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SUBMITTING STUDENT		
SURNAME Dearlove	GIVEN NAMES Cole	STUDENT NUMBER 22896684
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DATE/TIME DUE		DATE/TIME SUBMITTED

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FOR GROUP ASSIGNMENTS ONLY		STUDENT NUMBER
NAME		
1.	Himanshu Rana	22975507
2.	Qinming Wu	22861414
3.	Cole Dearlove	22896684
4.	Cameron	22968675
5.	Richard Wu	22581235
6.		
7.		
8.		

Unless other arrangements have been made it will be assumed that all group members have contributed equally to group assignments/laboratory reports

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ENSC2004 – Engineering Mechanics

Statics Laboratory: Truss Analysis and Failure

By:

Qinming Wu (22861414)
Cole Dearlove (22896684)
Cameron Nguyen (22968675)
Himanshu Rana (22975507)
Richard Wu (22581235)

Stream Number: 5

Lab Day and Time: Wednesday, 2pm - 4pm

Lab Facilitator: Liam

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1. Introduction

In this lab we look at three different truss types: Pratt, Howe and Warren. Each truss type differs through the different arrangement of members.

A Pratt truss (Fig. 1) consists of diagonal members that slope towards the middle with vertical members occurring between each diagonal. The internal diagonal members are in tension, while the vertical members are in compression. A Pratt truss is the best at dissipating

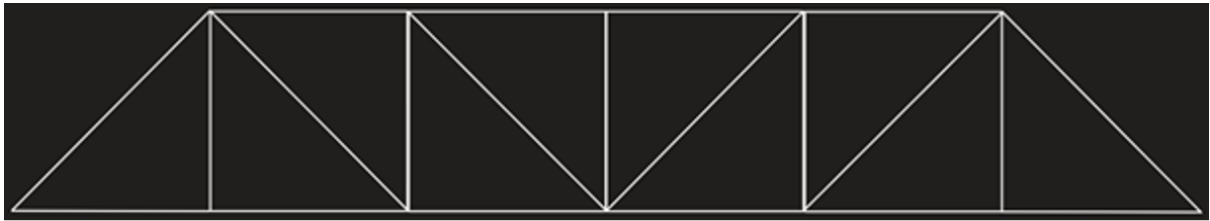


Figure 1. A simple diagram of a Pratt truss, with 21 members.

force out of all three Truss types. It is not the best truss if load is applied horizontally [1].

The Howe truss (Fig. 2) is very similar to the Pratt truss, but the diagonal slopes away from the middle. This also means that the diagonal members are in compression, and the vertical

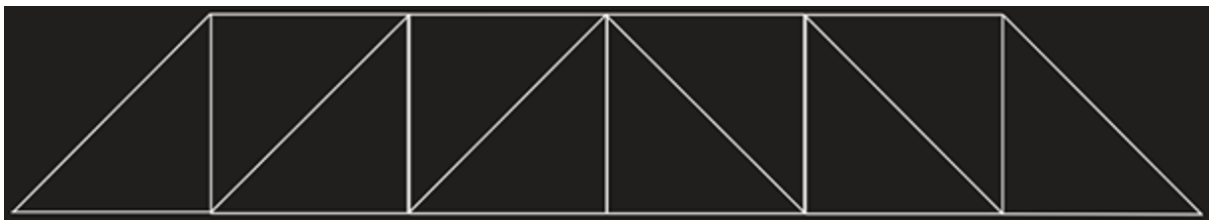


Figure 2. A simple diagram of a Howe truss, with 21 members.

members are in tension.

The Warren truss (Fig. 3) consists of equilateral triangles that alternate orientation in order to make the bridge. Due to all of its members being the same size and angle to each other the main advantage of the Warren truss is its ability to equally distribute the applied force across multiple members; however it works only with distributed loads. With point loads however the concentrated application of force makes the Warren truss ineffective and weak at single joints [1].

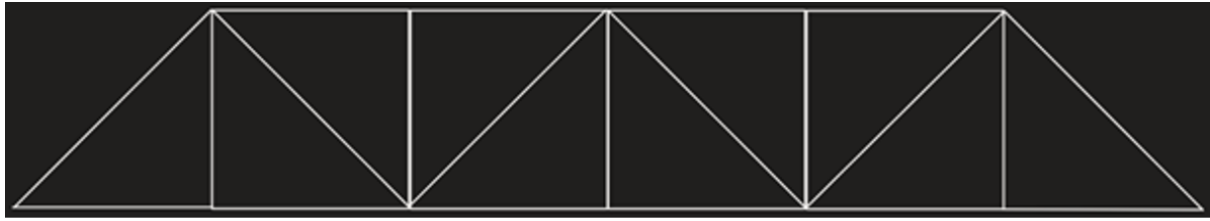


Figure 3. A simple diagram of a Warren truss, with 21 members.

1.1. Acronyms and Abbreviations

PTLL – Purely Theoretical Live Load

ATLL – Actual Theoretical Live Load

ELL – Experimental Live Load

NTSB – National Transportation Safety Board

2. Methodology

The truss type assigned was the Howe truss with dead load to be attached at joint D, and live load to be attached at joint E (Fig.1). The theoretical dead load assigned 50N. The critical is to be replaced with a sacrificial plastic member (PMMA).

