01-D2

绪论

复杂度分析：迭代

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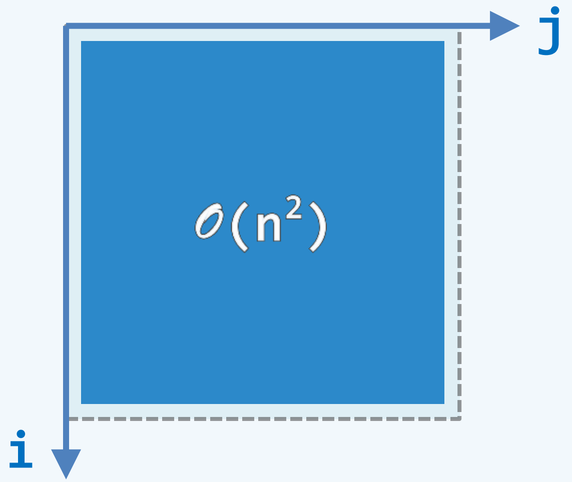
Go To Statement Considered Harmful.

- E. Dijkstra, 1968

迭代 + 算术级数

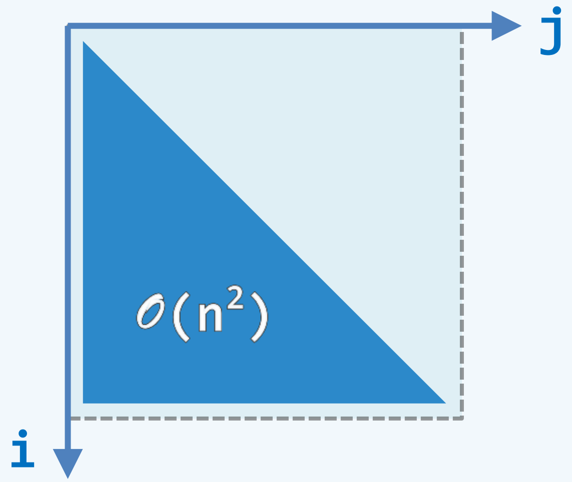
```
for( int i = 0; i < n; i++ )
  for( int j = 0; j < n; j++ )
    O1op(const i, const j);
```

$$\sum_{i=0}^{n-1} n = n \times n = \mathcal{O}(n^2)$$




```
for( int i = 0; i < n; i++ )
  for( int j = 0; j < i; j++ )
    O1op(const i, const j);
```

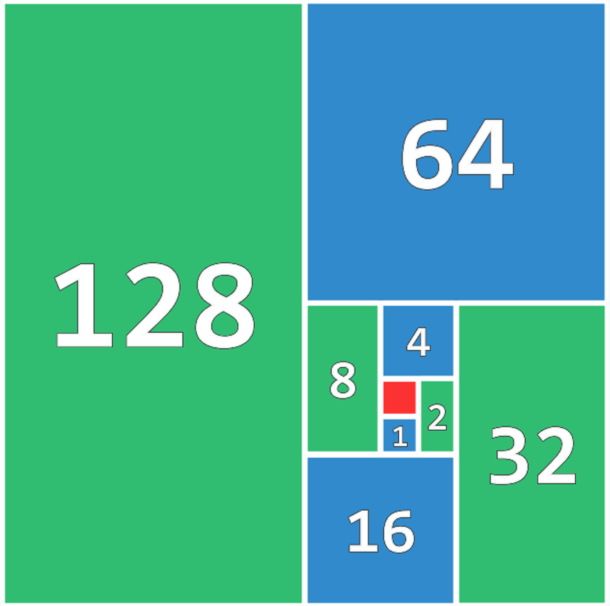
$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2} = \mathcal{O}(n^2)$$



迭代 vs. 级数

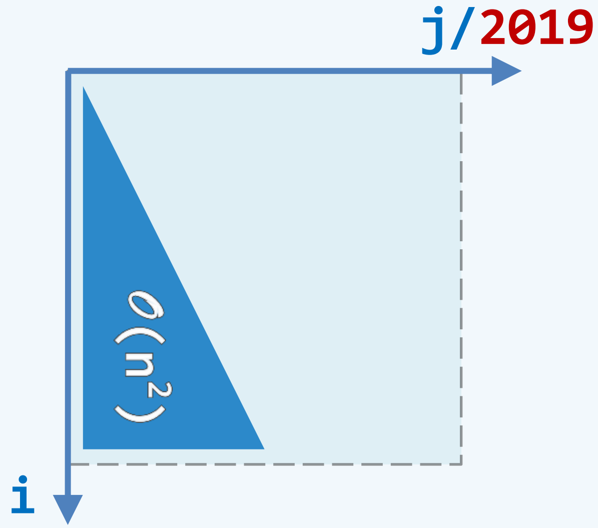
```
❖ for( int i = 1; i < n; i <<= 1 )
    for( int j = 0; j < i; j++ )
       O1op( const i, const j );
```

```
❖ for( int i = 0; i < n; i++ )
    for( int j = 0; j < i; j += 2019 )
       O1op( const i, const j );
```



$$1 + 2 + 4 + \cdots + 2^{\lfloor \log_2 (n-1) \rfloor}$$

$$= \ 2^{\lceil \log_2 n \rceil} - 1 \ = \ \mathcal{O}(n)$$



迭代 + 复杂级数

```
❖ for( int i = 0; i <= n; i++ )
    for( int j = 1; j < i; j += j )
      O1op( const i, const j );
```

$$T(n) = \sum_{i=0}^{n} \lceil \log_2 i \rceil = \mathcal{O}(n \log n)$$

$$T(n) \approx \int_{x=0}^{n} \ln(x)dx = n \cdot \ln(n) - \int_{x=0}^{n} x \cdot d(\ln(x)) = n \cdot \ln(n) - n$$

$$T(n) = \sum_{k=1}^{\log n} k \cdot 2^{k-1} = \sum_{k=1}^{\log n} \sum_{i=1}^{k} 2^{k-1} = \sum_{i=1}^{\log n} \sum_{k=i}^{\log n} 2^{k-1}$$

$$T(n) \leq \sum_{i=1}^{\log n} \sum_{k=1}^{\log n} 2^{k-1} \leq \sum_{i=1}^{\log n} 2^{\log n} = \sum_{i=1}^{\log n} n = n \cdot \log n$$

$$T(n) \geq \sum_{i=1}^{\log n} \sum_{k=\log n}^{\log n} 2^{k-1} \geq \sum_{i=1}^{\log n} 2^{\log n - 1} = \sum_{i=1}^{\log n} n/2 = n/2 \cdot \log n$$