

EE536 HW6

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Classical PI

```
clear
clc

syms i_ast i v_ast v omega e
syms s K_p K_i K_v L_p R_p K_e K_hat_e b_p T_L J T_em K_t

% Classical PI System
eq1_1 = v_ast == K_p*(i_ast-i) + (1/s)*K_i*(i_ast-i) + omega*K_hat_e
```

```
eq1_1 =
v^* =  $\hat{K}_e \omega - K_p (i - i^*) - \frac{K_i (i - i^*)}{s}$ 
```

```
eq1_2 = v == K_v * v_ast
```

```
eq1_2 = v =  $K_v v^*$ 
```

```
eq1_3 = i == (v-e-i*R_p) * (1/L_p) * (1/s)
```

```
eq1_3 =
i =  $-\frac{e - v + R_p i}{L_p s}$ 
```

```
eq1_4 = e == omega*K_e
```

```
eq1_4 = e =  $K_e \omega$ 
```

```
eq1_5 = T_em == i * K_t
```

```
eq1_5 = T_em =  $K_t i$ 
```

```
eq1_6 = omega == (1/J) * (1/s) * (T_em - T_L - omega*b_p)
```

```
eq1_6 =
omega =  $-\frac{T_L - T_{em} + b_p \omega}{J s}$ 
```

```
eqs1 = [eq1_1, eq1_2, eq1_3, eq1_4, eq1_5, eq1_6];
```

```
% Current Regulator Command Tracking TF
S = solve(eqs1, [i omega v_ast v T_em e]);
```

```
EQ1_1 = i == collect(S.i, [i_ast,T_L])
```

```
EQ1_1 =
```

$$i = \frac{K_i K_v b_p + J K_p K_v s^2 + J K_i K_v s + K_p K_v b_p s}{\sigma_1} i^* + \frac{K_e s - \hat{K}_e K_v s}{\sigma_1} T_L$$

where

$$\sigma_1 = K_i K_v b_p + K_e K_t s + R_p b_p s + J L_p s^3 + J R_p s^2 + L_p b_p s^2 + J K_p K_v s^2 + J K_i K_v s - \hat{K}_e K_t K_v s + K_p.$$

```
subtheta1_1 = children(rhs(EQ1_1));
EQ1_2 = i/i_ast == subtheta1_1{1}/i_ast
```

```
EQ1_2 =
```

$$\frac{i}{i^*} = \frac{K_i K_v b_p + \sigma_1 + J K_i K_v s + K_p K_v b_p s}{K_i K_v b_p + K_e K_t s + R_p b_p s + J L_p s^3 + J R_p s^2 + L_p b_p s^2 + \sigma_1 + J K_i K_v s - \hat{K}_e K_t K_v s + K_p K_v b_p s}$$

where

$$\sigma_1 = J K_p K_v s^2$$

Active Resistance

```
syms R_hat_p R_a
```

```
% Active Resistance System
```

```
eq2_1 = v_ast == R_a*(i_ast-i) + omega*K_hat_e + i*R_hat_p
```

$$eq2_1 = v^* = \hat{K}_e \omega + \hat{R}_p i - R_a (i - i^*)$$

```
eq2_2 = v == K_v * v_ast
```

$$eq2_2 = v = K_v v^*$$

```
eq2_3 = i == (v-e-i*R_p) * (1/L_p) * (1/s)
```

```
eq2_3 =
```

$$i = -\frac{e - v + R_p i}{L_p s}$$

```
eq2_4 = e == omega*K_e
```

$$eq2_4 = e = K_e \omega$$

```
eq2_5 = T_em == i * K_t
```

```
eq2_5 = T_em = K_t i
```

```
eq2_6 = omega == (1/J) * (1/s) * (T_em - T_L - omega*b_p)
```

```
eq2_6 =
```

$$\omega = -\frac{T_L - T_{em} + b_p \omega}{J s}$$

```
eqs2 = [eq2_1, eq2_2, eq2_3, eq2_4, eq2_5, eq2_6];
```

```
% Current Regulator Command Tracking TF
```

```
S = solve(eqs2, [i omega v_ast v T_em e]);  
EQ2_1 = i == collect(S.i, [i_ast, T_L])
```

```
EQ2_1 =
```

$$i = \frac{K_v R_a b_p + J K_v R_a s}{\sigma_1} i^* + \frac{K_e - \hat{K}_e K_v}{\sigma_1} T_L$$

where

$$\sigma_1 = R_p b_p + K_e K_t - \hat{K}_e K_t K_v + K_v R_a b_p - K_v \hat{R}_p b_p + J R_p s + L_p b_p s + J L_p s^2 + J K_v R_a s - J K_v \hat{R}_p s$$

```
subtheta2_1 = children(rhs(EQ2_1));  
EQ2_2 = i/i_ast == subtheta2_1{1}/i_ast
```

```
EQ2_2 =
```

$$\frac{i}{i^*} = \frac{K_v R_a b_p + J K_v R_a s}{R_p b_p + K_e K_t - \hat{K}_e K_t K_v + K_v R_a b_p - K_v \hat{R}_p b_p + J R_p s + L_p b_p s + J L_p s^2 + J K_v R_a s - J K_v \hat{R}_p s}$$

c1. Frequency Response of Current Regulator - Classical PI

```
J = 1e-5
```

```
J = 1.0000e-05
```

```
b_p = 1e-5
```

```
b_p = 1.0000e-05
```

```
K_t = 0.2
```

```
K_t = 0.2000
```

```
K_e = 0.2
```

```
K_e = 0.2000
```

```
L_p = 1e-3
```

```
L_p = 1.0000e-03
```

```
R_p = 1
```

```
R_p = 1
```

```
K_v = 1
```

```
K_v = 1
```

```
f_desired = 1000
```

```
f_desired = 1000
```

```
% With perfect decoupling
```

```
K_i = 2*pi*f_desired*R_p
```

```
K_i = 6.2832e+03
```

```
K_p = 2*pi*f_desired*L_p
```

```
K_p = 6.2832
```

```
opts = bodeoptions;  
opts.FreqUnits = 'Hz';  
opts.Grid = 'on';  
opts.XLimMode = 'manual';  
opts.XLim = [0.1,1e+4];
```

```
% zero speed
```

```
K_e = 0;  
K_hat_e = 0;  
EQ1_tf = subs(EQ1_2)
```

```
EQ1_tf =
```

$$\frac{i}{i^*} = \frac{\frac{\pi s^2}{50000} + \frac{1001 \pi s}{50000} + \frac{\pi}{50}}{\frac{s}{100000} + \frac{\pi}{50} + \frac{1001 \pi s}{50000} + \frac{\pi s^2}{50000} + \frac{1001 s^2}{100000000} + \frac{s^3}{100000000}}$$

```
tf = sym2tf(simplify(rhs(EQ1_tf)))
```

```
tf =
```

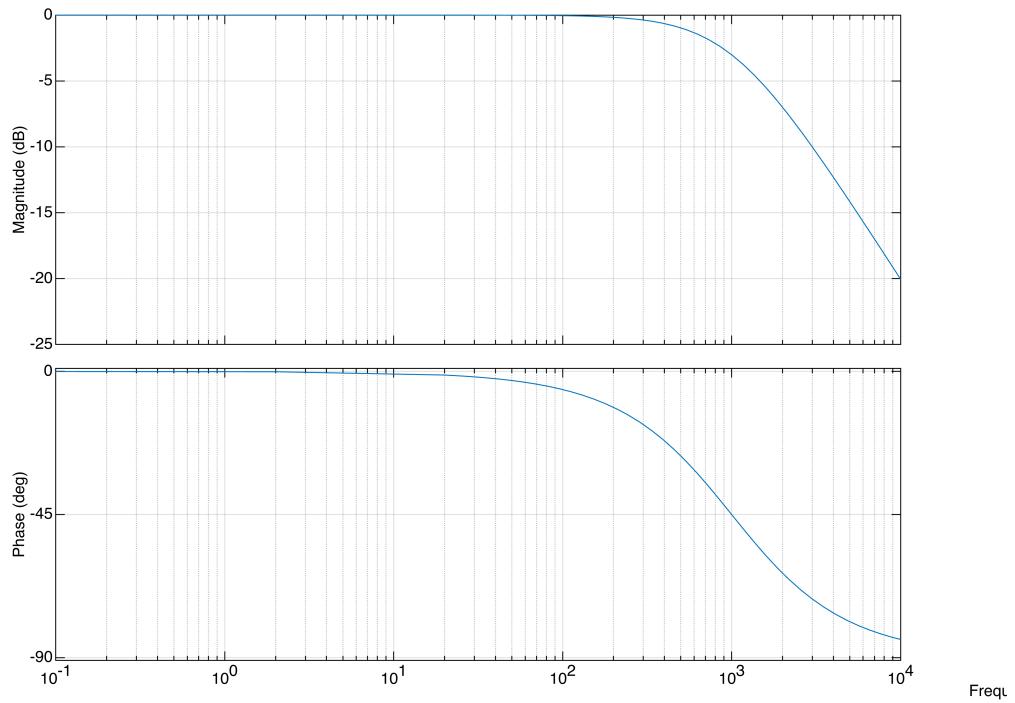
$$\frac{6283}{s + 6283}$$

```
Continuous-time transfer function.
```

```

figure
bodeplot(tf,opts)
title('Classicle PI with Zero Speed')