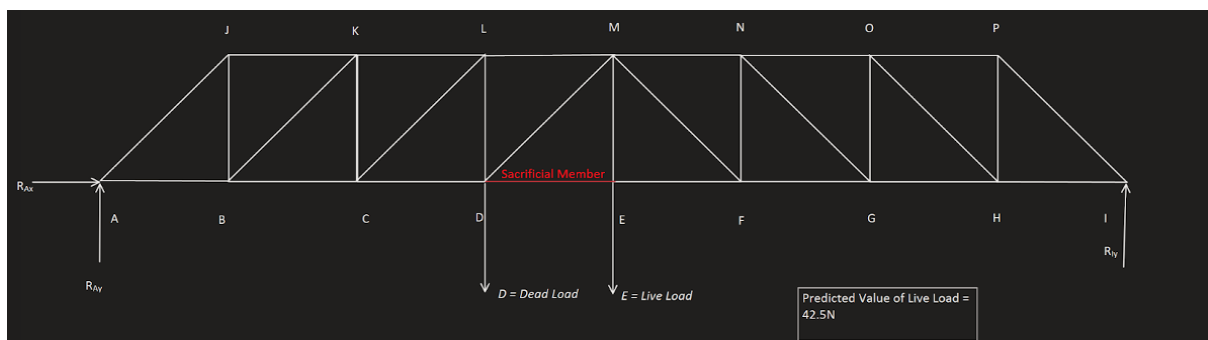


Figure 4. A FBD of the Howe truss our group analysed, with dead load, live load, and the sacrificial member.

3. Modelling

In this section of the report, the mathematical model of the Howe truss analysis will be built to support the conceptual understanding of the truss structure and evaluating the internal forces within the truss members. Specifically, the following processes will be detailed.

- The process of identifying the critical member
- The calculations of the Expected Failure Load

- The plots for horizontal and vertical truss members under tension

3.1. Identifying the Critical Member

The critical member in a truss is the one most likely to fail under certain mechanical conditions. To identify the position of critical members, the general strategy that has been adopted in the experiment is by analysing the internal force within each truss member. Since the truss is in equilibrium, the internal force can be represented as a function in terms of fixed dead load and changing dead load by applying the equations of equilibrium. The first truss member with internal force reaching the expected failure load is the suspected critical member. The complete 5-step process is detailed below:

1. The Free Body Diagram of the Howe truss

Based on the dimension, supporting condition and applying external forces of the Howe truss, the FBD with appropriate labelling has been drawn to assist analysis.

2. The identification of zero force members to simplify truss structure

To simplify the analysis, the second step is reducing the truss structure by removing the zero force members. Zero force members present in the manner of two members are connected without external load acts along either member or three members are connected in an orthogonal way with no external force. In the specific case of Howe truss with external force exerting on point D and E, no zero force members present.

3. The formation of internal force within each truss member as a function of live load and dead load.

By applying the equations of equilibrium at each joint of the truss, internal force functions are obtained to quantify the analysis.

4. The calculation of Expected failure load

The fourth step of the process is to determine the critical condition of failure, which is the expected failure load based on the nominal cross-sectional area of the specimen and the standard Stress-strain curve.

5. The interpretation of the internal forces with the Expected failure load

The terminal internal force within each truss member can be predicted from the functions obtained from step 3. The first truss member with internal force meeting the condition of expected failure load is the suspected critical member.

3.2. Calculations of the Expected Failure Load

a. The determination of failure point in Stress-Strain curve

Since the weight of live load is constantly increasing until the failure in the experiment, the ultimate tensile strength is adopted as a sensible failure point to be processed.

b. The calculation of the Expected failure load based on nominal dimension

1. Strategy: The presentation of used formula

$$\sigma = F / A \text{ (N / mm}^2\text{)}$$

2. Solution: the process of plugging in values and yielding out answer

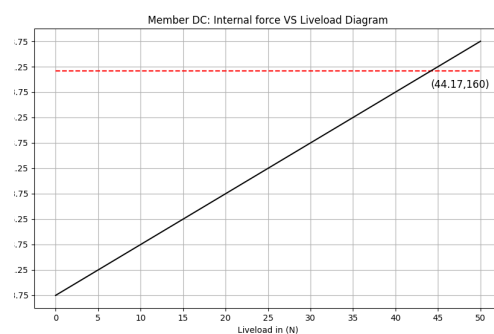
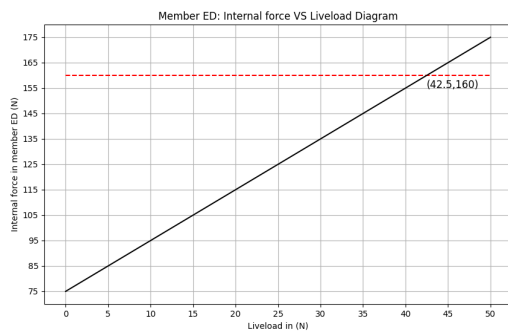
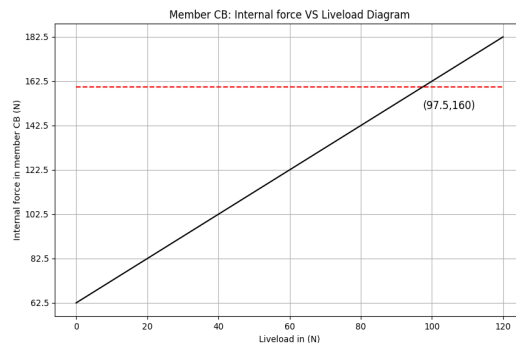
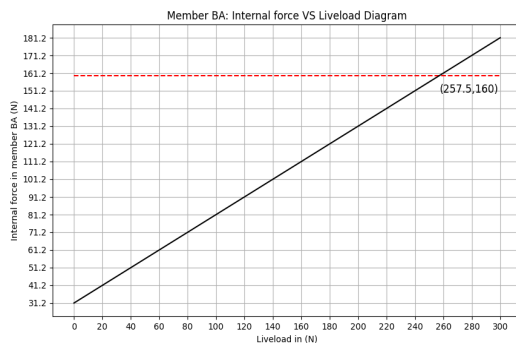
$$\sigma = 80 \text{ (N / mm}^2\text{)}$$

