Stoikov (AS) model for market-making strategies. While the AS model primarily focuses on optimizing the spread based on inventory risk and market volatility, the ASQ model introduces additional considerations such as order queue dynamics. Specifically, the ASQ model incorporates the probability of order execution based on an agent's position in the limit order book, providing a more nuanced view of market microstructure. This enhancement allows for a more detailed analysis of the order execution process, considering the time-varying nature of liquidity and the impact of other market participants. In essence, while the AS model offers a foundational framework for market-making under inventory risk, the ASQ model expands this framework to include the dynamics of order queues, making it more applicable to real-world high-frequency trading environments where order book position is crucial.

Modified Optimal Bid and Ask Prices The optimal bid (b_t) and ask (a_t) prices, accounting for the queue dynamics, are given by:

$$b_t = p_t - \delta(q_t, \theta_t) \tag{13}$$

$$a_t = p_t + \delta(q_t, \theta_t) \tag{14}$$

where p_t is the mid-price, q_t is the inventory level, and $\delta(q_t, \theta_t)$ is the dynamically adjusted spread that now includes the queue position dynamics represented by θ_t.

Queue Position Dynamics The queue position dynamics, denoted by θ_t, influence the spread function δ, indicating the complexity of real-world order execution:

$$\delta(q_t, \theta_t) = \alpha e^{-\beta\theta_t} + \gamma q_t^2 \tag{15}$$

where α, β, and γ are model parameters, θ_t represents the queue position dynamics, and q_t is the inventory level. This function suggests that the spread increases with inventory level and decreases as the position in the queue improves.

Probability of Order Execution The probability of order execution, which is central to the ASQ model, can be mathematically represented, though its specific formulation varies based on the market microstructure and model calibration. It typically involves modeling the order fill probability as a function of queue position and other market factors.

6.2.1 Backtest Results

6.2.2 Without transaction cost

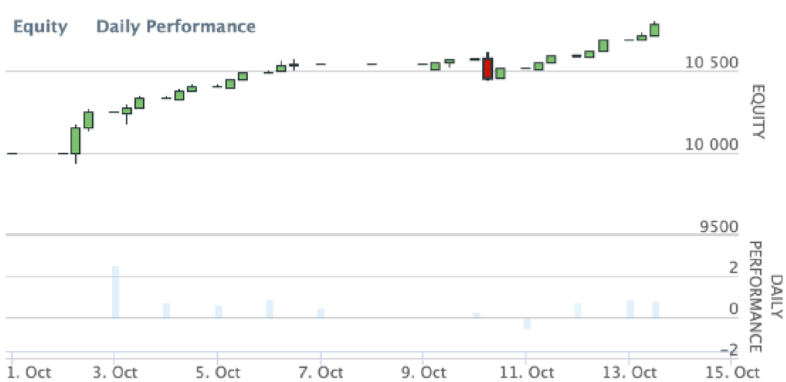

Figure 39: ASQ Model without Transaction Fee

6.2.3 With transaction cost

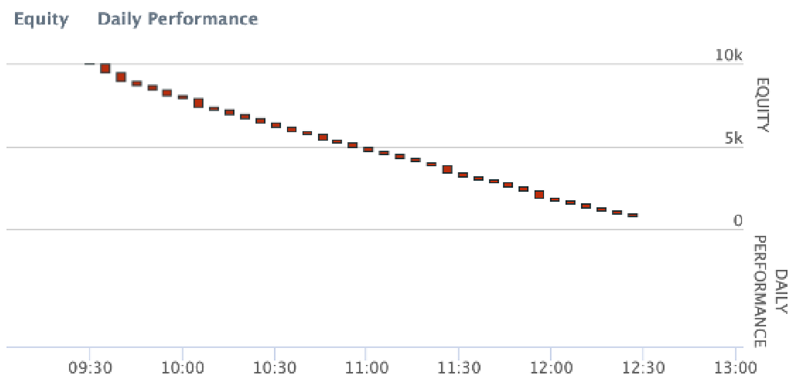

Figure 40: ASQ Model with Transaction Fee

6.3 Discussion

As evidenced by our analysis, high-frequency market making can indeed be profitable, particularly in scenarios without transaction costs. However, it's crucial to note the model's pronounced sensitivity to costs, such as transaction fees, and slippage — these factors can significantly erode potential profits. Additionally, the extensive time invested in backtesting the high-frequency market-making strategy has had a considerable impact on the progress of our project. Given the substantial resources required and the potential risks associated with such a strategy, we are considering the possibility of excluding it from our portfolio. This decision is driven by a need to balance profitability with the practical constraints and inherent uncertainties of high-frequency trading.

7 Slippage and Margin Model

7.1 Brokerage Margin Model

In aligning our portfolio management strategy with realistic trading conditions, we adopted margin requirements and transaction fee structures based on the Interactive Brokers model. This approach ensured our portfolio complied with industry-standard margin requirements, providing a more accurate reflection of actual brokerage constraints. By emulating this widely-recognized brokerage framework, we aimed to enhance the practical relevance of our strategy's performance and risk assessment, accounting for factors such as leverage limitations and the cost implications of trading activities. This methodological choice underscores our commitment to developing a strategy that is not only theoretically sound but also viable in the real-world trading environment.

