```
% EEP557_HW2
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% 10/16/2022
```
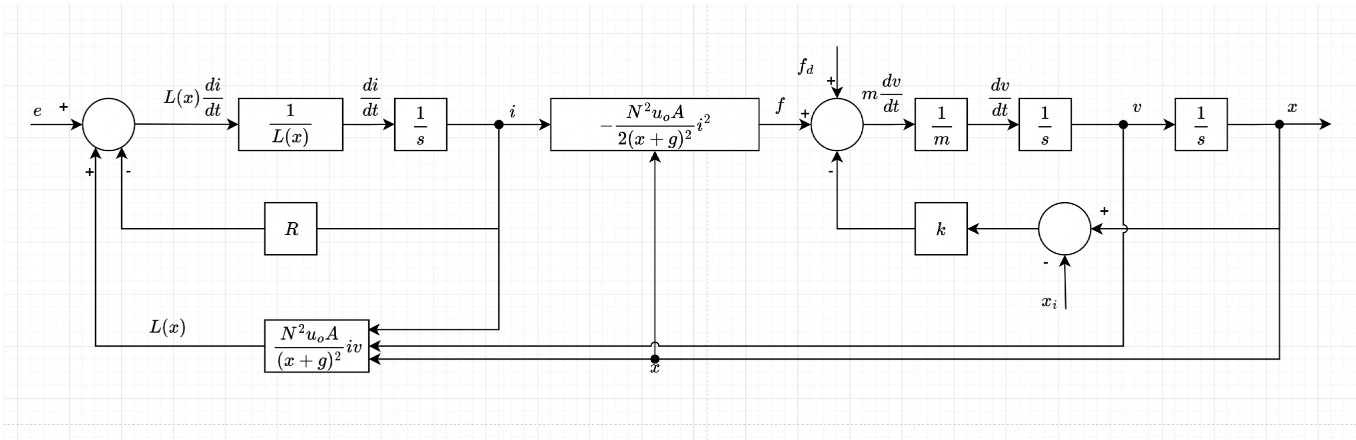
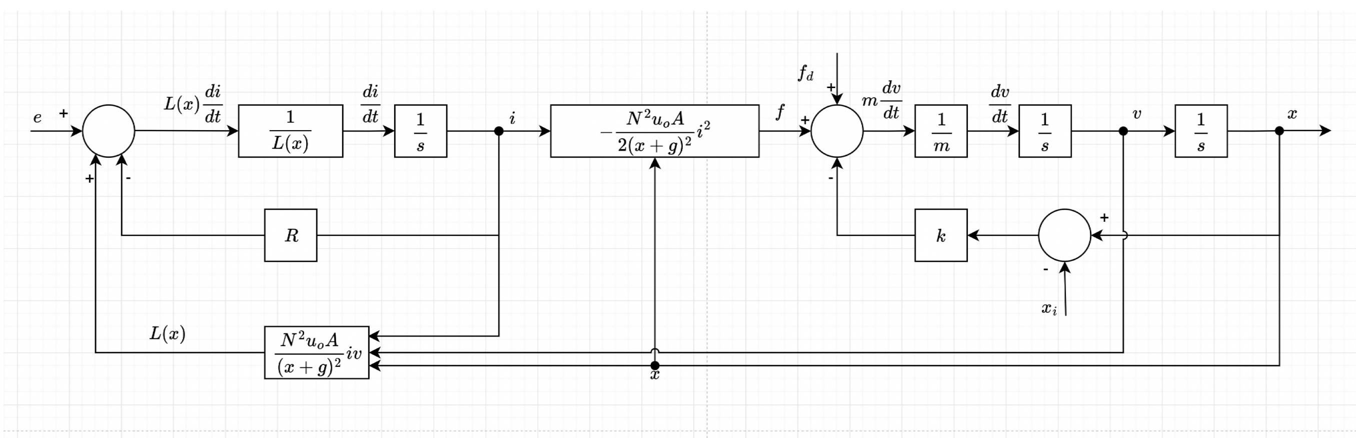

## Nonlinear State Space Block Diagram

```
imshow("HW2/EEP557_HW2P1.png")
```




## Operating Point Block Diagram

```
imshow("HW2/EEP557_HW2P1.png")
```




## Analyze

```
syms i i_dot x x_dot v v_dot e f_d e N mu_0 A x g R m x_i K
syms Delta_I Delta_V Delta_X Delta_E Delta_Fd s e_op i_op v_op x_op e_op fd_op fr
```

```
% Nonlinear State Space Equations
L_x = N^2*mu_0*A / (x + g)
```