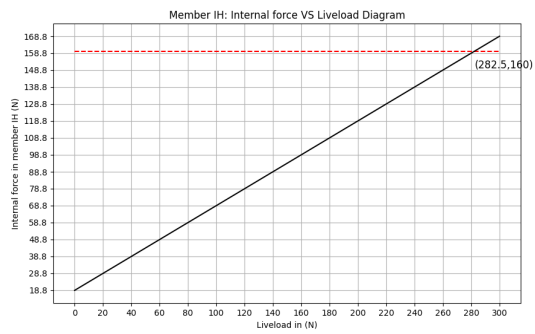
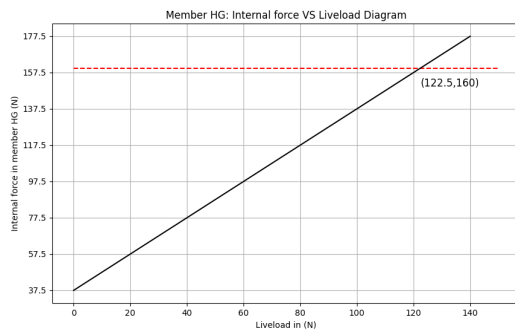
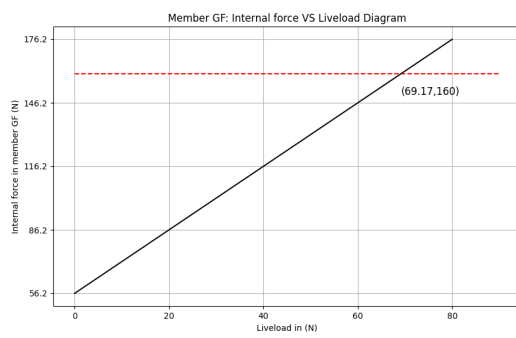
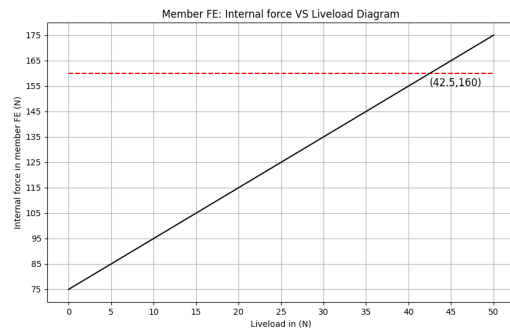
$$A = 2 \text{ (mm}^2\text{)}$$

$$F = \sigma \cdot A = 160 \text{ (N)}$$

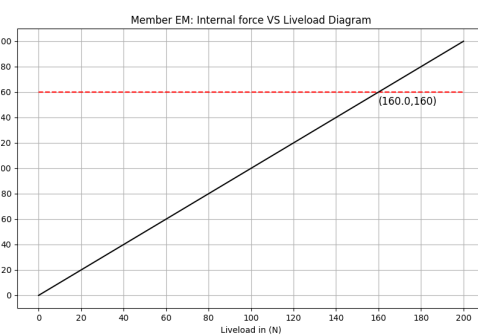
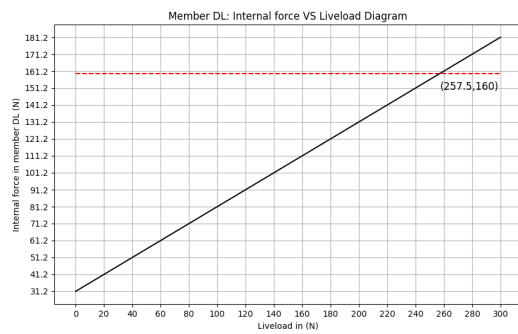
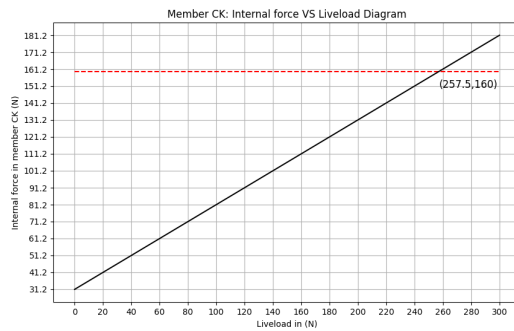
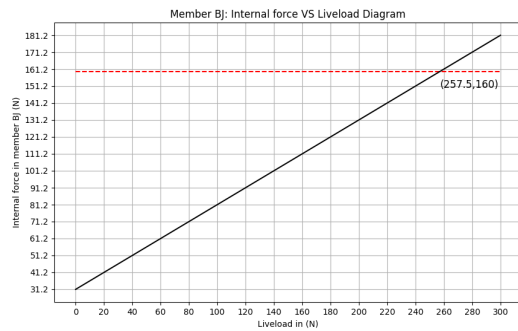
3.3. Plots for Horizontal and Vertical Truss Members in Tension

