```
% EEP557_HW2
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% 10/16/2022
```

Nonlinear State Space Block Diagram

```
imshow("HW2/EEP557_HW2P1.png")
```

Operating Point Block Diagram

```
imshow("HW2/EEP557_HW2P2.png")
```

Analyze

```
syms i i_dot x x_dot v v_dot e f_d e N mu_0 A x g R m x_i K
syms Delta_I Delta_V Delta_X Delta_E Delta_Fd s e_op i_op v_op x_op e_op fd_op fr

% Nonlinear State Space Equations
L_x = N^2*mu_0*A / (x + g)
```

L_x =

$$\frac{A\,N^2\,\mu_0}{g+x}$$

```
eq1 = e == i*R + L_x*i_dot - L_x*i*v/(x+g)              % Voltage loop
```

eq1 =

$$e = R\,i + \frac{A\,N^2\,\dot{i}\,\mu_0}{g+x} - \frac{A\,N^2\,i\,\mu_0\,v}{(g+x)^2}$$

```
eq2 = m*v_dot == f_d - (1/2)*L_x*i^2/(x+g) - K*(x-x_i)  % Newton's Law
```

eq2 =

$$m\,\dot{v} = f_d - K\,(x - x_i) - \frac{A\,N^2\,i^2\,\mu_0}{2\,(g+x)^2}$$

```
% i_dot, v_dot
i_dot = simplify(solve(eq1, i_dot))
```

i_dot =

$$\frac{(g+x)\left(e - R\,i + \frac{A\,N^2\,i\,\mu_0\,v}{(g+x)^2}\right)}{A\,N^2\,\mu_0}$$

```
v_dot = solve(eq2, v_dot)
```

v_dot =

$$-\frac{K\ (x-x_i)-f_d+\frac{A\,N^2\,i^2\,\mu_0}{2\ (g+x)^2}}{m}$$

```

```

Operating Point

```
% Partial
i_dot_di = simplify(expand(diff(i_dot, i)))
```

i_dot_di =

$$-\frac{-A\,\mu_0\,v\,N^2+R\,g^2+2\,R\,g\,x+R\,x^2}{A\,N^2\,\mu_0\ (g+x)}$$

```
i_dot_dv = expand(diff(i_dot, v))
```

i_dot_dv =

$$\frac{i}{g+x}$$

```
i_dot_dx = expand(diff(i_dot, x))
```

i_dot_dx =

$$\frac{e}{A\,N^2\,\mu_0}-\frac{i\,v}{g^2+2\,g\,x+x^2}-\frac{R\,i}{A\,N^2\,\mu_0}$$

```
i_dot_de = simplify(expand(diff(i_dot, e)))
```

i_dot_de =

$$\frac{g+x}{A\,N^2\,\mu_0}$$

```
i_dot_df_d = expand(diff(i_dot, f_d))
```

i_dot_df_d = 0

```

v_dot_di = simplify(expand(diff(v_dot, i)))
```

v_dot_di =

$$-\frac{A\,N^2\,i\,\mu_0}{m\ (g+x)^2}$$

```
v_dot_dv = expand(diff(v_dot, v))
```

v_dot_dv = 0

```
v_dot_dx = diff(v_dot, x)
```

v_dot_dx =

$$-\frac{K-\frac{A\,N^2\,i^2\,\mu_0}{(g+x)^3}}{m}$$

```
v_dot_de = expand(diff(v_dot, e))
```

v_dot_de = 0

```
v_dot_df_d = expand(diff(v_dot, f_d))
```

v_dot_df_d =

$$\frac{1}{m}$$

```
% Operating point LaPlace domain equations
eq3 = s*Delta_I == Delta_I*i_dot_di + Delta_V*i_dot_dv + Delta_X*i_dot_dx + Delta_E*i_(
```

eq3 =

$$\Delta_I\,s=\frac{\Delta_V\,i}{g+x}-\Delta_X\left(\frac{i\,v}{g^2+2\,g\,x+x^2}-\frac{e}{\sigma_1}+\frac{R\,i}{\sigma_1}\right)+\frac{\Delta_E\,(g+x)}{\sigma_1}-\frac{\Delta_I\,\left(-A\,\mu_0\,v\,N^2+R\,g^2+2\,R\,g\,x+R\,x^2\right)}{A\,N^2\,\mu_0\,(g+x)}$$