L_x =

$$\frac{A\,N^2\,\mu_0}{g+x}$$

```
eq1 = e == i*R + L_x*i_dot - L_x*i*v/(x+g)              % Voltage loop
```

eq1 =

$$e = R\,i + \frac{A\,N^2\,\dot{i}\,\mu_0}{g+x} - \frac{A\,N^2\,i\,\mu_0\,v}{(g+x)^2}$$

```
eq2 = m*v_dot == f_d - (1/2)*L_x*i^2/(x+g) - K*(x-x_i)  % Newton's Law
```

eq2 =

$$m\,\dot{v} = f_d - K\,(x - x_i) - \frac{A\,N^2\,i^2\,\mu_0}{2\,(g+x)^2}$$

```
% i_dot, v_dot
i_dot = simplify(solve(eq1, i_dot))
```

i_dot =

$$\frac{(g+x)\left(e - R\,i + \frac{A\,N^2\,i\,\mu_0\,v}{(g+x)^2}\right)}{A\,N^2\,\mu_0}$$

```
v_dot = solve(eq2, v_dot)
```

v_dot =

$$-\frac{K\,(x - x_i) - f_d + \frac{A\,N^2\,i^2\,\mu_0}{2\,(g+x)^2}}{m}$$

## Operating Point

```
% Partial
i_dot_di = simplify(expand(diff(i_dot, i)))
```

i_dot_di =

$$-\frac{-A\,\mu_0\,v\,N^2 + R\,g^2 + 2\,R\,g\,x + R\,x^2}{A\,N^2\,\mu_0\,(g+x)}$$

```
i_dot_dv = expand(diff(i_dot, v))
```

i_dot_dv =

$$\frac{i}{g + x}$$

```
i_dot_dx = expand(diff(i_dot, x))
```

i_dot_dx =

$$\frac{e}{A\,N^2\,\mu_0} - \frac{i\,v}{g^2 + 2\,g\,x + x^2} - \frac{R\,i}{A\,N^2\,\mu_0}$$

```
i_dot_de = simplify(expand(diff(i_dot, e)))
```

i_dot_de =

$$\frac{g + x}{A\,N^2\,\mu_0}$$

```
i_dot_df_d = expand(diff(i_dot, f_d))
```

i_dot_df_d = $0$

```

v_dot_di = simplify(expand(diff(v_dot, i)))
```

v_dot_di =

$$-\frac{A\,N^2\,i\,\mu_0}{m\,(g + x)^2}$$

```
v_dot_dv = expand(diff(v_dot, v))
```

v_dot_dv = $0$

```
v_dot_dx = diff(v_dot, x)
```

v_dot_dx =

$$-\frac{K - \frac{A\,N^2\,i^2\,\mu_0}{(g + x)^3}}{m}$$

```
v_dot_de = expand(diff(v_dot, e))
```

v_dot_de = $0$

```
v_dot_df_d = expand(diff(v_dot, f_d))
```

v_dot_df_d =

$$\frac{1}{m}$$

```


% Operating point LaPlace domain equations
```

```
eq3 = s*Delta_I == Delta_I*i_dot_di + Delta_V*i_dot_dv + Delta_X*i_dot_dx + Delta_E*i_
```

eq3 =

$$\Delta_I\, s = \frac{\Delta_V\, i}{g+x} - \Delta_X \left(\frac{i\, v}{g^2+2\,g\,x+x^2} - \frac{e}{\sigma_1} + \frac{R\, i}{\sigma_1}\right) + \frac{\Delta_E\, (g+x)}{\sigma_1} - \frac{\Delta_I\, \left(-A\,\mu_0\, v\, N^2 + R\, g^2 + 2\,R\,g\,x + R\,x^2\right)}{A\,N^2\,\mu_0\,(g+x)}$$

where

$$\sigma_1 = A\,N^2\,\mu_0$$

```
eq4 = s*Delta_V == Delta_I*v_dot_di + Delta_V*v_dot_dv + Delta_X*v_dot_dx + Delta_Fd*v_
```