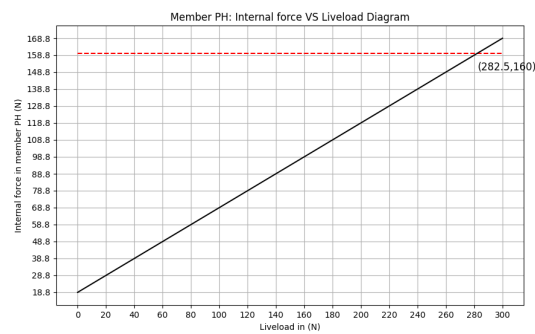
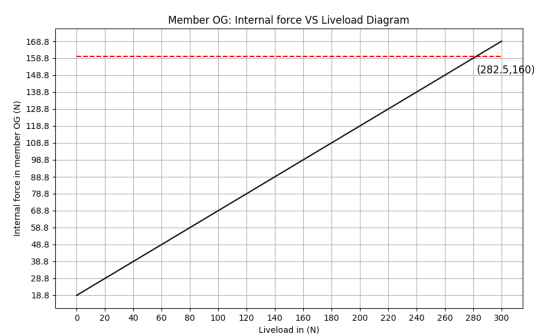
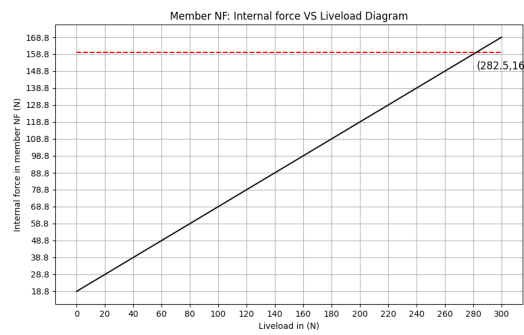
Section A: The plots for horizontal truss members under tension





Section B: The plots for vertical truss members under tension





4. Experiment

This section will detail the experiment and examine sources of uncertainty that may contribute to a difference between the theoretical and experimental results.

4.1. Experimental Procedure

The first step in the lab is to acquire a sacrificial PMMA member and use callipers to determine the cross-sectional area at the narrowest point along the member. Next, the PMMA member replaces the critical member in the truss, and the truss is delicately transported to the testing rig, making sure not to place the sacrificial member under compression to avoid premature buckling.

Once the truss is in the testing rig, carefully hang the 50N or 20N dead load as close to point D as possible and hang the bucket for the live load as close to point E as possible. Record the live load value you anticipate the sacrificial member to fail at. Slowly add sand to the live load bucket, being careful to keep clear of the underside of the truss at all times.

When the sacrificial member fails, detach the live load and measure the total mass of the live load and record the data. Estimate and comment on the uncertainty in the measurement of the live load.

4.2. Uncertainties

There are a few points of uncertainty that generated a larger overall uncertainty.

The callipers used to measure the cross-sectional area had a $\pm 0.2\text{mm}$ uncertainty, which resulted in the cross-sectional area having a $\pm 0.54\text{mm}$ (or 30.0%) uncertainty. This was our major source of uncertainty.

The expected failure load is another source of uncertainty. The given ultimate tensile strength (σ_{UTS}) did not have an uncertainty associated with it. As a result, the uncertainty in the expected failure load comes purely from the cross-sectional area. The expected failure load had a total uncertainty of $\pm 43.2\text{N}$ (or 30%).

Mass of the dead load, predicted live load, and experimental live loads introduce more uncertainty. The digital scale we used had an uncertainty of $\pm 0.0025\text{kg}$. The dead load had a total uncertainty of $\pm 0.05\text{N}$. The predicted live load had a total uncertainty of $\pm 22\text{N}$ (or $\pm 70.4\%$).

5. Discussion

The value our group calculated the maximum theoretical live load at the moment of failure to be 42.5N (based on the nominal values provided in the lab guide; “purely theoretical live load”) or $31 \pm 22\text{N}$ (or 31N with 70.4% percentage uncertainty) (based on the actual measurements done in the lab; “actual theoretical live load”). The value our group determined for the experimental live load at the moment of failure was $47.9 \pm 0.025\text{N}$ (or $47.9\text{N} \pm 0.05\%$).

Comparing the PTLL with the ELL, we find that the ELL is $1.13 \pm 0.05\%$ times larger than the PTLL.

Comparing the ATLL with the ELL, we find that the ELL is $1.55 \pm 71.0\%$ times larger than the ATLL.

Despite the ATLL being a more realistic representation of the expected live load, the PTLL is more accurate and precise.

The terminal live load (F_E) (5.1) is based on the dead load (F_D) and the total load in the critical member (F_{ED}), and F_{ED} (5.2) is based on ultimate tensile strength (σ_{UTS}) and the cross-sectional area (A) of the member in question.

$$F_E = \frac{1}{2}F_{ED} - \frac{3}{4}F_D - \quad (5.1)$$

$$F_{ED} = \sigma_{\text{UTS}} \times A - \quad (5.2)$$

Combining equations (5.1) and (5.2):

$$F_E = \frac{1}{2}(\sigma_{UTS} \times A) - \frac{3}{4}F_D - \quad (5.3)$$

From equation (5.3), we can see that the terminal live load is proportional to the cross-sectional area of the critical member:

$$F_E \propto A - \quad (5.4)$$

Thus the terminal live load increases as the cross-sectional area of the critical member increases.