where

$$\sigma_1=A\,N^2\,\mu_0$$

```
eq4 = s*Delta_V == Delta_I*v_dot_di + Delta_V*v_dot_dv + Delta_X*v_dot_dx + Delta_Fd*v_
```

eq4 =

$$\Delta_V\,s=\frac{\Delta_{\mathrm{Fd}}}{m}-\frac{\Delta_X\left(K-\frac{A\,N^2\,i^2\,\mu_0}{(g+x)^3}\right)}{m}-\frac{A\,\Delta_I\,N^2\,i\,\mu_0}{m\,(g+x)^2}$$

```
eq5 = s*Delta_X == Delta_V
```

eq5 = $\Delta_X\,s=\Delta_V$

```
% Solve Delta_X
eqs = [eq3, eq4, eq5]
```

eqs =

$$\left(\Delta_I\, s = \frac{\Delta_V\, i}{g+x} - \Delta_X \left(\frac{i\, v}{g^2+2\,g\,x+x^2} - \frac{e}{\sigma_1} + \frac{R\, i}{\sigma_1} \right) + \frac{\Delta_E\, (g+x)}{\sigma_1} - \frac{\Delta_I\, \left(-A\,\mu_0\, v\, N^2 + R\, g^2 + 2\, R\, g\, x + R\, x^2\right.}{A\, N^2\, \mu_0\, (g+x)} \right.$$

where

$$\sigma_1 = A\, N^2\, \mu_0$$

```
S = subs(solve(eqs, [Delta_I, Delta_V, Delta_X]), [x i v e f_d], [x_op i_op v_op e_op f
```

```
S = struct with fields:
    Delta_I: (A*Delta_Fd*mu_0*N^2*g*i_op*s - A*Delta_E*mu_0*N^2*i_op^2 + A*Delta_Fd*mu_0*N^2*i_op*s*x_op -
    Delta_V: (s*(g + x_op)*(Delta_Fd*R*g^2 + Delta_Fd*R*x_op^2 + 2*Delta_Fd*R*g*x_op - A*Delta_E*N^2*i_op*
    Delta_X: ((g + x_op)*(Delta_Fd*R*g^2 + Delta_Fd*R*x_op^2 + 2*Delta_Fd*R*g*x_op - A*Delta_E*N^2*i_op*mu
```

```
EQ1 = Delta_X == collect(S.Delta_X, [Delta_E, Delta_Fd])
```

EQ1 =

$$\Delta_X = \left(-\frac{A\, N^2\, i_{\mathrm{op}}\, \mu_0\, (g+x_{\mathrm{op}})}{\sigma_1} \right) \Delta_E + \frac{(g+x_{\mathrm{op}})\, \left(A\, \mu_0\, s\, N^2\, g + A\, \mu_0\, s\, N^2\, x_{\mathrm{op}} - A\, \mu_0\, v_{\mathrm{op}}\, N^2 + R\, g^2 + 2\, R\, g\, x \right.}{\sigma_1}$$

where

$$\sigma_1 = A\, m\, \mu_0\, N^2\, g^2\, s^3 + A\, K\, \mu_0\, N^2\, g^2\, s + 2\, A\, m\, \mu_0\, N^2\, g\, s^3\, x_{\mathrm{op}} - A\, m\, \mu_0\, v_{\mathrm{op}}\, N^2\, g\, s^2 + 2\, A\, K\, \mu_0\, N^2\, g\, s\, x_{\mathrm{op}} - $$

```
% Delta_Fd/Delta_X
subDelta_X = children(rhs(EQ1))
```

subDelta_X = 1×2 cell

...

	1
1	(-(A*N^2*i_op*mu_0*(g + x_op))/(A*m*mu_0*N^2*g^2*s^3 + A*K*mu_0*N^2*g^2*s...

```
EQ2 = Delta_Fd/Delta_X == Delta_Fd/subDelta_X{2}
```