```



```

drawnow

% non-zero speed with error
K_e = 0.2;
for error = [1, 0.9, 1.1]
    K_hat_e = error*K_e;
    EQ1_tf = subs(EQ1_2)

    tf = sym2tf(simplify(rhs(EQ1_tf)))

    figure
    bodeplot(tf,opts)
    title(['Classicle PI with ', num2str((error-1)*100), '% Back EMF Error'])
    drawnow
end

```

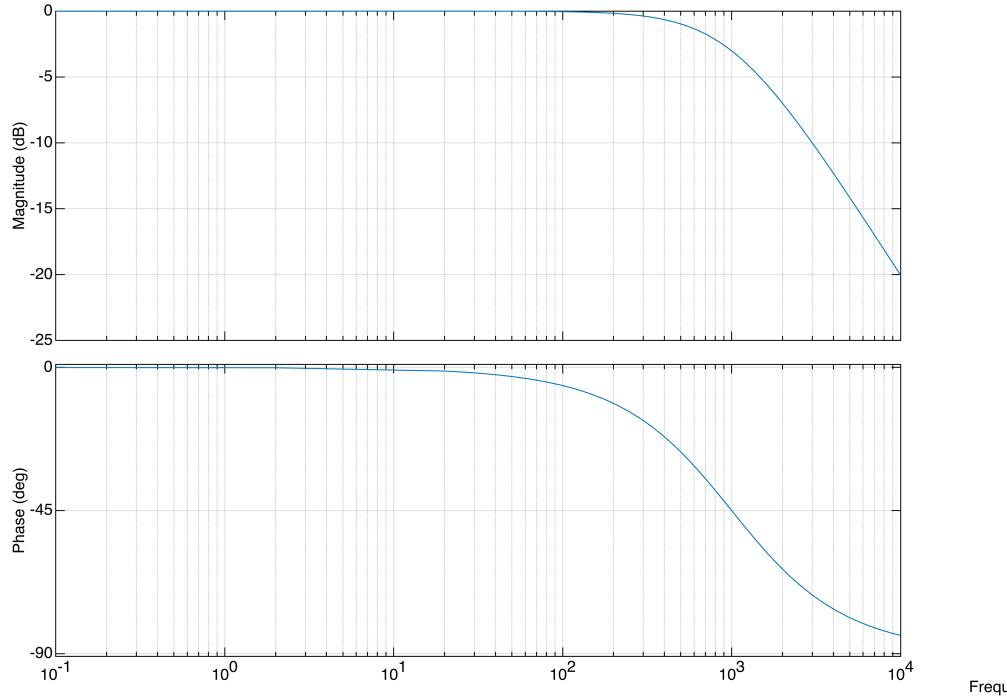
$$\frac{i}{i^*} = \frac{\frac{\pi s^2}{50000} + \frac{1001 \pi s}{50000} + \frac{\pi}{50}}{\frac{s}{1000000} + \frac{\pi}{50} + \frac{1001 \pi s}{50000} + \frac{\pi s^2}{50000} + \frac{1001 s^2}{100000000} + \frac{s^3}{100000000}}$$

tf =

$$6283$$

$$\frac{6283}{s + 6283}$$

Continuous-time transfer function.



EQ1_tf =

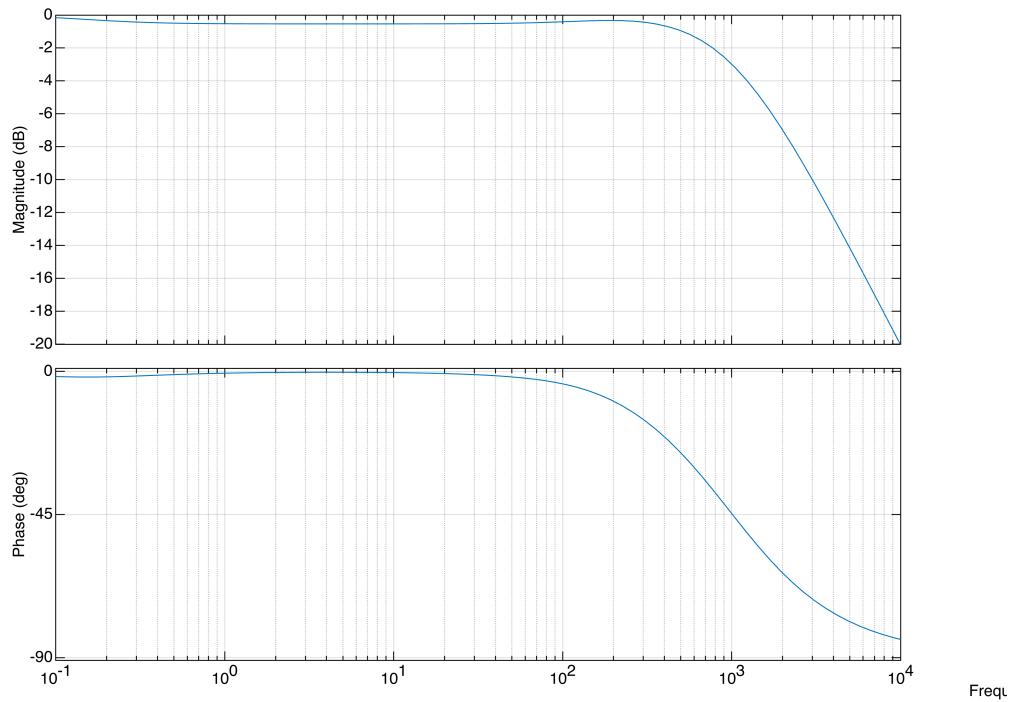
$$i = \frac{\frac{\pi s^2}{50000} + \frac{1001 \pi s}{50000} + \frac{\pi}{50}}{\frac{401 s}{100000} + \frac{\pi}{50} + \frac{1001 \pi s}{50000} + \frac{\pi s^2}{50000} + \frac{1001 s^2}{100000000} + \frac{s^3}{100000000}}$$

tf =

$$6283 s^2 + 6.289e06 s + 6.283e06$$

$$\frac{6283 s^2 + 6.289e06 s + 6.283e06}{s^3 + 7284 s^2 + 6.69e06 s + 6.283e06}$$

Continuous-time transfer function.



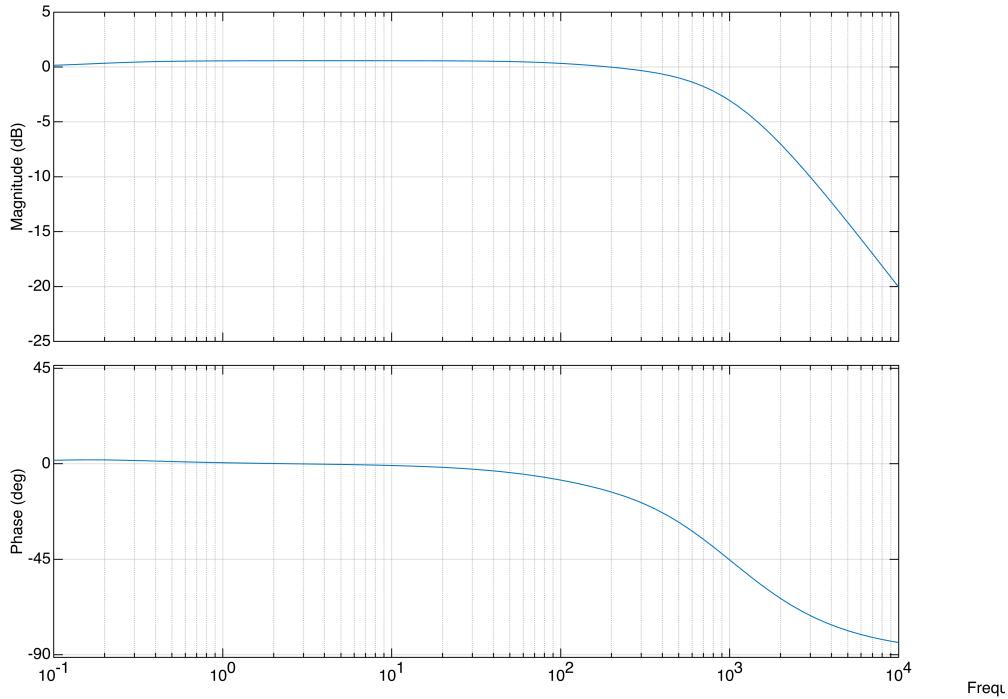
EQ1_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi s^2}{50000} + \frac{1001 \pi s}{50000} + \frac{\pi}{50}}{\frac{\pi}{50} - \frac{399 s}{100000} + \frac{1001 \pi s}{50000} + \frac{\pi s^2}{50000} + \frac{1001 s^2}{100000000} + \frac{s^3}{100000000}}$$

tf =

$$\frac{6283 s^2 + 6.289e06 s + 6.283e06}{s^3 + 7284 s^2 + 5.89e06 s + 6.283e06}$$