Material properties refer to the mechanical and physical properties of a material [2] and predict how that material reacts under certain circumstances. Variations in material properties such as purity, tensile strength, and structure formation change the result of the experiment. If the specimen is impure (i.e. air bubbles), its strength may be compromised, and thus the terminal live load will be reduced. Increasing or decreasing tensile strength increases or decreases the terminal live load, respectively. If the specimen is formed too quickly, then the molecular structure of the specimen will be compromised, thus weakening the member and decreasing the terminal live load.

Assuming the ultimate tensile strength (σ_{UTS}) has an uncertainty of $\pm 15\%$, using (5.3), we can recalculate the uncertainty. The value(s) for the cross-sectional area remain the same; $1.8 \pm 0.54\text{mm}$ ($1.8\text{mm} \pm 30\%$). The value(s) determined from calculating the total load (F_{ED}) however, do change. Originally, there was no uncertainty in σ_{UTS} , resulting in the value(s) of $144 \pm 43.2\text{N}$ ($144\text{N} \pm 30\%$) for F_{ED} . With the $\pm 15\%$ uncertainty in σ_{UTS} , however, the values for F_{ED} become $144 \pm 64.8\text{N}$ ($144\text{N} \pm 45\%$). As a result, the terminal live load F_E becomes $31 \pm 32\text{N}$ ($31\text{N} \pm 104\%$). This tells us that the terminal live load is between 0N and 63N ; which is absurdly inaccurate.

In the experiment, the manual and theoretical calculations assumed the placement of the loads at the exact point (Point D for the dead load and point E for the live load). In practice, the loads were placed slightly off the point, due to the apparatus we used. As such, this would change the components of the forces experienced by the critical member, changing the terminal live load.

6. Case Study

In this case study, the I-5 Skagit River bridge located in Washington will be investigated to identify the causes of failure in engineering structures. This section specifically looks into what caused this bridge's detrimental failure in terms of its design, the impact that this failure

had on people and the environment, and finally what changes were made to engineering practice because of it.

6.1. Background

The bridge, between Mount Vernon and Burlington, Washington, is part of a road service between downtown Seattle and Vancouver. During its life, about 71,000 cars passed through it every day.

6.2. Causes of Failure

The NTSB released a preliminary report on the investigation of the bridge collapse on June 11, 2013. The report indicated that: “Just before the collapse, a Kenworth truck tractor and an aspen flatbed trailer with a casing shed (which is overloaded) were following a pilot vehicle south on Interstate 5. According to witnesses, when two cars approached the bridge, a large truck and a semi-trailer were overtaking on the road nearby, causing a huge load on the left (South) lane. The driver told the investigators that he felt the distance was not enough, so he drove the car to the right. When several heavy-duty vehicles pass through at the same time, the truss part on the top collides with the part of the viaduct door, and causes a certain degree of damage to the top structure, which eventually leads to the destruction of the northernmost bridge span and subsequent collapse.”

According to the data provided by the driver of the vehicle, the vehicle's clearance bar was 16 feet, 2 inches, higher than the 14 feet, 8 inches that researchers later detected in the lowest part of the bridge, which led to the accident [3].

6.3. Problems in Design and Construction Process

Sway braces are diagonal braces located at the top of a butted truss that resist horizontal forces, such as wind. Anti sway braces do not bear loads, but help to keep the components of the truss aligned. However, as pointed out in the report, the load-bearing members have been damaged, so the span has been reduced and the truck could not pass through [4]. At that time, bridge engineers used vernier calipers for measurement. As such, every calculation was very laborious, and there were few bridge engineers at that time. Thus, there were not enough designers to design the bridge as an uncertain structure.

6.4. Impact of the Failure on People and the Environment

Fortunately, the aftermath of this bridge failure only left a few people with minor injuries, and had a minimal impact on the environment. The bridge sits over the Skagit River in Washington that connects the highway between two cities, Mount Vernon and Burlington hence making it a very active and busy bridge having on average 71000 vehicles cross it daily [7]. The bridges collapse made detours hard to find as each one found consisted of unsafe

bridges and narrow roads that would lead to huge traffic jams on a busy highway. This huge inconvenience paused most of the travel and trading leaving the community in despair and irritation [7]. In addition to this, the bridge's failure caused most of the spans truss elements to fall into the water that ended staying there for a few days until it was cleaned up. The truss elements were primarily made from structural carbon and low alloy steels which both are quite corrosion resistant, meaning that the larger pieces of debris would not have made a significant impact to the environment considering the elements were only in the river for a short amount of time. As for the smaller pieces of debris that could not have been retrieved, were eventually spread out into the river which possibly could have a negative impact towards the wildlife and inevitably cause pollution as the materials continue to slowly corrode [6].

6.5. Subsequent Changes to the Engineering Practice

This significant bridge collapse led to new design features to be learnt and implemented into the fixed bridge which eventually made changes in the engineering of bridges. The I-5 skagit bridge collapse was a prime example of an outdated design which ultimately led to the collapse of it, the design features that were at fault were the elliptical overhang elements (Fig. 5) that made an uneven height difference from one lane to the other [5]. From this, a straight horizontal member was replaced for the old elliptical shaped members, which made a uniform maximum height along the width of the bridge that allowed tall trucks to safely change lanes. Another suggestion that would have made a beneficial change was to add a longitudinal member connecting all the overhanging elements so that the horizontal impact forces applied to a frame could be resolved into the truss [5]. This bridge's collapse led to many changes to be made within engineering practice as it proved that design features play a significant deal in the safety and strength of bridges [7].