EQ2 =

$$\frac{\Delta_{\mathrm{Fd}}}{\Delta_X} = \frac{A\, m\, \mu_0\, N^2\, g^2\, s^3 + A\, K\, \mu_0\, N^2\, g^2\, s + 2\, A\, m\, \mu_0\, N^2\, g\, s^3\, x_{\mathrm{op}} - A\, m\, \mu_0\, v_{\mathrm{op}}\, N^2\, g\, s^2 + 2\, A\, K\, \mu_0\, N^2\, g\, s\, x_{\mathrm{op}} - }{}$$

```
% Operation point
eq6 = subs((1/2)*L_x*i^2/(x+g) - K*(x-x_i) == 0, [x i], [x_op i_op])
```

eq6 =

$$K\ (x_i - x_{op}) + \frac{A\, N^2\, {i_{op}}^2\, \mu_0}{2\ (g + x_{op})^2} = 0$$

```
i_op_x = solve(eq6, i_op)
```

i_op_x =

$$\begin{pmatrix} \frac{\sqrt{2}\ \sqrt{-K}\ (g + x_{op})\ \sqrt{x_i - x_{op}}}{\sqrt{A}\ N\ \sqrt{\mu_0}} \\ -\frac{\sqrt{2}\ \sqrt{-K}\ (g + x_{op})\ \sqrt{x_i - x_{op}}}{\sqrt{A}\ N\ \sqrt{\mu_0}} \end{pmatrix}$$

```
i_op_x = i_op_x(1,1)
```

i_op_x =

$$\frac{\sqrt{2}\ \sqrt{-K}\ (g + x_{op})\ \sqrt{x_i - x_{op}}}{\sqrt{A}\ N\ \sqrt{\mu_0}}$$

```
e_op_x = i_op_x * R
```

e_op_x =

$$\frac{\sqrt{2}\ \sqrt{-K}\ R\ (g + x_{op})\ \sqrt{x_i - x_{op}}}{\sqrt{A}\ N\ \sqrt{\mu_0}}$$

```
eq7 = subs(EQ2,[e_op i_op v_op], [e_op_x i_op_x 0])          % Change the i_op, e_op wi
```

eq7 =

$$\frac{\Delta_{Fd}}{\Delta_X} = \frac{K\ R\, g^3 + K\ R\, {x_{op}}^3 + 3\ K\ R\, g\, {x_{op}}^2 + 3\ K\ R\, g^2\, x_{op} + 2\ K\ R\ (g + x_{op})^2\ (x_i - x_{op}) + R\, g^3\, m\, s^2 + R\, m\, s^2\, x_{o}}{(g + x}$$

```
% Transfer function with only x_op
% EQ2 = subs(eq7,[R x_i g N A mu_0 m K],[1 0.01 5*10^(-4) 100 4*10^(-4) 4*pi*10^(-5) 0

EQ2 = subs(eq7,[R x_i g N A mu_0 m K],[1 0.01 5*10^(-4) 100 4*10^(-4) 4*pi*10^(-5) 0.0
```

EQ2 =

$$\frac{\Delta_{Fd}}{\Delta_X} = \frac{\frac{3\, x_{op}}{40000} + \frac{3\, s^2\, {x_{op}}^2}{40000} + \frac{s^2\, {x_{op}}^3}{20} + \frac{\pi\, s}{250000000} - 200\, \left(x_{op} - \frac{1}{100}\right)\, \left(x_{op} + \frac{1}{2000}\right)^2 + \frac{\pi\, s^3}{500000000000} + \frac{3\,}{800}}{\left(x_{op} + \frac{1}{2000}\right)\, \left(\frac{x_{op}}{1000} + \frac{\pi\, s}{12500}\right.}$$

```
% FRF plot
fr = logspace(0,2)
```

```
fr = 1×50
    1.0000    1.0985    1.2068    1.3257    1.4563    1.5999    1.7575    1.9307 ⋯
```

```
s_f = 2*pi*fr*j
```

```
s_f = 1×50 complex
10^2 ×
   0.0000 + 0.0628i   0.0000 + 0.0690i   0.0000 + 0.0758i   0.0000 + 0.0833i ⋯
```

```
EQ3 = subs(EQ2, s, s_f)
```

EQ3 =

$$\left(\frac{\Delta_{\mathrm{Fd}}}{\Delta_X} = -\frac{\sigma_1 - \frac{3\,x_{\mathrm{op}}}{40000} - \sigma_2 - 100\,x_{\mathrm{op}}^3 + \frac{x_{\mathrm{op}}^3\,\pi^2}{5} - \frac{1}{80000000} + x_{\mathrm{op}}\,\pi^2\left(\frac{3}{20000000} - \frac{1}{31250}\,\mathrm{i}\right) + \frac{x_{\mathrm{op}}\,\pi^4\,\mathrm{i}}{15625000}\cdot}{\left(x_{\mathrm{op}} + \frac{1}{2000}\right)\left(\frac{x_{\mathrm{op}}}{1000} + x_{\mathrm{op}}^2 + \frac{}{40}\right.}\right.$$