Continuous-time transfer function.



```
% without EMF decoupling
K_e = 0.2;
K_hat_e = 0;
EQ1_tf = subs(EQ1_2)
```

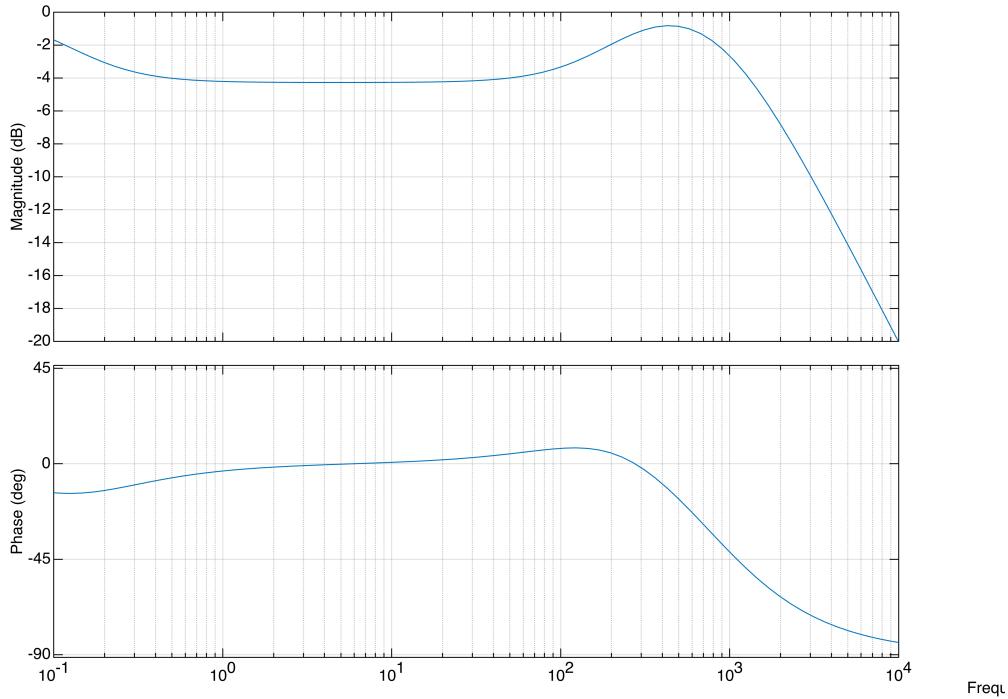
$$EQ1_tf = \frac{\frac{\pi s^2}{50000} + \frac{1001 \pi s}{50000} + \frac{\pi}{50}}{\frac{4001 s}{100000} + \frac{\pi}{50} + \frac{1001 \pi s}{50000} + \frac{\pi s^2}{50000} + \frac{1001 s^2}{100000000} + \frac{s^3}{100000000}}$$

```
tf = sym2tf(simplify(rhs(EQ1_tf)))
```

$$tf = \frac{6283 s^2 + 6.289e06 s + 6.283e06}{s^3 + 7284 s^2 + 1.029e07 s + 6.283e06}$$

Continuous-time transfer function.

```
figure
bodeplot(tf,opts)
title('Classicle PI Without Back EMF Decoupling')
```



```
drawnow
```

c2. Frequency Response of Current Regulator - Active Resistance

```
% With perfect decoupling
R_a = 2*pi*f_desired*L_p
```

```
R_a = 6.2832
```

```
% zero speed with different R_p error
K_e = 0;
K_hat_e = 0;

for Rerror = [1, 0.8, 1.2]
    R_hat_p = Rerror*R_p;

    EQ2_tf = subs(EQ2_2)
    tf = syms2tf(simplify(rhs(EQ2_tf)))

    figure
    bodeplot(tf,opts)
    title(['Active Resistance Zero Speed with ', num2str((Rerror-1)*100), '% Resistance'])
    drawnow
```

end

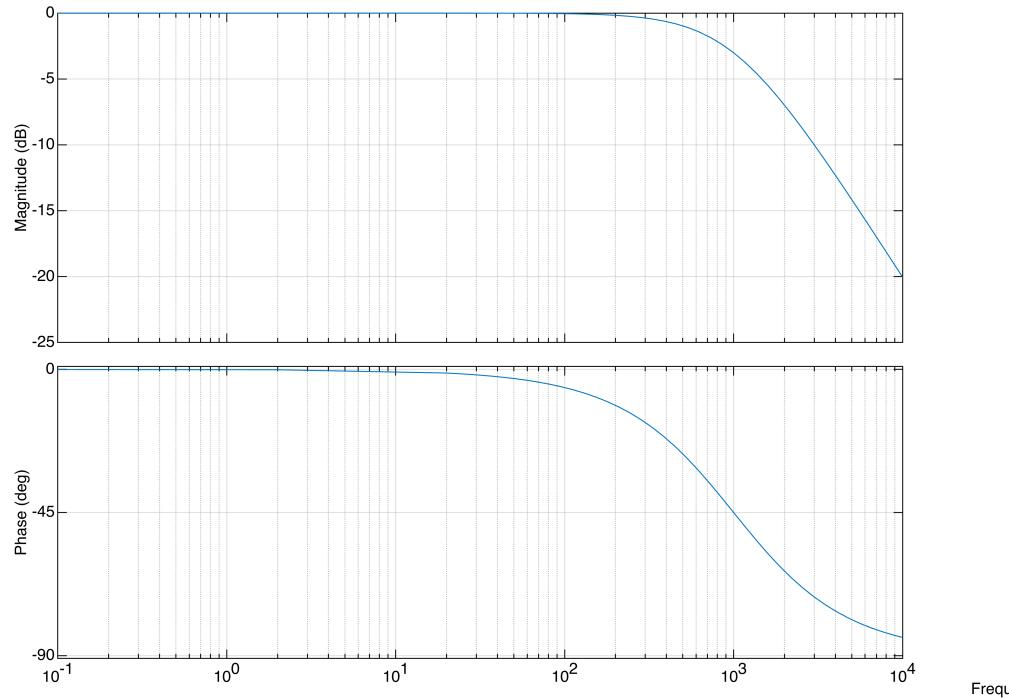
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{s}{100000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{100000000}}$$

tf =

$$\frac{6283}{s + 6283}$$

Continuous-time transfer function.



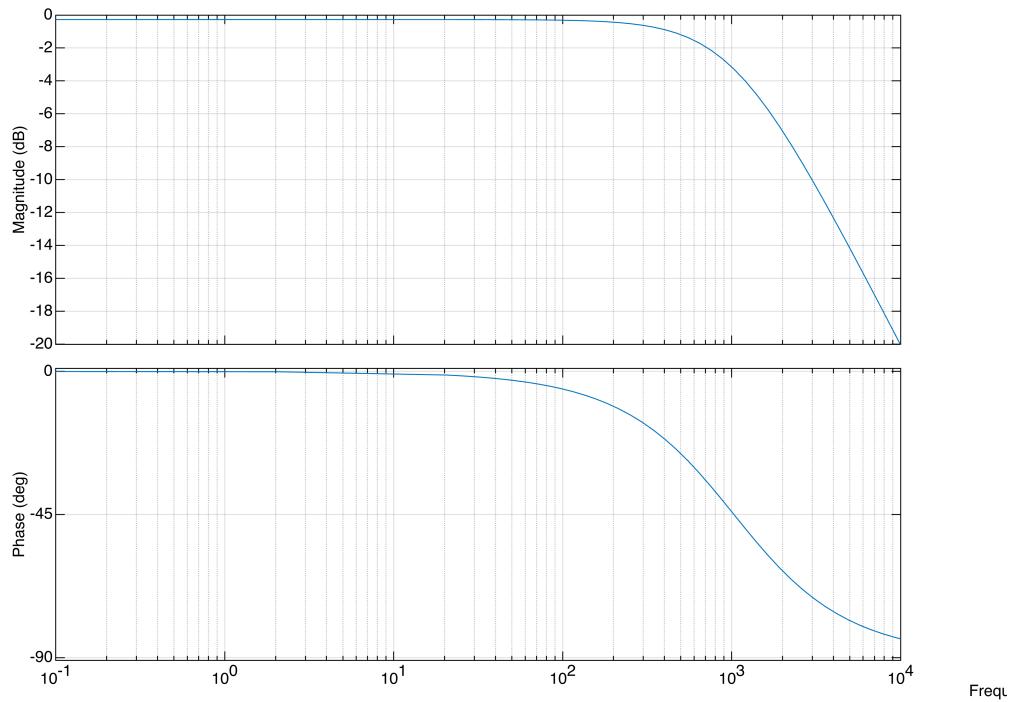
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{201 s}{100000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{100000000} + \frac{1}{500000}}$$

tf =

$$\frac{6283}{s + 6483}$$

Continuous-time transfer function.