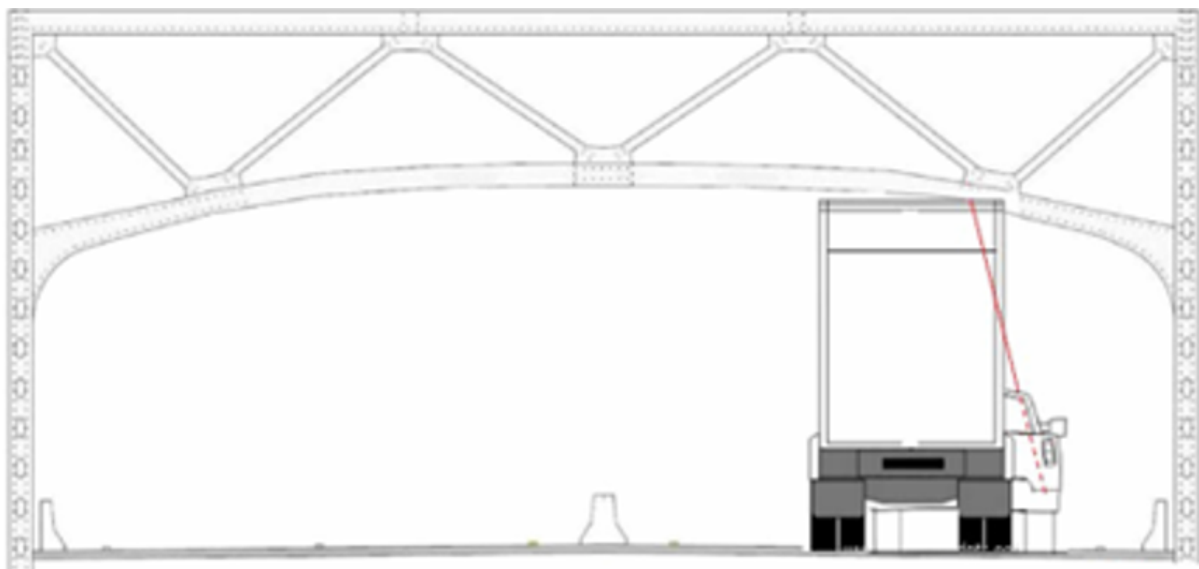


Figure 5. A cross-section of the bridge showing where the truck had impacted the bridge.

7. Appendix

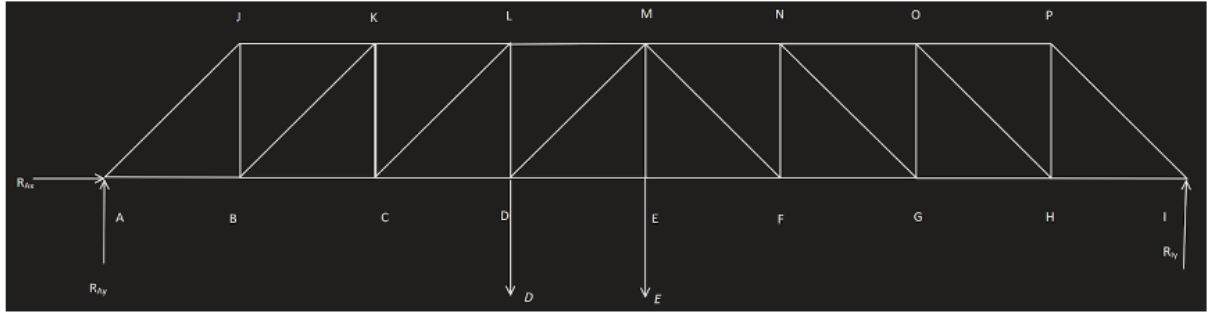


Figure 6. FBD for the Howe Truss under experimental conditions.

7.1. Detailed Calculations of the Reaction Forces

Strategy: $\rightarrow + \Sigma F_x = R_{Ax} = 0 \text{ (N)}$

$\uparrow + \Sigma F_y = R_{Ay} + R_{Iy} = 0 \text{ (N)}$

$\curvearrowright + \Sigma M_{(A)} = -D \cdot 3l - L \cdot 4l + R_{Iy} \cdot 8l = 0 \text{ (N)}$

Solution: $(R_{Ax}, R_{Ay}, R_{Iy}) = (0, \frac{5}{8}D + \frac{1}{2}E, \frac{3}{8}D + \frac{1}{2}E)$

7.2. Internal Force Functions of each Truss Member in terms of D and L

Joint I: Strategy: $\rightarrow + \Sigma F_x = -F_{IP} \cdot \cos(45^\circ) - F_{IH} = 0 \text{ (N)}$

$\uparrow + \Sigma F_y = R_{Iy} + F_{IP} \cdot \sin(45^\circ) = 0 \text{ (N)}$

Solution: $(F_{IH}, F_{IP}) = (\frac{3}{8}D + \frac{1}{2}E, \frac{-3\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E)$

Joint P: Strategy: $\rightarrow + \Sigma F_x = F_{PI} \cdot \sin(45^\circ) - F_{PO} = 0 \text{ (N)}$