where

$$\sigma_1 = 200\left(x_{\mathrm{op}} - \frac{1}{100}\right)\left(x_{\mathrm{op}} + \frac{1}{2000}\right)^2$$

$$\sigma_2 = \frac{3\,x_{\mathrm{op}}^2}{20}$$

```
frf = abs(rhs(EQ3))      % magnitude of transfer function
```

frf =

$$\left(\frac{\left|\sigma_1 - \frac{3\,x_{\mathrm{op}}}{40000} - \sigma_3 - 100\,x_{\mathrm{op}}^3 + \frac{x_{\mathrm{op}}^3\,\pi^2}{5} - \frac{1}{80000000} + x_{\mathrm{op}}\,\pi^2\left(\frac{3}{20000000} - \frac{1}{31250}\,\mathrm{i}\right) + \frac{x_{\mathrm{op}}\,\pi^4\,\mathrm{i}}{15625000} + \pi^2\left(\frac{}{40}\right.\right.}{\left|\frac{x_{\mathrm{op}}}{1000} + x_{\mathrm{op}}^2 + \frac{1}{4000000} + \frac{x_{\mathrm{op}}\,\pi^2\,\mathrm{i}}{3125}\cdot\right.}\right.$$

where

$$\sigma_1 = 200\left(x_{\mathrm{op}} - \frac{1}{100}\right)\left(x_{\mathrm{op}} + \frac{1}{2000}\right)^2$$

$$\sigma_2 = \left|x_{\mathrm{op}} + \frac{1}{2000}\right|$$

$$\sigma_3 = \frac{3\,x_{\mathrm{op}}^2}{20}$$

```
% fill in the x_op numbers
```

```
EQ3_1 = subs(frf,x_op,1*10^(-3))
```

EQ3_1 =

$$\left(\frac{2000\sqrt{\left(\frac{27\,\pi^2}{40000000000}-\frac{351}{80000000}\right)^2+\left(\frac{9\,\pi^2}{125000000}-\frac{9\,\pi^4}{62500000000}\right)^2}}{3\sqrt{\frac{9\,\pi^4}{39062500000000}+\frac{81}{16000000000000}}} \quad \frac{2000\sqrt{\frac{29963662947}{5092589940836215}}}{} \right.$$

```
EQ3_3 = subs(frf,x_op,3*10^(-3))
```

EQ3_3 =

$$\left(\frac{2000\sqrt{\left(\frac{343\,\pi^2}{40000000000}-\frac{343}{16000000}\right)^2+\left(\frac{49\,\pi^2}{125000000}-\frac{49\,\pi^4}{62500000000}\right)^2}}{7\sqrt{\frac{49\,\pi^4}{39062500000000}+\frac{2401}{16000000000000}}} \quad \frac{2000\sqrt{\frac{88818215723}{5092589940836215}}}{} \right.$$

```
EQ3_6 = subs(frf,x_op,6*10^(-3))
```

EQ3_6 =

$$\left(\frac{2000\sqrt{\left(\frac{2197\,\pi^2}{40000000000}-\frac{4901}{80000000}\right)^2+\left(\frac{169\,\pi^2}{125000000}-\frac{169\,\pi^4}{62500000000}\right)^2}}{13\sqrt{\frac{169\,\pi^4}{39062500000000}+\frac{28561}{16000000000000}}} \quad \frac{2000\sqrt{\frac{105653355238}{5092589940836215}}}{} \right.$$

```
EQ3_9 = subs(frf,x_op,9*10^(-3))
```

EQ3_9 =

$$\left(\frac{2000\sqrt{\left(\frac{6859\,\pi^2}{40000000000}-\frac{8303}{80000000}\right)^2+\left(\frac{361\,\pi^2}{125000000}-\frac{361\,\pi^4}{62500000000}\right)^2}}{19\sqrt{\frac{361\,\pi^4}{39062500000000}+\frac{130321}{16000000000000}}} \quad \frac{2000\sqrt{\frac{482085743078}{5092589940836215}}}{} \right.$$

```
loglog(fr, EQ3_1, fr, EQ3_3, fr, EQ3_6, fr, EQ3_9, 'LineWidth', 3)
grid

legend(["1mm" "3mm" "6mm" "9mm"], Location="best")

xlabel('Frequency[Hz]')
ylabel('\Delta F_d/ \Delta X[N/m]')
title('FRF Plot')
```