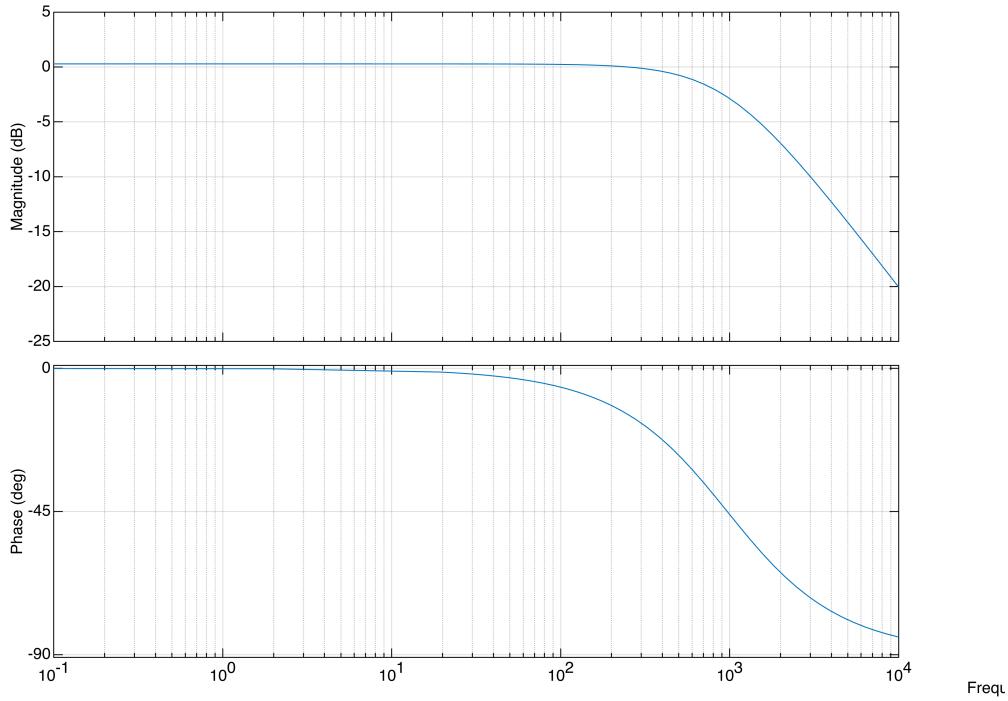
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{\pi}{50000} - \frac{199 s}{100000000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} - \frac{1}{500000}}$$

tf =

$$\frac{6283}{s + 6083}$$

Continuous-time transfer function.



```
% non-zero speed with EMF and resistance error
K_e = 0.2;
for Rerror = [1, 0.8, 1.2] % Resistance error
    for Kerror = [1, 0.9, 1.1] % EMF error
        R_hat_p = Rerror*R_p;
        K_hat_e = Kerror*K_e;

        EQ2_tf = subs(EQ2_2)
        tf = syms2tf(simplify(rhs(EQ2_tf)))

        figure
        bodeplot(tf,opts)
        title(['Active Resistance with ', num2str((Kerror-1)*100), '% Back EMF Error, '])
        drawnow
    end
end
```

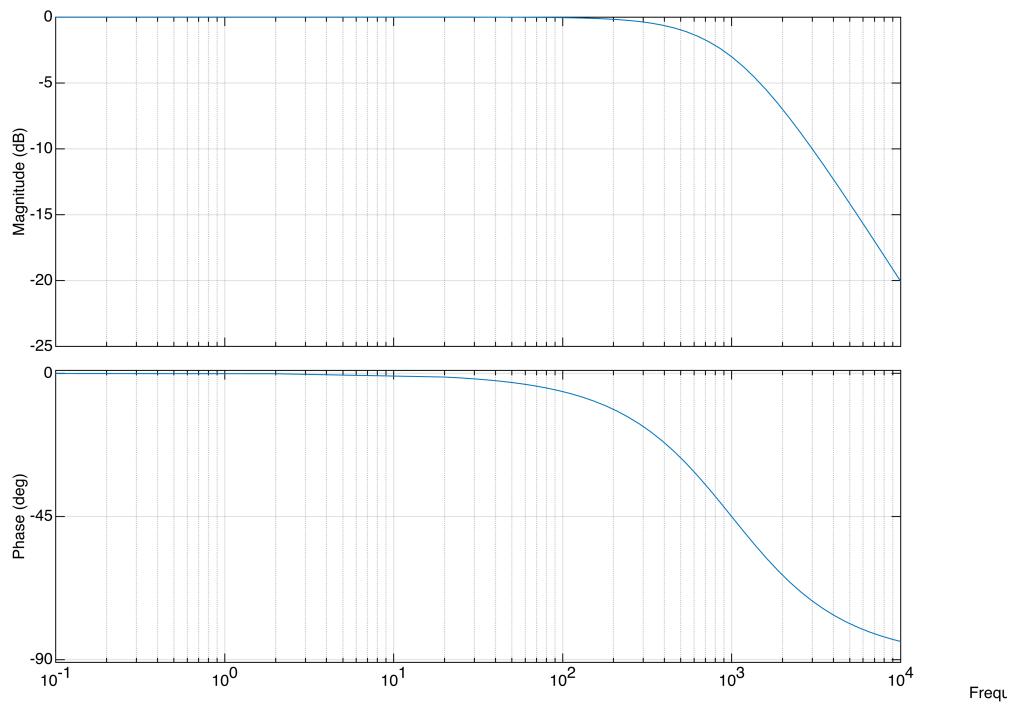
$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{s}{1000000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000}}$$

tf =

6283

s + 6283

Continuous-time transfer function.



EQ2_tf =

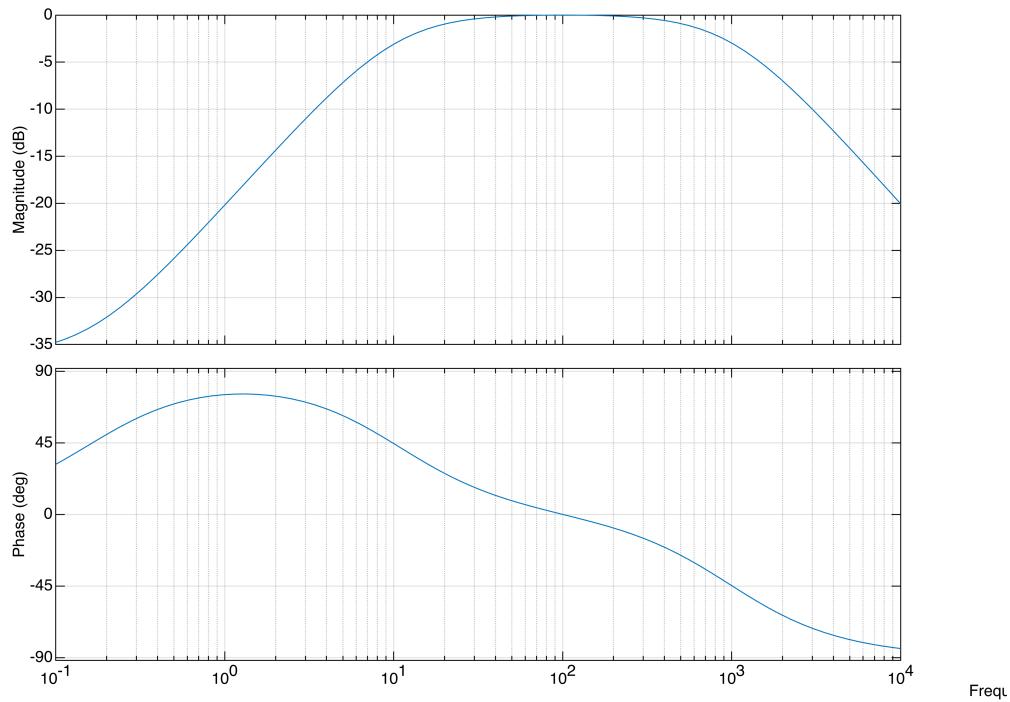
$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{s}{100000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{100000000} + \frac{1}{250}}$$

tf =

6283 s + 6283

s^2 + 6284 s + 4.063e05

Continuous-time transfer function.