$\uparrow + \Sigma F_y = -F_{PH} - F_{PI} \cdot \cos(45^\circ) = 0 \text{ (N)}$

Solution: $(F_{PO}, F_{PH}) = (\frac{-3}{8}D - \frac{1}{2}E, \frac{3}{8}D + \frac{1}{2}E)$

Joint H: Strategy: $\rightarrow + \Sigma F_x = F_{HI} - F_{HO} \cdot \cos(45^\circ) - F_{HG} = 0 \text{ (N)}$

$$\uparrow + \Sigma Fy = F_{HP} + F_{HO} \cdot \sin(45^\circ) = 0 (N)$$

Solution: $(F_{HD}, F_{HG}) = \left(\frac{-3\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E, \frac{3}{4}D + E\right)$

Joint O: Strategy: $\rightarrow + \Sigma Fx = F_{OP} + F_{OH} \cdot \sin(45^\circ) - F_{ON} = 0 (N)$

$$\uparrow + \Sigma Fy = -F_{OH} \cdot \cos(45^\circ) - F_{OG} = 0 (N)$$

Solution: $(F_{OG}, F_{PON}) = \left(\frac{3}{8}D + \frac{1}{2}E, \frac{-3}{4}D - E\right)$

Joint G: Strategy: $\rightarrow + \Sigma Fx = F_{GH} - F_{GF} - F_{GN} \cdot \cos(45^\circ) = 0 (N)$

$$\uparrow + \Sigma Fy = F_{GO} + F_{GN} \cdot \sin(45^\circ) = 0 (N)$$

Solution: $(F_{GN}, F_{GF}) = \left(\frac{-3\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E, \frac{9}{8}D + \frac{3}{2}E\right)$

Joint N: Strategy: $\rightarrow + \Sigma Fx = F_{NO} + F_{NG} \cdot \sin(45^\circ) - F_{NM} = 0 (N)$

$$\uparrow + \Sigma Fy = -F_{NG} \cdot \cos(45^\circ) - F_{NF} = 0 (N)$$

Solution: $(F_{NF}, F_{NM}) = \left(\frac{3}{8}D + \frac{1}{2}E, \frac{-9}{8}D - \frac{3}{2}E\right)$

Joint F: Strategy: $\rightarrow + \Sigma Fx = F_{FG} - F_{FM} \cdot \cos(45^\circ) - F_{FE} = 0 (N)$

$$\uparrow + \Sigma Fy = F_{FN} + F_{FM} \cdot \cos(45^\circ) = 0 (N)$$

Solution: $(F_{FM}, F_{FE}) = \left(\frac{-3\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E, \frac{3}{2}D + 2E\right)$

Joint E: Strategy: $\rightarrow + \Sigma Fx = F_{EF} - F_{ED} = 0 (N)$

$$\uparrow + \Sigma Fy = F_{EM} - E = 0 (N)$$

Solution: $(F_{EM}, F_{ED}) = (E, \frac{3}{2}D + 2E)$

Joint M: Strategy:

$$\rightarrow + \Sigma F_x = F_{MN} - F_{ML} + F_{MF} \cdot \sin(45^\circ) - F_{MD} \cdot \sin(45^\circ) = 0 \text{ (N)}$$

$$\uparrow + \Sigma F_y = -F_{MF} \cdot \cos(45^\circ) - F_{MD} \cdot \cos(45^\circ) - F_{ME} = 0 \text{ (N)}$$

Solution: $(F_{ML}, F_{MD}) = (\frac{-15}{8}D - \frac{3}{2}E, \frac{3\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E)$

Joint D: Strategy: $\rightarrow + \Sigma F_x = F_{DM} \cdot \cos(45^\circ) + F_{DE} - F_{DC} = 0 \text{ (N)}$

$$\uparrow + \Sigma F_y = F_{DL} - D + F_{DM} \cdot \sin(45^\circ) = 0 \text{ (N)}$$

Solution: $(F_{DL}, F_{DC}) = (\frac{5}{8}D + \frac{1}{2}E, \frac{15}{8}D + \frac{3}{2}E)$

Joint L: Strategy: $\rightarrow + \Sigma F_x = F_{LM} - F_{LK} - F_{LC} \cdot \sin(45^\circ) = 0 \text{ (N)}$

$$\uparrow + \Sigma F_y = -F_{LD} - F_{LC} \cdot \cos(45^\circ) = 0 \text{ (N)}$$

Solution: $(F_{LK}, F_{LC}) = (\frac{-5}{4}D - E, \frac{-5\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E)$

Joint C: Strategy: $\rightarrow + \Sigma F_x = F_{CD} - F_{CB} + F_{LC} \cdot \cos(45^\circ) = 0 \text{ (N)}$

$$\uparrow + \Sigma F_y = F_{CK} + F_{CL} \cdot \sin(45^\circ) = 0 \text{ (N)}$$

Solution: $(F_{CK}, F_{CB}) = (\frac{5}{8}D + \frac{1}{2}E, \frac{5}{4}D + E)$

Joint K: Strategy: $\rightarrow + \Sigma F_x = F_{KL} - F_{KJ} - F_{KB} \cdot \sin(45^\circ) = 0 \text{ (N)}$

$$\uparrow + \Sigma Fy = -F_{KC} - F_{KB} \cdot \cos(45^\circ) = 0 (N)$$

Solution: $(F_{KJ}, F_{KB}) = (\frac{-5}{8}D - \frac{1}{2}E, \frac{-5\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E)$

