Midterm 1 fixed

Wednesday, September 27, 2017 8:20 PM

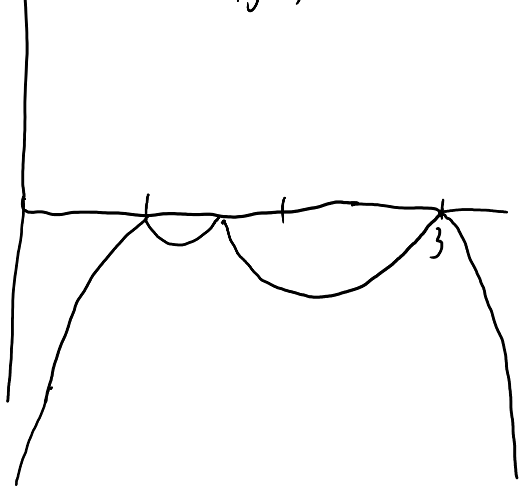
6) Since $q(x_{low})$ = negative #
$q(x_{high})$ = negative #.

bisection Cannot proceed

We will not be able to bracket a root, because we don't know if a root exists.

$r(x) = -|(x-1)(x-1.4)(x-3)|.$

$r(x_{low}) = r(0)$ = negative
$r(high) = r(2.4)$ = negative



x_{low} or x_{high}

We can't update because we don't know if we bracketed a root because x_{low} and x_{high} are both same sign.

The difference between bisection on p(x) and bisection on r(x) is that for p(x) we might be able to bracket a root because there are positive and negative values. However for r(x) all possible range is negative so we will never know whether we should update x_low, or x_high.

Question 7.

1.xlow = 0.0 , xhigh =10.5
f(0.0) = 5, positive.
f(10.5) = -3, negative.
Since the brackets are opposite sign, we can proceed with the bisection algorithm.
xnew = (0+10.5)/2 = 5.25
f(5.25) =5.
Since xnew is positive, we update the xlow to be xnew.
New bracket xlow = 5.25, xhigh = 10.5
(5.25+10.5)/2 = 7.875 = 7.875 =xnew
f(7.875) = negative, we update xhigh to be 7.875.
We see that there is no root between x=[5.25,7.875]
So the algorithm will get stuck at the discontinuity at 6.

2. Xlow =0.0 , xhigh = 5.5
   f(0.0) = positive , f(5.5) = positive,
   Since both end of bracket is same sign(positive),
   The algorithm cannot proceed.

3. Xlow = 1.0 , xhigh = 9.5
   f(1.0) = 1, f(9.5) = -1(negative)
   There's 2 possibilities,
   if the tolerance is <=1, we have bracketed the root at xlow =1.0.
   Else, the xnew = (1+9.5)/2=5.25
   f(5.25) = positive, new bracket = [5.25,9.5].
   Xnew = (5.25 +9.5)/2=7.375
   f(7.375) = negative, we update xhigh = xnew
   New bracket = [5.25,7.375]
   By inspection, we see that there is no root between [5.25,7.375]
   However it will converge to the discontinuity at 6

4.xlow = 2, xhigh = 12.
f(2) = -1, f(12) = -7
Since both is negative, the bisection cannot proceed.

5.xlow = 2.5, xhigh = 8
f(2.5) = -1,negative, f(8) = 0.
With the intial conditon check, we would not start the bisection algorithm, because we have already bracketed the root at xhigh =8.

6.xlow =3, xhigh= 5.5
f(3) = -1 negative, f(5.5) = 7 positive,
xnew = (3+5.5)/2=4.25
f(4.25) = positive , new bracket =[3,4.25]
As we can see, the only root we can bracket is Beta.
So the algorithm will bracket beta.

7.xlow = 4, xhigh =6.5
f(4) = 1 positive, f(6.5) = negative.
Xnew = (4+6.5)/2=5.25 positive
Xlow = 5.25
[5.25,6.5] , this algorithm will converge to discontinuity at 6.

8.xlow = 3, xhigh = 300000.
f(3) = negative, f(300000) = negative,
The bisection will not proceed.

9. Xlow = -49.5, xhigh = 6.5
   f(-49.5) = positive, f(6.5) = negative.
   Xnew = (-49.5 +6.5)/2=-21.5
   f(-21.5) = positive, new bracket [-21.5, 6.5]
   Xnew = (-21.5 + 6.5)/2=-7.5
   f(-7.5) = positive,[-7.5,6.5]
   Xnew = (-7.5+6.5)/2=-0.5
   f(-0.5) = positive ,[-0.5,6.5]
   Xnew = (-0.5 + 6.5)/2=3
   f(3) = negative , xhigh = 3,
   New bracket = [-0.5,3]
   Now the only root between the bracket is alpha, so it will
   Converge to alpha.

Goal: find xlow such that [xlow,7] will bracket root alpha.
2.any choice of xlow such that 2<= xlow <6 will not work because the initial bracket is [2<=xlow<6 , 7]. But the root alpha is before x=2. so we will never bracket alpha since the beginning.

3.any of xlow such that 6<xlow<7 will not work because of 2 reason. One reason is that f(xlow) is negative, f(7) is negative, so bisection can't proceed. Another reason is because the bracket will not bracket root alpha, because root alpha at around x=1.5

4. When xlow is less than equal to 1(xlow <=1)
   the bracket [xlow <=1 , 7] will bracket root alpha, root beta, and discontinuity at 6.
   Because root alpha occur at about x=1.5
   Root beta occur at about x = 3.6
   Discontinuity occur at x= 6.
   All 3 x's are bounded by the bracket [xlow<=1, 7]

5. let:

$$-3 \le x_{low} \le -1 \qquad x_{high} = 7$$

$$x_{mid} = \frac{(x_{low} + x_{high})}{2}$$

(-2.5 +7)/2=2.25
(-1.5 +7)/2=2.75

x_{low}	x_{mid}
-3.0	$\frac{-3+7}{2} = 2$
-2.5	$\frac{-2.5+7}{2} = 2.25$
-2.0	$\frac{-2+7}{2} = 2.5$
-1.5	$\frac{-1.5+7}{2} = 2.75$
-1.0	$\frac{-1+7}{2} = 3$

(a) $2 \le x_{mid} \le 3$

(b) Xhigh will be updated because

f(2< xmid <3) = always negative. So xhigh will update because xhigh is negative.

(c) only one root (alpha) will be bracketed because
[-3 <=xlow <=-1 , 2<= mid <= 3] will bracket only root alpha at about x= 1.5

6. When we select xlow such that -3 <=xlow <=-1, and xhigh to be 7.
   f(xlow) will be positive.
   f(7) will be negative.
   Now our goal is to move the bracket closer (another way of saying it is to decrease the bracket size).
   Choosing -3 <=xlow <=-1 will make sure the f(xlow) is always positive.
   We want

$$x_{mid} \leq 3 \leq \frac{x_{low} + 7}{2}$$

$$3(2) = x_{low} + 7$$

$$-1 \leq x_{low}$$

So as long as xlow<= -1, the xmid will always be <=3. because if xhigh greater than 3, lets say 4, root beta is also bracketed In there. Also we want to satisfy the condition that f(xmid is negative) so that the xhigh will update to xmid , because we don’t want to update xlow.