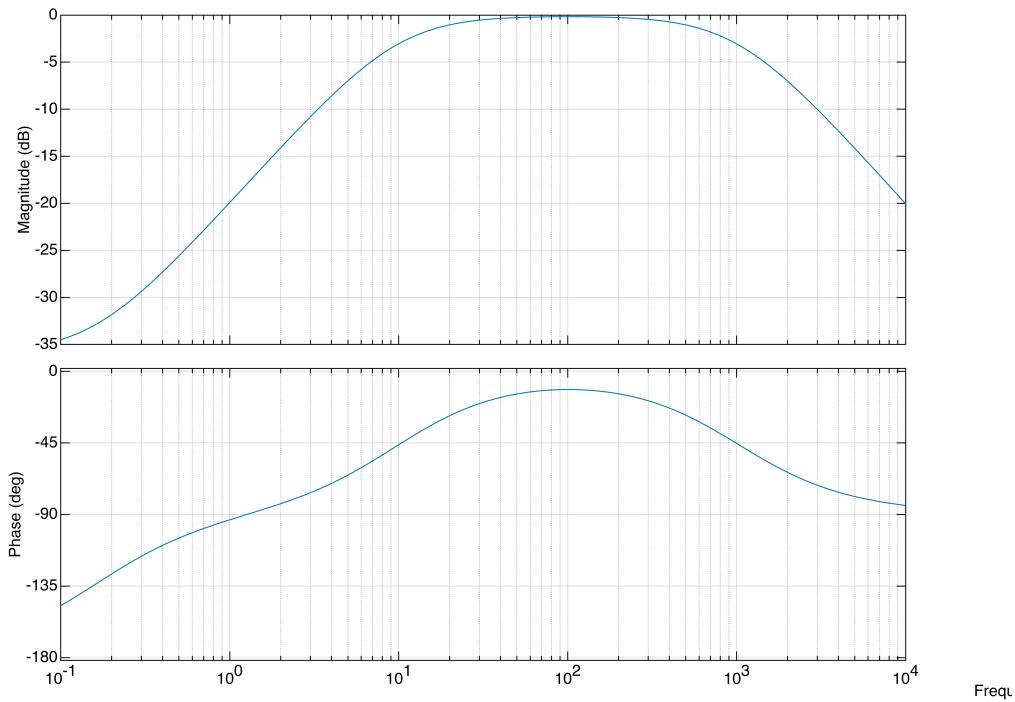
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{s}{1000000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} - \frac{1}{250}}$$

tf =

$$\frac{6283 s + 6283}{s^2 + 6284 s - 3.937e05}$$

Continuous-time transfer function.



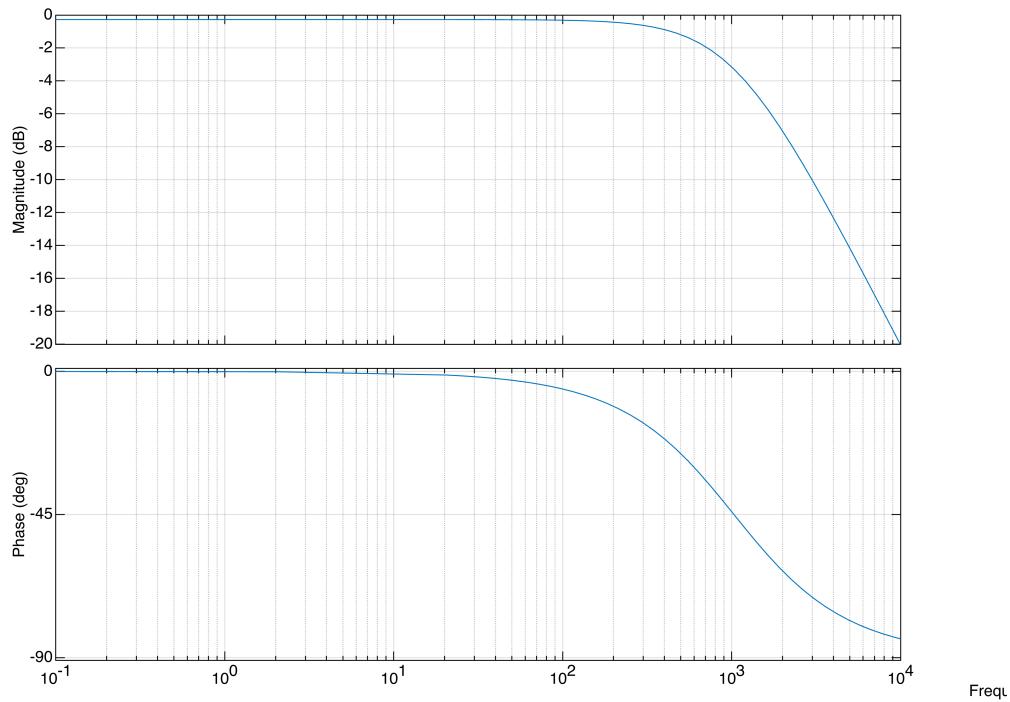
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{201 s}{1000000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} + \frac{1}{500000}}$$

tf =

$$\frac{6283}{s + 6483}$$

Continuous-time transfer function.



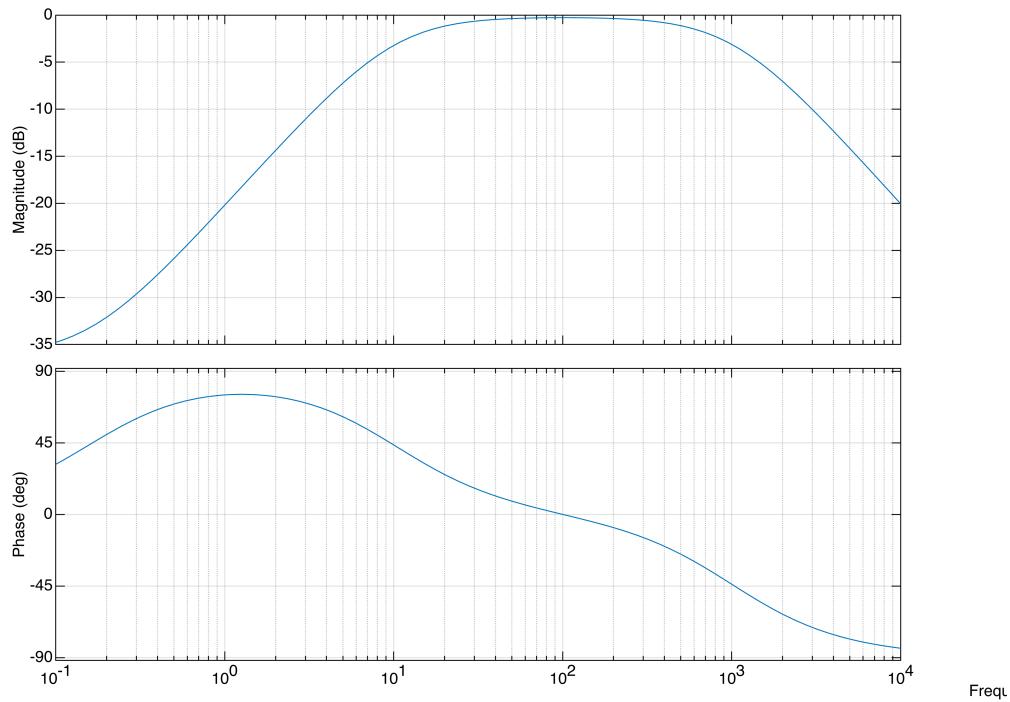
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{201 s}{1000000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} + \frac{2001}{500000}}$$

tf =

$$\frac{6283 s + 6283}{s^2 + 6484 s + 4.065e05}$$

Continuous-time transfer function.



