Variance reduction

Variance reduction is crucial in stochastic estimation and optimization problems.

In statistics, the antithetic variates method is a variance reduction technique used in Monte Carlo methods. Considering that the error in the simulated signal (using Monte Carlo methods) has a one-over square root convergence, a very large number of sample paths is required to obtain an accurate result. The antithetic variates method reduces the variance of the simulation results.

The antithetic sampling and stratification can be combined. For example, suppose that we are interested in estimating

$$\mu = E[h(U)]$$

where U is a uniform random variable on [0, 1].

Then antithetic sampling uses

$$\hat{h}(U) \doteq \frac{1}{2}[h(U) + h(1-U)]$$

as its estimate.

Therefore antithetic sampling is equivalent to simulating

$$\mu = E[\hat{h}(U)].$$

Now, in this code, we can use stratification method to further improve the performance.

```
r = 0.05;
sigma = 0.2;
S0 = 50;
K = 50;
T = 1;
N = 10000;
k1 = 25;
[v1, se1] = combo(r, sigma, S0, K, T, k1, N)
```

```
v1 = 5.2237
se1 = 0.0119
```

```

k2 = 100;
[v2, se2] = combo(r, sigma, S0, K, T, k2, N)
```

```
v2 = 5.2232
se2 = 0.0061
```

Functions

Combine antithetic sampling and stratification.

```
function [v, se] = combo(r, sigma, S0, K, T, k, N)
```

```
    n = zeros(1, k);
    for i = 1:k
        n(i) = 1/k * N; % use proportional allocation, p_i=1/k
    end
    mu = zeros(1, k);
    s = zeros(1, k);
    for i = 1:k
        h_of_U = zeros(1, n(i));
        for j = 1:n(i)
            V = rand;
            U = (i-1)/k + V/k;
            S1 = S0*exp((r-1/2*sigma^2)*T + sigma*sqrt(T)*norminv(U));
            X = exp(-r*T)*(max(S1-K, 0));
            S2 = S0*exp((r-1/2*sigma^2)*T - sigma*sqrt(T)*norminv(1-U));
            Y = exp(-r*T)*(max(S2-K, 0));
            h_of_U(j) = (X+Y)/2;
        end
       mu(i) = 1/n(i) * sum(h_of_U);
       s(i) = sqrt(1/(n(i)-1) * sum((h_of_U - mu(i)).^2));
    end
    v = 1/k * sum(mu);
    se = 1/k * sqrt(sum((1./n).*(s.^2)));

end
```