1 Normal Distribution

Example 1: Conduct 100 groups of coin toss experiments, 10 times per group, and record the number of heads. Represent the frequency of the 100 results (integers from 1 to 10) using a histogram.

- This is a "100 trials of 10-fold Bernoulli experiments."

Example 2: Estimate the average English test score of 5,000 freshmen by sampling 5 students each time, calculating the mean, repeating 1,000 times, and plotting the distribution of means in a histogram.

- This is a "1,000 trials of 5-fold Bernoulli experiments."

Conclusion: The results of a large number of n-fold Bernoulli experiments produce a normal distribution curve.

2 Hypothesis Testing

2.1 Basic Concepts

- **Scenario**: Determine whether there is a significant difference in the average English test scores between freshmen and sophomores.
- **Objective**: Determine whether there exists a statistically significant difference between the average college entrance examination scores in English for freshmen and sophomores, when complete academic records of the freshman cohort are unavailable.
- **Hypotheses**:
  - H_0 (Null Hypothesis): $\mu_1 = \mu_2$ (no significant difference)
  - H_1(Alternative Hypothesis): $\mu_1 \neq \mu_2$(significant difference exists)

2.2 Testing Process

1. Set the **significance level** $\alpha = 0.05$ (typically 0.01 or 0.001).
2. Determine the rejection region (critical region):
   - Two-tailed test: $\frac{\alpha}{2}$ on each side
   - One-tailed test: α on one side
3. Decision rule:
   - If a single sampling of freshmen yields a sample mean that falls within the extreme region (i.e., a statistically rare event occurs), this suggests potential issues with our initial hypothesis.
   - So, if the sample statistic falls in the rejection region ⇒ Reject H_0. Otherwise ⇒ Fail to reject H_0

3 t-Distribution and t-Test

To eliminate the necessity of performing repeated sampling for every hypothesis test, we standardize these distribution patterns.

3.1 t-Value Formula

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

Where:

- μ: Hypothesized population mean (e.g., sophomores' average score)
- $\overline{x}$: Sample mean
- s: Sample standard deviation
- n: Sample size

3.2 Example Test

Given $t = 1.77, n = 20, df = 19, \alpha = 0.05$ (one-tailed):

- The area where $t > 1.729$ is 0.05, which is called the p-value. If $p < alpha$, then reject H_0 ; otherwise, fail to reject H_0 .
- Critical value from table is 1.729. Since $1.77 > 1.729$ ⇒ Reject H_0

3.3 Definition of p-Value

"p-value is, under the assumption that the Null Hypothesis is True, the probability of obtaining test results At Least As Extreme As the Results actually observed."

3.4 Types of Hypothesis Tests

1. **One-Sample Two-tailed test**:
   - $H_0: \ \mu = x \quad H_1: \ \mu \neq x$
2. **One-Sample One-tailed test**:
   - $H_0: \ \mu \leq x \quad H_1: \ \mu > x$
   - $H_0: \ \mu \geq x \quad H_1: \ \mu < x$

4 Confidence Interval

4.1 Basic Concept

- **Definition**: An interval that contains the true parameter with a confidence level of $1 - \alpha$.
- **Example**: A 95 confidence interval corresponds to $\alpha = 0.05$.

4.2 Construction Method

Critical value formula:

$$X_{\text{critical}} = \mu_{\text{hypothesized population}} + t_{\text{critical}} \cdot \frac{s}{\sqrt{n}}$$

4.3 Interpretation

If infinite samples are drawn from the same population and confidence intervals are constructed:

- $(1-\alpha)\%$ of the intervals will contain the true population mean.
- A specific confidence interval has a $(1-\alpha)\%$ probability of containing the true parameter.
- https://seeing-theory.brown.edu/frequentist-inference/cn.html#section2

4.4 Application

When using a sample mean (point estimate) to estimate the population mean, the confidence interval provides the possible range of variation for the estimate.