eq4 =

$$\Delta_V\, s = \frac{\Delta_{\mathrm{Fd}}}{m} - \frac{\Delta_X \left(K - \frac{A\,N^2\,i^2\,\mu_0}{(g+x)^3}\right)}{m} - \frac{A\,\Delta_I\,N^2\,i\,\mu_0}{m\,(g+x)^2}$$

```
eq5 = s*Delta_X == Delta_V
```

eq5 = $\Delta_X\, s = \Delta_V$

```
% Solve Delta_X
eqs = [eq3, eq4, eq5]
```

eqs =

$$\left(\Delta_I\, s = \frac{\Delta_V\, i}{g+x} - \Delta_X \left(\frac{i\, v}{g^2+2\,g\,x+x^2} - \frac{e}{\sigma_1} + \frac{R\, i}{\sigma_1}\right) + \frac{\Delta_E\, (g+x)}{\sigma_1} - \frac{\Delta_I\, \left(-A\,\mu_0\, v\, N^2 + R\, g^2 + 2\,R\,g\,x + R\,x^2\right)}{A\,N^2\,\mu_0\,(g+x)}\right.$$

where

$$\sigma_1 = A\,N^2\,\mu_0$$

```
S = subs(solve(eqs, [Delta_I, Delta_V, Delta_X]), [x i v e f_d], [x_op i_op v_op e_op
```

```
S = struct with fields:
    Delta_I: (A*Delta_Fd*mu_0*N^2*g*i_op*s - A*Delta_E*mu_0*N^2*i_op^2 + A*Delta_Fd*mu_0*N^2*i_op*s*x_op
    Delta_V: (s*(g + x_op)*(Delta_Fd*R*g^2 + Delta_Fd*R*x_op^2 + 2*Delta_Fd*R*g*x_op - A*Delta_E*N^2*i_op
    Delta_X: ((g + x_op)*(Delta_Fd*R*g^2 + Delta_Fd*R*x_op^2 + 2*Delta_Fd*R*g*x_op - A*Delta_E*N^2*i_op*mu
```

```
EQ1 = Delta_X == collect(S.Delta_X, [Delta_E, Delta_Fd])
```

EQ1 =

$$\Delta_X = \left(-\frac{A\,N^2\,i_{\mathrm{op}}\,\mu_0\,(g+x_{\mathrm{op}})}{\sigma_1}\right)\Delta_E + \frac{(g+x_{\mathrm{op}})\,\left(A\,\mu_0\,s\,N^2\,g + A\,\mu_0\,s\,N^2\,x_{\mathrm{op}} - A\,\mu_0\,v_{\mathrm{op}}\,N^2 + R\,g^2 + 2\,R\,g\,x\right.}{\sigma_1}$$

where

$$\sigma_1 = A\,m\,\mu_0\,N^2\,g^2\,s^3 + A\,K\,\mu_0\,N^2\,g^2\,s + 2\,A\,m\,\mu_0\,N^2\,g\,s^3\,x_{\mathrm{op}} - A\,m\,\mu_0\,v_{\mathrm{op}}\,N^2\,g\,s^2 + 2\,A\,K\,\mu_0\,N^2\,g\,s\,x_{\mathrm{op}} -$$

```
% Delta_Fd/Delta_X
subDelta_X = children(rhs(EQ1))
```

subDelta_X = 1×2 cell

...

| | 1 |
|---|---|
| 1 | (-(A*N^2*i_op*mu_0*(g + x_op))/(A*m*mu_0*N^2*g^2*s^3 + A*K*mu_0*N^2*g^2*s... |

```
EQ2 = Delta_Fd/Delta_X == Delta_Fd/subDelta_X{2}
```

EQ2 =

$$\frac{\Delta_{\mathrm{Fd}}}{\Delta_X} = \frac{A\,m\,\mu_0\,N^2\,g^2\,s^3 + A\,K\,\mu_0\,N^2\,g^2\,s + 2\,A\,m\,\mu_0\,N^2\,g\,s^3\,x_{\mathrm{op}} - A\,m\,\mu_0\,v_{\mathrm{op}}\,N^2\,g\,s^2 + 2\,A\,K\,\mu_0\,N^2\,g\,s\,x_{\mathrm{op}} -}{}$$

```
% Operation point
eq6 = subs((1/2)*L_x*i^2/(x+g) - K*(x-x_i) == 0, [x i], [x_op i_op])
```

eq6 =

$$K\,(x_i - x_{\mathrm{op}}) + \frac{A\,N^2\,{i_{\mathrm{op}}}^2\,\mu_0}{2\,(g+x_{\mathrm{op}})^2} = 0$$

```
i_op_x = solve(eq6, i_op)
```