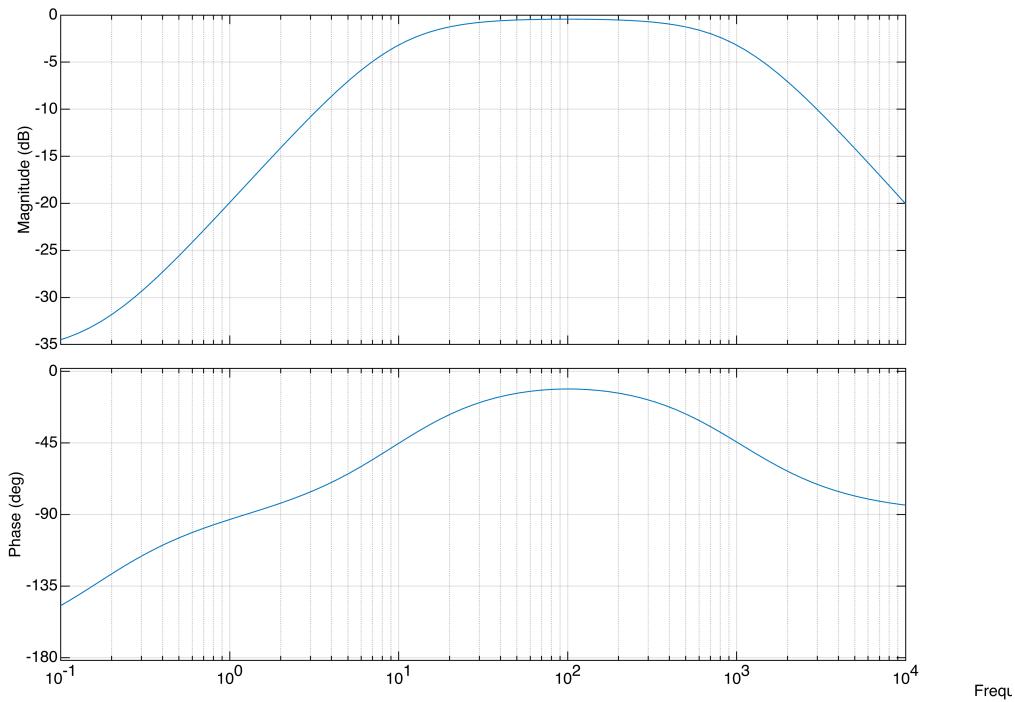
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{201 s}{1000000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} - \frac{1999}{500000}}$$

tf =

$$\frac{6283 s + 6283}{s^2 + 6484 s - 3.935e05}$$

Continuous-time transfer function.



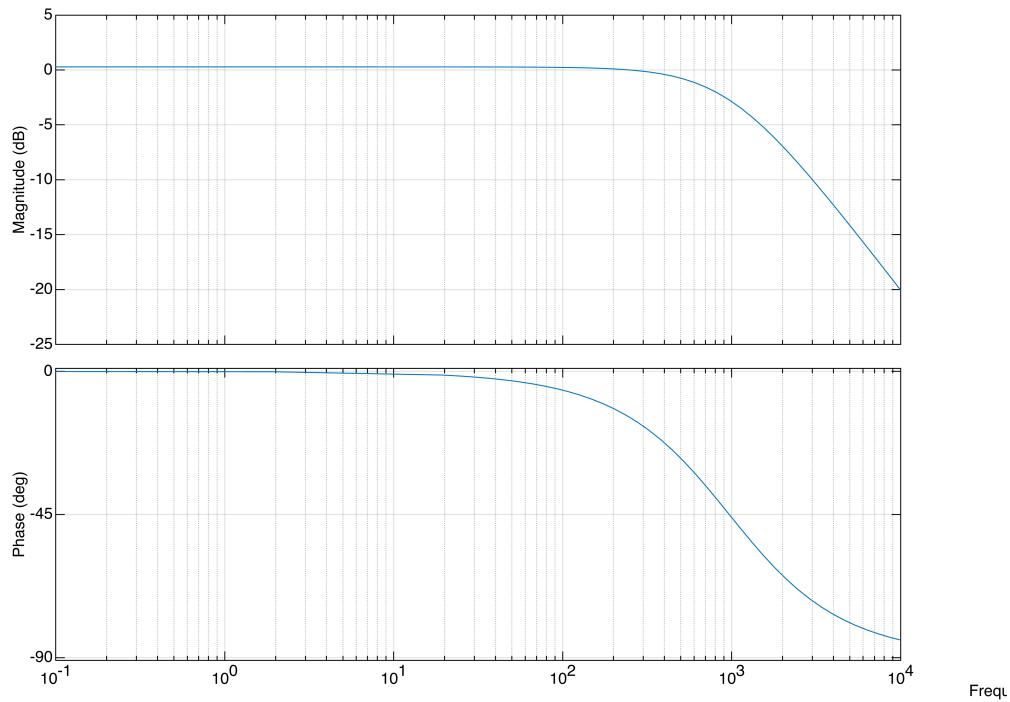
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{\pi}{50000} - \frac{199 s}{100000000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} - \frac{1}{500000}}$$

tf =

$$\frac{6283}{s + 6083}$$

Continuous-time transfer function.



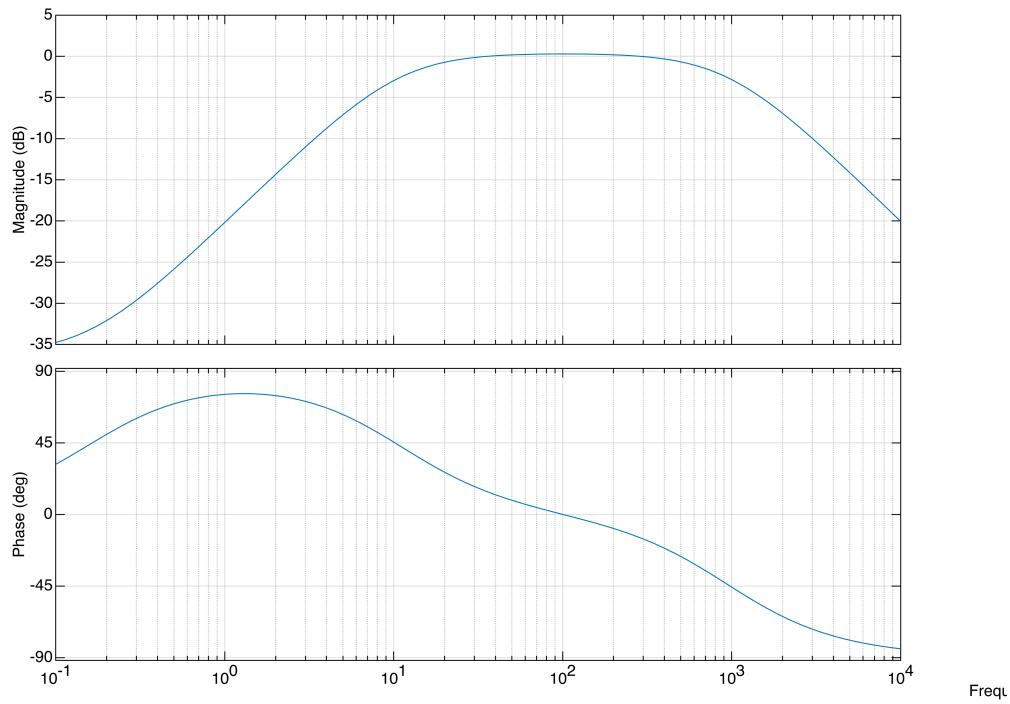
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{\pi}{50000} - \frac{199 s}{100000000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} + \frac{1999}{500000}}$$

tf =

$$\frac{6283 s + 6283}{s^2 + 6084 s + 4.061e05}$$

Continuous-time transfer function.