7.2 Slippage Model

The Volume Share Model, as outlined, is chosen for its nuanced approach to capturing market impact. This model scales the slippage by the square of the order's proportion of total volume, which is a realistic reflection of how large orders can disproportionately affect market prices – a phenomenon well-understood by seasoned traders.

By implementing a minimum function that considers the lesser of the order quantity over bar volume and a predefined volume limit squared, we're acknowledging and bounding the influence of an order. This prevents the model from overestimating the impact of an exceptionally large trade in a high-volume market, ensuring that our slippage estimates remain grounded and practical.

Moreover, the use of custom input variables for volume limit and price impact allows us to tailor the model to specific asset characteristics and market conditions. It's adaptable – crucial when dealing with diverse asset classes and the varying liquidity profiles inherent to them.

In essence, the Volume Share Model is selected because it mirrors the complex and often non-linear nature of market dynamics, a testament to our commitment to accuracy and realism in trade execution simulation.

8 Risk Management

We manage downward risk by integrating the risk management framework at Quantconnect.

8.1 Operational Risk Management

In our application of the Quantconnect Platform's Portfolio Framework, we encapsulated various strategies as alpha models, each representing distinct predictive signals for market opportunities. The allocation of weights across these strategies was governed by a portfolio construction model, which was tasked with the crucial role of adhering to specified leverage constraints and margin requirements. This model ensured that the collective leverage of the portfolio did not exceed the predetermined leverage limit, thus maintaining compliance with risk management mandates and preserving capital adequacy. By integrating these components within Quantconnect's robust infrastructure, we achieved a harmonized system that combines the orders from multiple alpha models while controlling for overall portfolio risk.

9 Portfolio Optimization

9.1 In-Sample Performance Analysis

After finishing the whole in-sample backtesting, we find the correlation among our three selected strategies is low.

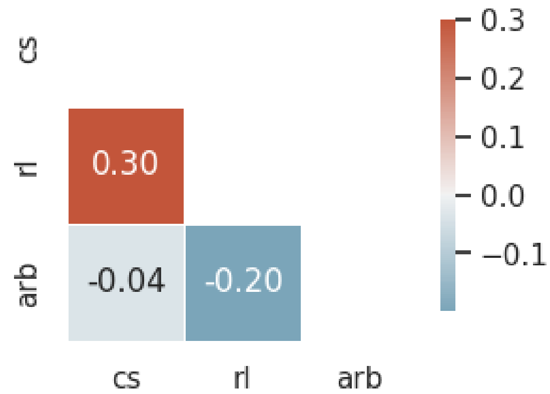

Figure 41: cs: AVE strategy; rl: A2C strategy; arb: Pairs Trading

Based on the diverified return performance, we organize them together as a portfolio to manage the downward risk.

9.2 Return and Risk Attribution

At the beginning of the backtesting, we assign equal weights to each strategies.

	AVE Strategy	A2C Strategy	Pairs Trading	Portfolio
Return Attribution	0.26	0.31	0.089	0.67
Risk Attribution	0.19%	0.20%	0.05%	0.44%

According to the table, we can find that the pairs trading attributes the lowest risk into the portfolio but A2C strategy attribute the most risk. However, in terms of return, Paris trading attribute the lowest return, A2C attribution the highest return. To manage the risk, we need to trade off between risk and return. In there, we utilize the Risk Parity Portfolio Optimization Approach to get the optimal weight.

9.3 Optimization of Portfolio Risk Using Risk Parity Approach

We utilized a risk parity approach for strategic reallocation to manage and control overall portfolio risk. The optimization process for achieving an equal risk contribution among all assets involved the following steps:

1. **Volatility Estimation**: Calculate the historical volatility σ_i for each asset i, which is a standard measure of risk.
2. **Correlation Matrix**: Construct a correlation matrix $\mathbf{C}$ where each element c_{ij} represents the correlation between assets i and j.
3. **Risk Contribution Calculation**: Determine the individual risk contribution RC_i of each asset, defined as $RC_i = w_i \cdot \sigma_i \cdot \sum_{j=1}^{N} w_j \cdot c_{ij}$, where w_i is the weight of asset i.
4. **Optimization Algorithm**: Utilize a numerical optimizer to solve the following problem:

$$\min_{\mathbf{w}} \left\{ \sum_{i=1}^{N} \left(RC_i - \frac{1}{N} \right)^2 \right\}$$

subject to the constraints:

$$\sum_{i=1}^{N} w_i = 1,$$
$$w_i \geq 0 \quad \forall i \in \{1, \ldots, N\}.$$

5. **Leverage Management**: Adjust the portfolio weights to achieve the targeted risk parity condition by applying leverage to the assets accordingly.

These steps are formulated to distribute risk equally among assets, ensuring a balanced portfolio that minimizes idiosyncratic risk and maximizes diversification benefits.