i_op_x =

$$\begin{pmatrix} \dfrac{\sqrt{2}\,\sqrt{-K}\,(g+x_{\mathrm{op}})\,\sqrt{x_i - x_{\mathrm{op}}}}{\sqrt{A}\,N\,\sqrt{\mu_0}} \\ -\dfrac{\sqrt{2}\,\sqrt{-K}\,(g+x_{\mathrm{op}})\,\sqrt{x_i - x_{\mathrm{op}}}}{\sqrt{A}\,N\,\sqrt{\mu_0}} \end{pmatrix}$$

```
i_op_x = i_op_x(1,1)
```

i_op_x =

$$\frac{\sqrt{2}\,\sqrt{-K}\,(g+x_{\mathrm{op}})\,\sqrt{x_i - x_{\mathrm{op}}}}{\sqrt{A}\,N\,\sqrt{\mu_0}}$$

```
e_op_x = i_op_x * R
```

e_op_x =

$$\frac{\sqrt{2}\ \sqrt{-K}\ R\ (g+x_{\text{op}})\ \sqrt{x_i - x_{\text{op}}}}{\sqrt{A}\ N\ \sqrt{\mu_0}}$$

```
eq7 = subs(EQ2,[e_op i_op v_op], [e_op_x i_op_x 0])          % Change the i_op, e_op wi
```

eq7 =

$$\frac{\Delta_{\text{Fd}}}{\Delta_X} = \frac{K\ R\ g^3 + K\ R\ x_{\text{op}}^3 + 3\ K\ R\ g\ x_{\text{op}}^2 + 3\ K\ R\ g^2\ x_{\text{op}} + 2\ K\ R\ (g+x_{\text{op}})^2\ (x_i - x_{\text{op}}) + R\ g^3\ m\ s^2 + R\ m\ s^2\ x_{\text{o}}}{(g + x_{\text{}}}$$

```
% Transfer function with only x_op
EQ2 = subs(eq7,[R x_i g N A mu_0 m K],[1 0.01 5*10^(-4) 100 4*10^(-4) 4*pi*10^(-5) 0.0
```

EQ2 =

$$\frac{\Delta_{\text{Fd}}}{\Delta_X} = \frac{\frac{3\ x_{\text{op}}}{40000} + \frac{3\ s^2\ x_{\text{op}}^2}{40000} + \frac{s^2\ x_{\text{op}}^3}{20} + \frac{\pi\ s}{250000000} - 200\ \left(x_{\text{op}} - \frac{1}{100}\right)\ \left(x_{\text{op}} + \frac{1}{2000}\right)^2 + \frac{\pi\ s^3}{500000000000} + \frac{3\ \text{}}{800}}{\left(x_{\text{op}} + \frac{1}{2000}\right)\ \left(\frac{x_{\text{op}}}{1000} + \frac{\pi\ s}{12500}\right.}$$

```
% FRF plot
fr = logspace(0,2)
```

fr = 1×50

```
    1.0000    1.0985    1.2068    1.3257    1.4563    1.5999    1.7575    1.9307 ···
```

```
s_f = 2*pi*fr*j
```

s_f = 1×50 complex

```
10^2 ×
   0.0000 + 0.0628i   0.0000 + 0.0690i   0.0000 + 0.0758i   0.0000 + 0.0833i ···
```

```
EQ3 = subs(EQ2, s, s_f)
```

EQ3 =

$$\left(\frac{\Delta_{\mathrm{Fd}}}{\Delta_X} = -\frac{\sigma_1 - \frac{3\,x_{\mathrm{op}}}{40000} - \sigma_2 - 100\,x_{\mathrm{op}}{}^3 + \frac{x_{\mathrm{op}}{}^3\,\pi^2}{5} - \frac{1}{80000000} + x_{\mathrm{op}}\,\pi^2\left(\frac{3}{20000000} - \frac{1}{31250}\,\mathrm{i}\right) + \frac{x_{\mathrm{op}}\,\pi^4\,\mathrm{i}}{15625000}\cdot}{\left(x_{\mathrm{op}} + \frac{1}{2000}\right)\left(\frac{x_{\mathrm{op}}}{1000} + x_{\mathrm{op}}{}^2 + \frac{}{40}\right.}\right.$$