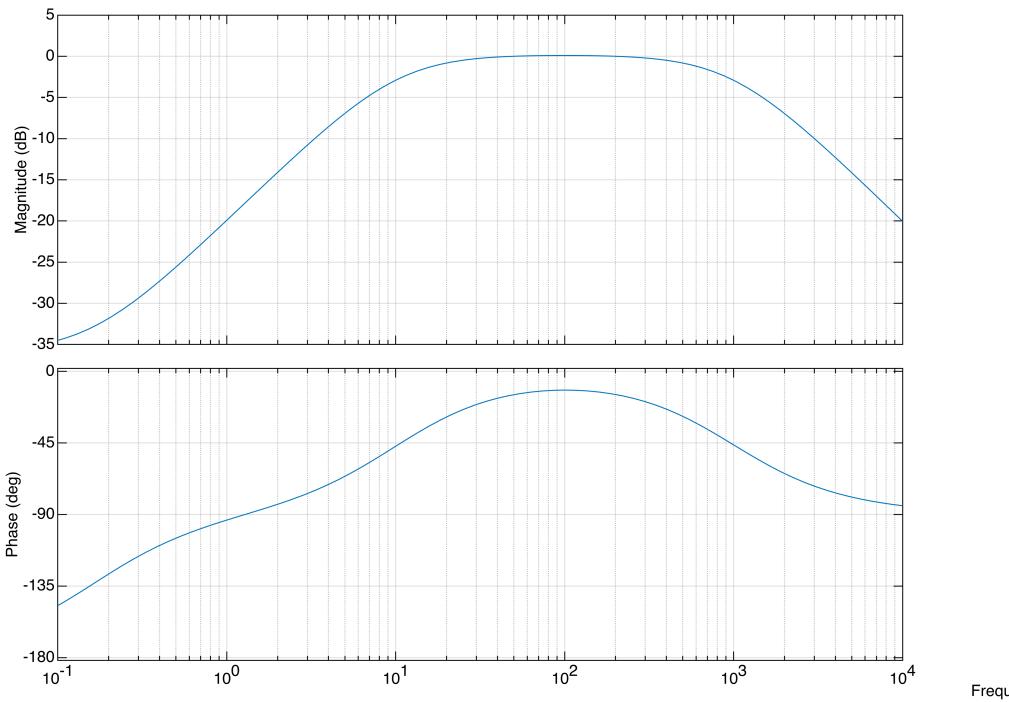
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{\pi}{50000} - \frac{199 s}{100000000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} - \frac{2001}{500000}}$$

tf =

$$\frac{6283 s + 6283}{s^2 + 6084 s - 3.939e05}$$

Continuous-time transfer function.



```
% without EMF decoupling
K_e = 0.2;
K_hat_e = 0;
for Rerror = [1, 0.8, 1.2]
    R_hat_p = Rerror*R_p;

    EQ2_tf = subs(EQ2_2)
    tf = sym2tf(simplify(rhs(EQ2_tf)))

    figure
    bodeplot(tf,opts)
    title(['Active Resistance Zero Speed without Back EMF Decoupling, and ', num2str((1-K_hat_e)/K_hat_e)])
    drawnow
end
```

```
EQ2_tf =

$$\frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{s}{100000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{100000000} + \frac{1}{25}}$$

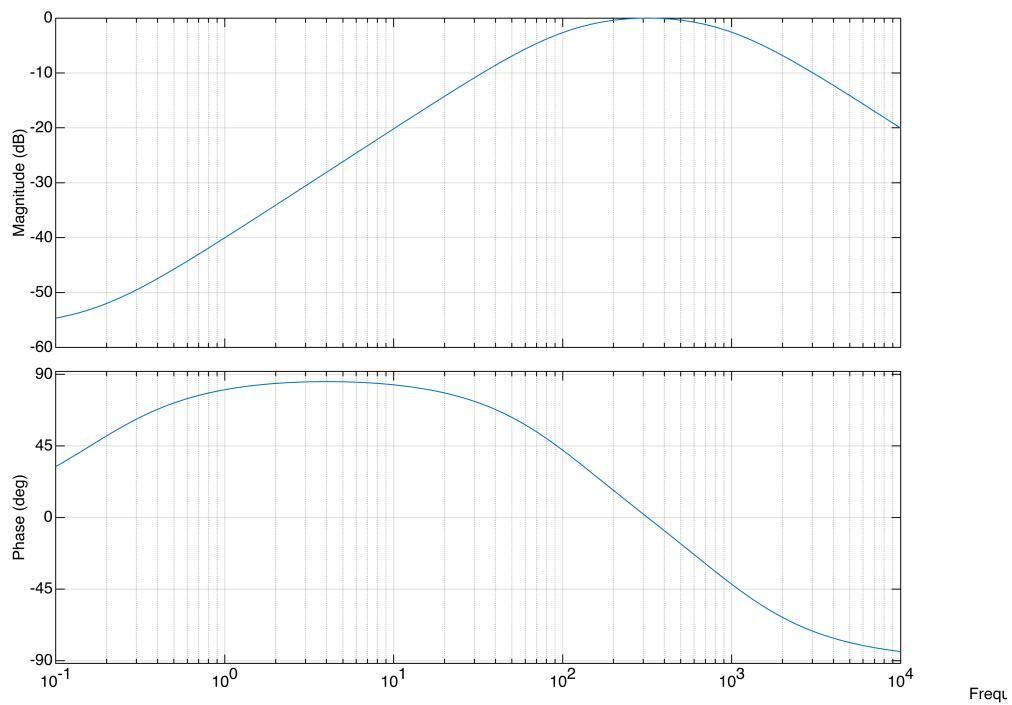
tf =

$$\frac{6283 s + 6283}{-----}$$

```

$$s^2 + 6284 s + 4.006e06$$

Continuous-time transfer function.



EQ2_tf =

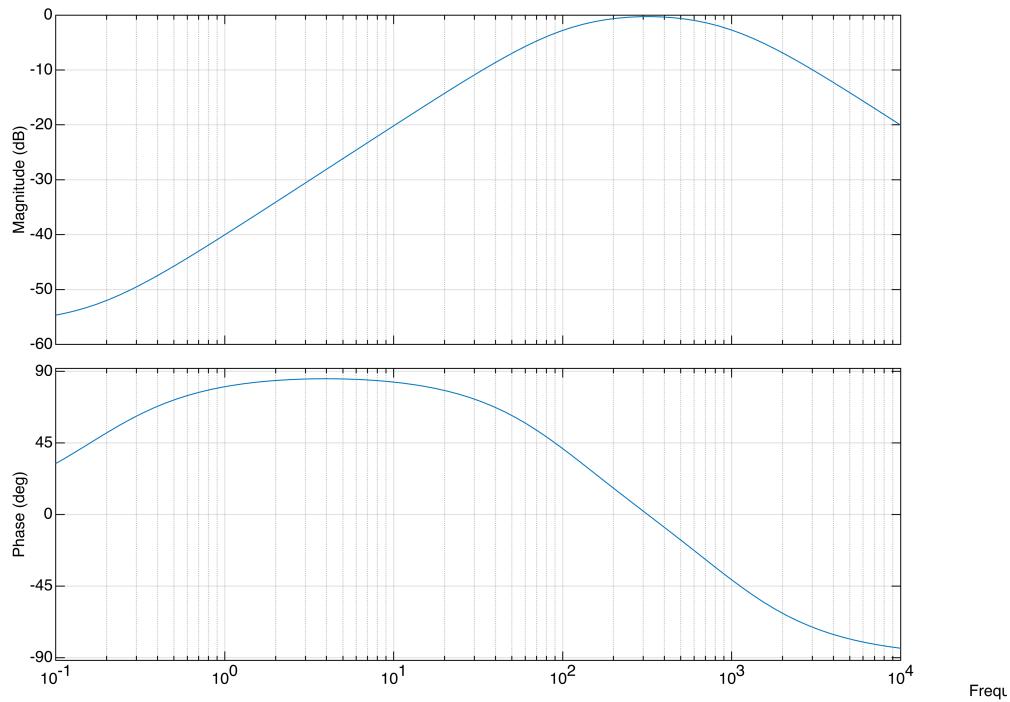
$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{201 s}{100000000} + \frac{\pi}{50000} + \frac{\pi s}{50000} + \frac{s^2}{100000000} + \frac{20001}{500000}}$$

tf =

$$6283 s + 6283$$

$$\frac{6283 s + 6283}{s^2 + 6484 s + 4.006e06}$$

Continuous-time transfer function.



