

CSE 417: Homework 4

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Problem 3:

(a)

Sort intervals by start time so $s_1 \leq s_2 \leq \dots \leq s_n$,

$d \leftarrow 0$

for lectures $j = 1$ to n {

 set a new room label $r = 1$

 for classrooms $k = 1$ to d {

 if last lecture's finishing time $f'_k < s_j$

 add lecture j in classroom set C_k

$f'_k \leftarrow f_j$, change the last lecture's finishing time for classroom k to
 the lecture j 's finishing time f_j

$r = 0$

 Break the classrooms searching for loop

 }

 if $r = 1$

$d++$

 make a new set C_d for a new classroom d

 add lecture j in classroom set C_d

 store the lecture j 's finishing time f_j in f'_d

 }

(b)

The worst case for this algorithm is every lecture is overlap with others.

In this case, every lecture need a new classroom. But before the algorithm assign a new classroom to a lecture, it need compare all the lectures added before. Then creat a new classroom list and assign the lecture in this classroom.

For the i 's sorted lecture, this lecture i will compare all the lectures before, which are $i - 1$ number of lectures. So the total run time of this algorithm will be

$$a \times (1 + 2 + \dots + (n - 1)) \\ = a \times \frac{n(n - 1)}{2}$$

where a is a constant for the steps to assign lecture to the classroom.

In this case, no matter what n is, the lower bound of this algorithm will be $\Omega(n^2)$

(c)

The basic idea is: when I have an empty classroom. Using binary search algorithm to search all the sorted lectures to find the easiest capable lecture in this classroom. Then repeat this step, still use a binary search algorithm to search all the sorted lectures to find the next earliest one which start time is after 1st lecture's end time. Repeat this step until this classroom is full and no lecture can be scheduled in this classroom. Then open a new classroom and repeat the steps above.

Sort intervals by start time so $s_1 \leq s_2 \leq \dots \leq s_n$, and store all lecture in a list A

$j \leftarrow 1$

Create a list C_1 as a classroom

While the list A is not empty

 Using binary search algorithm to search all the sorted lectures to find

 the easiest capable lecture L_i

 if find L_i , store in the C_j and remove it from list A

 if cannot find, open a new classroom list C_j

Endwhile

In this loop, the complex for binary search is $O(\log n)$, and every lecture will have one binary search. And every time add a new classrom, will have one more binary search. Even the worst case, every lecture need a new room, the run time for this worst case is $O(n \log n + n)$, which lecture number is equal to classroom number. It is still $O(n \log n)$