

CSCI-SHU 210 Data Structures

Assignment 2 Complexity

This assignment will be manually graded. Please submit on NYU Classes only.

Question 1 (Count Primes):

Implement function, `count_primes(n)`, to count the number of prime numbers less than a non-negative number, `n`.

Example:

Input: `count_primes(10)`
Returns: 4
Explanation: There are four prime numbers less than 10: 2, 3, 5, 7

Important:

- What is the runtime for your program? Mention your runtime in your .py file.

Question 2 (Merge):

Write a `merge(l1,l2)` function that takes two iterable objects and merges them alternately, once one runs out it continues from the other. Your algorithm should take $O(n)$ time. For example, it should work as follows:

```
print([i for i in merge( range(5),range(100,105))])  
print([i for i in merge( range(5),range(100,101))])  
print([i for i in merge( range(1),range(100,105))])
```

should output:

```
[0, 100, 1, 101, 2, 102, 3, 103, 4, 104]  
[0, 100, 1, 2, 3, 4]  
[0, 100, 101, 102, 103, 104]
```

Question 3 (Largest Ten):

Implement an efficient algorithm (Python code) for finding the **ten largest elements** in a list of size n .

```
Input: largest_ten([9,8,6,4,22,68,96,212,52,12,6,8,99,128])
Returns: [212, 128, 99, 96, 68, 52, 22, 12, 9, 8] # Order doesn't matter.
```

Important:

- You should avoid modifying original sequence.
- You can assume input list size is always greater than 10.
- What is the runtime for your program? Mention your runtime in your .py file.

Question 4 (Min-Max):

Implement an algorithm (Python code) for finding both the minimum and maximum of n numbers using fewer than $3n/2$ comparisons. Show that total number of comparisons is less than or equal to $3n/2$.

(Hint1: First, construct a group of candidate minimums and a group of candidate maximums.).

(Hint2: For simplicity, list sizes are even only).

```
Input: min_max([6,2,5,8,1,-4,6,12,78,21,55,62,1,0])
Returns: (-4, 78) # Order matters. First term is min, second term is max.
           # You can return a list, or a tuple to contain your results.
```

Question 5 (Three-Way Disjoint problem):

Given three sets of items, A, B, and C, they are **Three-Way Set Disjoint** if there is no element common to all three sets, i.e., there exists no x such that x is in A, B, and C. In the text book, two solutions of **Three-Way Set Disjointness** is described which run time complexity is $O(n^3)$ and $O(n^2)$.

Implement an algorithm (Python Code) that solves the **Three Way Set Disjoint** problem using $O(n \log n)$ time. (Hint: Use $O(n \log n)$ sorting algorithm).

```
l1 = [1,2,3,4,5]
l2 = [6,7,8,9,10,11,12]
l3 = [5,13,14,15,16]
l4 = [5,6,7,8,9,10,11]

Input1:three_way_disjoint(l1,l2,l3)
Returns: True

Input2:three_way_disjoint(l1,l4,l3))
Returns: False
```

Question 6 (Why is $O(n^2)$ faster than $O(n \log n)$ sometimes?):

Al and Bob are arguing about their algorithms. Al claims his $O(n \log n)$ -time method is always faster than Bob's $O(n^2)$ -time method. To settle the issue, they perform a set of experiments. To Al's dismay, they find that if $n < 100$, the $O(n^2)$ -time algorithm runs faster, and only when $n \geq 100$, $O(n \log n)$ -time one runs faster. Explain how this is possible.

Important:

- You can submit a .txt file for this question.