Branching model in spatial spread of COVID-19

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1 Motivation

Empirical studies have found transmission is highly variable from person to person for several respiratory viruses, including SARS-CoV-2.

2 Model

2.1 Branching process

Galton-Watson branching process

Galton-Watson branching process (GWB) is a stochastic process that describes the growth of a population in which offspring are generated at random. The number of offspring of each individual is a random variable with a fixed distribution. The offspring of each individual are independent of each other. The population size at time t is denoted by $N(t)$. The population size at time $t+1$ is given by

$$N(t+1) = \sum_{i=1}^{N(t)} X_i, \tag{1}$$

where X_i is the number of offspring of individual i at time t. The distribution of X_i is assumed to be fixed and independent of i. The population size at time t is a random variable with a distribution $P(N(t))$. The distribution of $N(t)$ is given by.

Age-dependent branching process

Age-dependent branching process (ADBP) is a generalization of the GWB. The number of offspring of each individual is a random variable with a distribution that depends on the age of the individual. The distribution of X_i is given by

$$P(X_i = x) = \sum_{j=1}^{x} \frac{1}{j} \pi_j, \tag{2}$$

where π_j is the probability that an individual of age j has x offspring. The distribution of $N(t)$ is given by

$$P(N(t)) = \prod_{i=1}^{N(t)} \sum_{x=1}^{\infty} \frac{1}{x} \pi_x P(X_i = x). \tag{3}$$

Bellman-Harris branching process

Bellman-Harris branching process (BHBP) is a special case of the ADBP. The number of offspring of each individual is a random variable with a negative binomial distribution. The distribution of X_i is given by

$$P(X_i = x) = \frac{1}{x} \frac{\Gamma(x+r)}{\Gamma(x+1)\Gamma(r)} (1-p)^r p^x, \tag{4}$$

where p is the probability that an individual has x offspring and r is the dispersion parameter. The distribution of $N(t)$ is given by

$$P(N(t)) = \prod_{i=1}^{N(t)} \sum_{x=1}^{\infty} \frac{1}{x} \frac{\Gamma(x+r)}{\Gamma(x+1)\Gamma(r)} (1-p)^r p^x. \tag{5}$$

Negative binomial distribution

In the branching process model, the number of infections caused by each infected person (i.e., secondary infections) is represented by a negative binomial distribution $\mathcal{NB}(R_0, r)$ with a mean reproductive number R_0 and a dispersion parameter $r \in (0, +\infty)$. For a fixed , a smaller means a larger variation in secondary infections, that is, rarer but more explosive superspreading events. By varying the dispersion parameter , we can control individual transmission heterogeneity in the model.

A sequence of independent random variables $X_1, X_2, \ldots$ is said to have a negative binomial distribution with parameters r and p if the probability mass function is given by

$$f(x) = \binom{x+r-1}{r-1} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots \tag{6}$$

where $\binom{x+r-1}{r-1} = \frac{(x+r-1)!}{x!(r-1)!}$ is the binomial coefficient. The mean and variance of the distribution are given by

$$\mathbb{E}[X] = \frac{rp}{1-p}, \quad \mathrm{Var}[X] = \frac{rp}{(1-p)^2}. \tag{7}$$

$$P(X_i = x) = \frac{1}{x} \frac{\Gamma(x+r)}{\Gamma(x+1)\Gamma(r)} (1-p)^r p^x, \tag{8}$$

where $p = R_0/(R_0+1)$ and r is the dispersion parameter. The distribution of $N(t)$ is given by

$$P(N(t)) = \prod_{i=1}^{N(t)} \sum_{x=1}^{\infty} \frac{1}{x} \frac{\Gamma(x+r)}{\Gamma(x+1)\Gamma(r)} (1-p)^r p^x. \tag{9}$$

2.2 Spatial spread

Qing's comments

The spatial spread of the disease is modeled as a branching process on a network. The network is a graph with nodes representing individuals and edges representing contacts between different locations. The number of secondary infections caused by each infected person is a random variable with a distribution that depends on the age of the individual. The distribution of X_i is given by

$$P(X_i = x) = \sum_{j=1}^{x} \frac{1}{j} \pi_j, \tag{10}$$