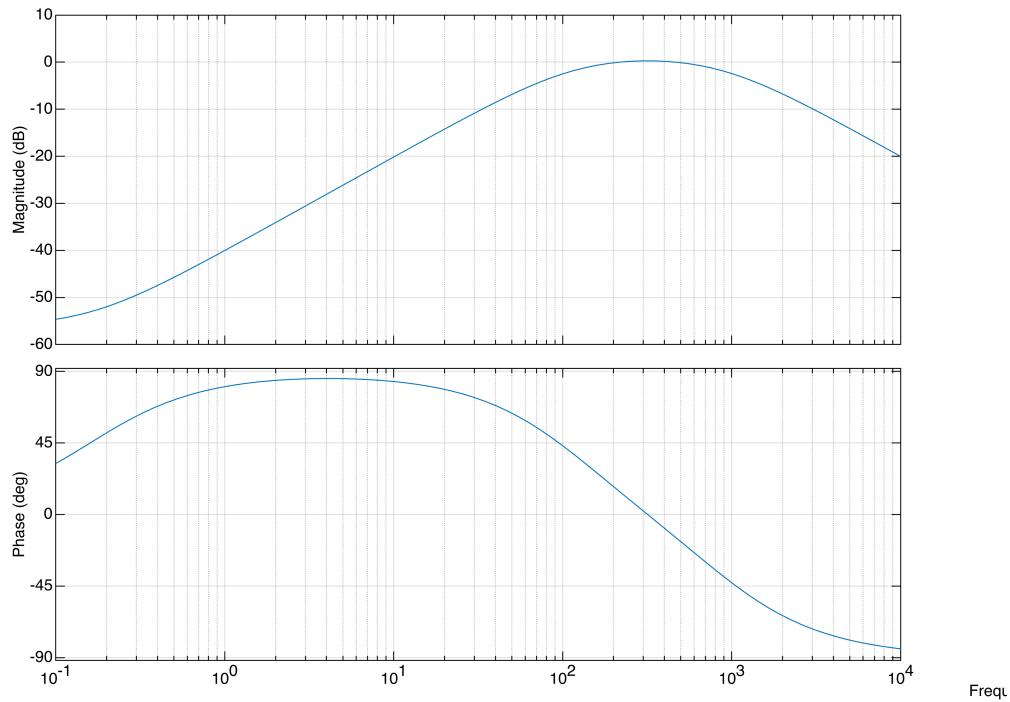
EQ2_tf =

$$\frac{i}{i^*} = \frac{\frac{\pi}{50000} + \frac{\pi s}{50000}}{\frac{\pi}{50000} - \frac{199 s}{100000000} + \frac{\pi s}{50000} + \frac{s^2}{1000000000} + \frac{19999}{500000}}$$

tf =

$$\frac{6283 s + 6283}{s^2 + 6084 s + 4.006e06}$$

Continuous-time transfer function.



d1. Current Response - Classical PI

```

syms f_c t

K_e = 0.2;
I_0 = 5;

for f_c = [1, 10, 100, 1000]
    % with EMF decoupling
    K_hat_e = K_e;
    EQ1_tf = subs(EQ1_2);
    tf1 = syms2tf(simplify(rhs(EQ1_tf)));

    % without EMF decoupling
    K_hat_e = 0;
    EQ1_tf = subs(EQ1_2);
    tf2 = syms2tf(simplify(rhs(EQ1_tf)));

    t = 0:0.001/f_c:(5/f_c);
    i_ast_t = I_0 * sin(2*pi*f_c*t);

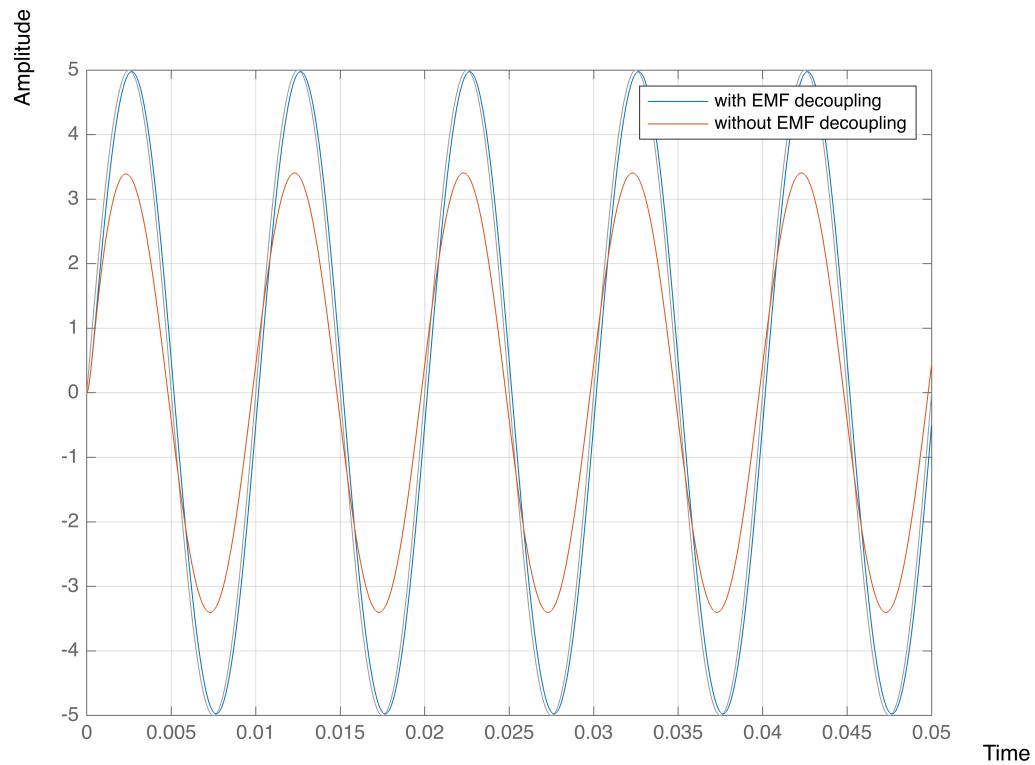
    figure
    lsim(tf1,tf2, i_ast_t,t)
    legend('with EMF decoupling', 'without EMF decoupling')

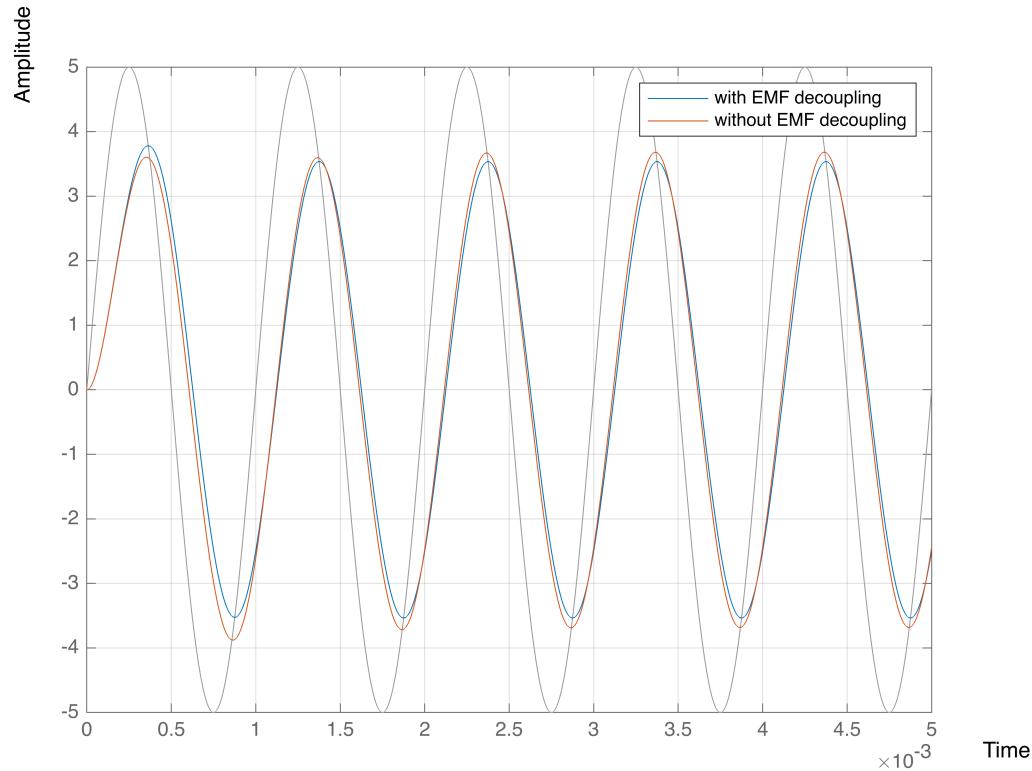
```

```

grid on
title(['Classical PI Current Response I(t) with fc =',num2str(f_c),'Hz'])
drawnow
end

```





d2. Current Response - Active Resistance

```

K_e = 0.2;
R_hat_p = R_p;
I_0 = 5;

for f_c = [1, 10, 100, 1000]
    % with EMF decoupling
    K_hat_e = 0.2;
    EQ2_tf = subs(EQ2_2);
    tf1 = syms2tf(simplify(rhs(EQ2_tf)));

    % without EMF decoupling
    K_hat_e = 0;
    EQ2_tf = subs(EQ2_2);
    tf2 = syms2tf(simplify(rhs(EQ2_tf)));

    t = 0:0.001/f_c:(5/f_c);
    i_ast_t = I_0 * sin(2*pi*f_c*t);

    figure
    lsim(tf1,tf2, i_ast_t,t)
    legend('with EMF decoupling', 'without EMF decoupling')

```

```

grid on
title(['Active Resistance Current Response I(t) with fc =',num2str(f_c), 'Hz'])
drawnow
end

```