Joint B: Strategy: $\rightarrow + \Sigma Fx = F_{BC} - F_{BA} + F_{BK} \cdot \cos(45^\circ) = 0 (N)$

$$\uparrow + \Sigma Fy = F_{BJ} + F_{BK} \cdot \sin(45^\circ) = 0 (N)$$

Solution: $(F_{BJ}, F_{BA}) = (\frac{5}{8}D + \frac{1}{2}E, \frac{5}{8}D + \frac{1}{2}E)$

Joint J: Strategy: $\rightarrow + \Sigma Fx = F_{JK} - F_{JA} \cdot \sin(45^\circ) = 0 (N)$

$$\uparrow + \Sigma Fy = -F_{JB} - F_{JA} \cdot \cos(45^\circ) = 0 (N)$$

Solution: $F_{JA} = \frac{-5\sqrt{2}}{8}D - \frac{\sqrt{2}}{2}E$

7.3. The Sacrificial PMMA Member

7.3.1. Calculations of Force on the PMMA Member at Failure Based on the Measure Cross-Sectional Area and Tensile Strengths

Assumption:

Gravity: $g = 9.81 (N / Kg)$

Known:

Live load: $m_{bucket\ and\ sand} = 4.885 \pm 0.0025 (Kg)$

Dead load component 1: $m_{hook} = 0.450 \pm 0.0025 (Kg)$

Dead load component 2: $m_{weight} = 5.095 \pm 0.0025 (Kg)$

Specimen width: $l_{width} = 1.2 \pm 0.2 (mm)$

Specimen height: $l_{height} = 1.5 \pm 0.2 (mm)$

Find: The internal force within sacrificial PMMA member at failure

based on measured dimension and strength characteristics.

Strategy:

$$\text{Real dead load: } D_{real} = (m_{hook} + m_{weight}) \cdot g \text{ (N)}$$

$$\text{Real live load: } E_{real} = m_{bucket \text{ and sand}} \cdot g \text{ (N)}$$

$$\text{Real cross-sectional area: } A_{real} = l_{width} \cdot l_{height} \text{ (mm}^2\text{)}$$

Real failure internal force:

$$F_{real \text{ failure internal force}} = \frac{3}{2} D_{real} + 2 E_{real} \text{ (N)}$$

$$\text{Real UTS: } \sigma_{real \text{ UTS}} = \frac{F_{real \text{ failure internal force}}}{A_{real}} \text{ (N / mm}^2\text{)}$$

Solution:

$$\text{Real dead load: } D_{real} = 54.4 \pm 0.05 \text{ (N)}$$

$$\text{Real live load: } E_{real} = 47.9 \pm 0.02 \text{ (N)}$$

$$\text{Real cross-sectional area: } A_{real} = 1.8 \pm 0.5 \text{ (mm}^2\text{)}$$

Real failure internal force:

$$F_{real \text{ failure internal force}} = 177.4 \pm 0.115 \text{ (N)}$$

$$F_{real \text{ failure internal force}} \text{ is quoted as } 177.4 \pm 0.1 \text{ (N)}$$

$$\text{Real UTS is quoted as : } \sigma_{real \text{ UTS}} = 99 \pm 27 \text{ (N / mm}^2\text{)}$$

7.3.2. Determination of whether calculated UTS falls into $\pm 15\%$ range of the given UTS of the PMMA member

$$\text{The nominal UTS: } \sigma_{nominal \text{ UTS}} = 80 \text{ (N / mm}^2\text{)}$$

Given the $\pm 15\%$ range of the nominal UTS,

The upper bound of

$$\sigma_{nominal \text{ UTS max}} = \sigma_{nominal \text{ UTS}} \cdot (1 + 15\%)$$

$$\sigma_{nominal\ UTS\ max} = 92(N / mm^2)$$

The lower bound of

$$\sigma_{nominal\ UTS\ min} = \sigma_{nominal\ UTS} \cdot (1 - 15\%)$$

$$\sigma_{nominal\ UTS\ min} = 68(N / mm^2)$$

The range of real UTS: (72 , 126) (N / mm²)

The range of $\pm 15\%$ given UTS: (68 , 92) (N / mm²)

Approximate 38% real UTS falls into The range of $\pm 15\%$ given UTS.

7.4. Calculations of live load of the sacrificial truss member

$$\text{Real dead load: } D_{real} = (m_{hook} + m_{weight}) \cdot g (N)$$

$$\text{Real live load: } E_{real} = m_{bucket\ and\ sand} \cdot g (N)$$

$$\text{Real cross-sectional area: } A_{real} = l_{width} \cdot l_{height} (mm^2)$$

Real failure internal force:

$$F_{real\ failure\ internal\ force} = \frac{3}{2}D_{real} + 2E_{real} (N)$$

$$\text{Real dead load: } D_{real} = 54.4 \pm 0.05 (N)$$

$$\text{Real live load: } E_{real} = 47.9 \pm 0.02 (N)$$

$$\text{Real cross-sectional area: } A_{real} = 1.8 \pm 0.5(mm^2)$$

Real failure internal force:

$$F_{real\ failure\ internal\ force} = 177.4 \pm 0.115 (N)$$

$$F_{real\ failure\ internal\ force} \text{ is quoted as } 177.4 \pm 0.1 (N)$$

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