9.4 Portfolio Performance

9.4.1 Equal Weights Asset Allocation

	Sharpe Ratio	Max Drawdown
IS 2017.1.1-2021.1.1	1.34	17.14%
OS 2016.1.1-2017.1.1	1.27	4.97%
OS 2022.1.1-2022.11.1	0.84	10.08%
OS 2023.3.10-2023.10.10	1.37	5.5%

Table 8: Portfolio Statistics

9.4.2 Risk Parity Asset Allocation

	Sharpe Ratio	Max Drawdown
IS 2017.1.1-2021.1.1	1.34	14.76%
OS 2016.1.1-2017.1.1	1.13	4.97%
OS 2022.1.1-2022.11.1	1.12	4.97%
OS 2023.3.10-2023.10.10	1.41	4.64%

Table 9: Portfolio Statistics

By comparing the above tables, we can find that risk parity portfolio construction is effective in reducing downside risk. As a result, our Max Drawdown has been reduced significantly. We will use the risk parity portfolio optimization method as a way to construct our portfolio in the future.

- Backtesting Link

10 Discussion

Turning to deep learning models, while their underlying concepts hold considerable promise, the prerequisite of extensive feature engineering prior to data ingestion signifies a substantial investment of time. Absent a robust set of features, the model's predictive potency wanes significantly. Moreover, computing features in real-time as new data slices become available imposes a non-trivial computational burden.

Given these considerations, we advise against the deployment of large-scale deep learning models for the task of cross-sectional return prediction, particularly within platforms like QuantConnect that do not offer dynamic universe selection in their research environments. Such models necessitate the storage of voluminous dynamic data, potentially leading to prohibitive memory consumption.

The A2C (Advantage Actor-Critic) trading algorithm in our strategy showcases significant strengths, particularly in capturing profit opportunities during favorable market trends. This is evidenced by the impressive Sharpe Ratio, nearly 2, during the 2016-2017 period, indicating superior risk-adjusted returns compared to many standard benchmarks. Such performance underscores the algorithm's efficacy in leveraging market upswings to maximize gains.

While proficient in capitalizing on favorable market trends, exhibits notable vulnerabilities during volatile market conditions, often resulting in considerable drawdowns. This susceptibility largely stems from the risk of overfitting due to its complex parameterization, leading the model to misinterpret market noise as significant trends, thus diminishing its predictive accuracy in new market scenarios. Future improvements may involving regularization to sharpen the model's ability to differentiate between genuine market trends and noise, and to manage risks more effectively during downturns.

Our findings also suggest that within the domain of portfolio optimization, the allocation weights assigned to various strategies critically influence their performance outcomes. A singular strategy might execute trades unfeasible for those with lesser weight allocations, resulting in observable discrepancies in performance across a multi-strategy framework.

Lastly, the application of leverage is an aspect warranting meticulous exploration. Leverage can serve as a double-edged sword, amplifying profits when judiciously applied, yet it also escalates risk. Future research endeavors might productively explore optimal leverage ratios, seeking to maximize profit margins while maintaining an acceptable risk profile.

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A Appendix

A.1 Stock Selection from Pairs Trading

- ETF:
  VDE, USO, XES, XOP, UNG, ICLN, ERX, UCO, AMJ, BNO, AMLP, TAN, GLD, IAU, SLV, GDX, AGQ, PPLT, NUGT, JNUG, QQQQ, QQQ, IGV, QTEC, FDN, FXL, TECL, SOXL, SKYY, KWEB, IEF, SHY, TLT, IEI, TLH, BIL, SPTL, TMF, SCHO, SCHR, SPTS, GOVT, SPLV, UVXY, EEMV, EFAV, USMV, XLB, XLE, XLF, XLI, XLK, XLP, XLU, XLV, XLY, SPY, MDY, IEV, EEM, ILF, VIXY, ERY, SCO, DGAZ, DUST, JDST, TECS, SOXS, SHV, TBT, TBF, TMV, SVXY, SQQQ, TQQQ, VGSH, VGIT, VGLT Serivces:
  AAPL, MSFT, INTC, IBM, ORCL, CSCO, QCOM, TXN, ACN, AVGO
- Communication Serivces:
  GOOCV, FB, SBC, BEL, DIS, CMCSA, CHTR, NFLX, PCS, BCE
- Financial Service:
  WFC, JPM, BRKB, NOB, NB, V, MA, RY, GS, TD, USB

- Basic Material:
  DOW, LYB, ECL, PX, APD, CRHCY, PPG, SHW , PCU, FCX, IYM
- Health Care Sector:
  JNJ, PFE, MRK, UNH, AMGN, ABBV, MDT, BMY, GILD, WAG
- Utility Sector:
  FPL, DUK, SO, D, PE, AEP, SRE, EIX, PPL, ED, PEG, NSP, WEC, DTE, NU
- Energy Sector:
  XON, SLB, IPPIF, P, EOG, SU, EPD, OXY, PSX, CED