where

$$\sigma_1 = 200\left(x_{\mathrm{op}} - \frac{1}{100}\right)\left(x_{\mathrm{op}} + \frac{1}{2000}\right)^2$$

$$\sigma_2 = \frac{3\,x_{\mathrm{op}}{}^2}{20}$$

```
frf = abs(rhs(EQ3))     % magnitude of transfer function
```

```
frf =
```

$$\left(\frac{\left|\sigma_1 - \frac{3\,x_{\mathrm{op}}}{40000} - \sigma_3 - 100\,x_{\mathrm{op}}{}^3 + \frac{x_{\mathrm{op}}{}^3\,\pi^2}{5} - \frac{1}{80000000} + x_{\mathrm{op}}\,\pi^2\left(\frac{3}{20000000} - \frac{1}{31250}\,\mathrm{i}\right) + \frac{x_{\mathrm{op}}\,\pi^4\,\mathrm{i}}{15625000} + \pi^2\left(\frac{}{40}\right.\right.}{\left|\frac{x_{\mathrm{op}}}{1000} + x_{\mathrm{op}}{}^2 + \frac{1}{4000000} + \frac{x_{\mathrm{op}}\,\pi^2\,\mathrm{i}}{3125}\cdot\right.}\right.$$

where

$$\sigma_1 = 200\left(x_{\mathrm{op}} - \frac{1}{100}\right)\left(x_{\mathrm{op}} + \frac{1}{2000}\right)^2$$

$$\sigma_2 = \left|x_{\mathrm{op}} + \frac{1}{2000}\right|$$

$$\sigma_3 = \frac{3\,x_{\mathrm{op}}{}^2}{20}$$

```
% fill in the x_op numbers
EQ3_1 = subs(frf,x_op,1*10^(-3))
```

```
EQ3_1 =
```

$$\left(\frac{2000\,\sqrt{\left(\frac{27\,\pi^2}{40000000000} - \frac{351}{80000000}\right)^2 + \left(\frac{9\,\pi^2}{125000000} - \frac{9\,\pi^4}{62500000000}\right)^2}}{3\,\sqrt{\frac{9\,\pi^4}{39062500000000} + \frac{81}{16000000000000}}}\quad \frac{2000\,\sqrt{\frac{2996366294}{5092589940836215}}}{}\right.$$

```
EQ3_3 = subs(frf,x_op,3*10^(-3))
```

EQ3_3 =

$$\left(\frac{2000\sqrt{\left(\frac{343\,\pi^2}{40000000000}-\frac{343}{16000000}\right)^2+\left(\frac{49\,\pi^2}{125000000}-\frac{49\,\pi^4}{62500000000}\right)^2}}{7\sqrt{\frac{49\,\pi^4}{39062500000000}+\frac{2401}{16000000000000}}} \quad \frac{2000\sqrt{\frac{88818215723}{5092589940836215}}}{}\right.$$

```
EQ3_6 = subs(frf,x_op,6*10^(-3))
```

EQ3_6 =

$$\left(\frac{2000\sqrt{\left(\frac{2197\,\pi^2}{40000000000}-\frac{4901}{80000000}\right)^2+\left(\frac{169\,\pi^2}{125000000}-\frac{169\,\pi^4}{62500000000}\right)^2}}{13\sqrt{\frac{169\,\pi^4}{39062500000000}+\frac{28561}{16000000000000}}} \quad \frac{2000\sqrt{\frac{105653355238}{5092589940836215}}}{}\right.$$

```
EQ3_9 = subs(frf,x_op,9*10^(-3))
```

EQ3_9 =

$$\left(\frac{2000\sqrt{\left(\frac{6859\,\pi^2}{40000000000}-\frac{8303}{80000000}\right)^2+\left(\frac{361\,\pi^2}{125000000}-\frac{361\,\pi^4}{62500000000}\right)^2}}{19\sqrt{\frac{361\,\pi^4}{39062500000000}+\frac{130321}{16000000000000}}} \quad \frac{2000\sqrt{\frac{482085743078}{5092589940836215}}}{}\right.$$

```
loglog(fr, EQ3_1, fr, EQ3_3, fr, EQ3_6, fr, EQ3_9, 'LineWidth', 3)
grid

legend(["1mm" "3mm" "6mm" "9mm"], Location="best")

xlabel('Frequency[Hz]')
ylabel('\Delta F_d/ \Delta X[N/m]')
title('FRF Plot')
```