Market Entry Strategies for City-Based Platforms

Qing Wei *

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Abstract

In this paper, I build a theoretical framework to study the relative importance of multiple factors on entry decision of city-based platforms with homogeneous products. I find that besides high network effect, high switching cost, low market size, and low post-entry market return could also lead to market concentration. The equilibrium in the two-player static game implies that the multi-sided and transaction efficiency property related to a platform-based market could make large cities more attractive to the second mover than in a non-platform–based market. Then I extend the static game to a dynamic game. In addition to the findings in the static game, the dynamic game allows for multiple entries under different timing; it is also able to plot the expansion path of both players within the structure of a platform-based market, and the conditions for a second mover to take over the market leadership become more stringent. The capability of capturing the largest city ahead of its rival competitor is crucial in winning the market leadership for both players. If a second mover lost the opportunity of capturing the largest market, it might have to raise a huge amount of money to overcome its disadvantage in the later competition.

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1 Introduction

Today, more and more businesses are organized around platforms, via which multiple sides interact to conduct transactions, such as social media (Facebook), online trading platform (eBay), ride-sharing app (Uber), etc. Most of these platforms are viewed as two-sided or multi-sided (K. Boudreau 2010; K. J. Boudreau 2012). This is different from the traditional business model. Because of information sharing and preference transparency, service provider and consumer seem to be more evenly matched with each other on platforms, and tradings in such markets are generally more frequent, which can result in greater transaction volume in a platform-based market (Gale and Shapley 1962; Kelso Jr and Crawford 1982; Kamecke 1992) .

Other important research streams studying business operations via platform are focused on network effect and switching cost (Klemperer 1987; Lam 2017). The number of total users active on one platform could be a positive component in a user's utility function, such as social media (Facebook and Twitter); because these platforms are highly concentrated, the network effect makes it difficult for a second mover to takeover the market leadership (Katz and Shapiro 1994; Anderson 1998). While some platforms failed to maintain the market leadership against later entrants, such as when Yahoo! lost the market leadership in search engine service to later entrant Google.

Some platforms have a lower switching cost, customers can easily home in on multiple platforms, such as Amazon and eBay, purchasing merchandise from both. Some platforms have higher switching cost, making it difficult for users to practice multi-homing,(e.g., cellphone users—once a cellphone is purchased, it can only support apps from one operating system; or video game players—games are incompatible between different consoles).

Another important feature, sometimes mentioned in previous platform-based or multi-sided literature is the "boundary" of markets (Eisenmann, Parker, and Van Alstyne 2006). The majority of the platforms such as eBay, YouTube, and Xbox, compete in one aggregate market, so their entry decision has broader boundaries; once an entry decision is made, soon it will open access to all users on both sides; in this way, we may only observe one entry movement at one specific time. Some platforms have narrower market boundaries, such as Groupon and Uber. These platforms need to make contracts with customers and suppliers from a local base (Table 2 lists several examples of city-based platforms.), so their expansion paths are quite different from the platforms I mentioned earlier, in that the entry decision could be more discrete and more dynamic. One may observe several entries with different timing. In this way, cities' characteristics and timing of decisions (Dowell and Swaminathan 2006; Fuentelsaz, Gomez, and Polo 2002) will have a big influence on a platform's final performance, and the traditional model, which studies the interaction in only one aggregate market, may not apply here.

City-based platforms are emerging businesses attracting growing attention. On the one hand, debates over government regulations on platforms such as Uber and Lyft are never ending (Kottasova 2016; Isaac 2016), and ride-sharing services are still being treated as illegal in some countries (such as Denmark and Bulgaria); on the other hand, vast commercial potential attracts global investors to the market in swarms.Therefore, re-examining market entry strategies under city-based, platform-based market frameworks is of critical importance for regulators and for investors.

In this paper I build a theoretical framework for market-entry strategies of city-based platforms. Besides static equilibriums, I also aim to model the entry dynamic between two players in multiple markets. Consistent with previous research (e.g. Farrell and Klemperer 2007), I find that large network effect, insufficient market size, and large multi-homing cost are all lead to market concentration. Moreover, uncertainty in market return may give the second mover more of a chance to catch up with the first mover in a dynamic game. I also find that the transaction efficiency in platform-based markets will make the first several largest cities more attractive to a second mover than a non-platform-based market. In this way, occupying the largest city becomes crucial. For a second mover to take over market leadership, he either needs sufficient initial funds to capture the largest market before the first mover does, or the market is very open for a second mover with low network effect and adept at multi-homing.

Although, there have been substantial increases in the literature studying multi-sided markets (e.g., Armstrong 2006; Rochet and Tirole 2003, 2006; Rysman 2009; Jullien 2011.), most of the theoretical work focuses on optimal pricing strategies and interactions between sides. This work complements this theoretical stream by offering a dynamic approach to interaction between firms.

This work also contributes to the literature studying the order-of-entry problem (e.g., Lambkin 1988; Mitchell 1989; Lilien and Yoon 1990; Mitchell 1991; Golder and Tellis 1993; Lee 2008) by examining the effect of timing on platforms' entries in multiple segregate markets.

Some other related works on platform entry problems are concluded as follow: Zhu and Iansiti (2012) built a theoretical model and found that an entrant's success depends on the strength of indirect network effects and on the consumer expectation of future applications. They empirically examined the model applicability by investigating the video game industry. Dewenter, Rösch, et al. (2012) analyzed the impact of indirect network effects in emerging two-sided markets on market entry, and found that, when network effect is strong, market entry will no longer occur, this leads to a natural monopoly. Seamans and Zhu (2013) empirically investigated the impact of Craigslist's entry on local newspaper; and Kim, Lee, and Park (2013) empirically studied the two-sided market entry strategies in the online daily deals promotion industry. This paper differs from the above research in that previous researchers mostly focused on post-entry competeting strategies or empirically investigated the welfare implications brought from an entrant platform. While this paper provides a theoretical approach for dynamic entry decisions of platforms into segregated markets.

Although the model in this research applies to many city-based platforms, the motivation of starting this research comes from several interesting unexplained observations of the major ride-sharing platforms, Uber and Lyft. First, both Uber and Lyft have a strong preference for large cities. Uber made its first entry into San Francisco in 2010; then New York, Seattle, and Chicago in 2011; and San Diego, Los Angeles, Philadelphia, Atlanta, and so on, in 2012. Two years later, Lyft also launched its first ride in San Francisco, then Los Angeles, Seattle, Chicago, and so on. If we look at the first several entries of both platforms (Table 3), we might find that both of them have a strong preference for large cities. Things may be easily understood for Uber, since a large city means a large market. But if we look at Lyft's entry path, we notice that Lyft seems to have followed the same entry path as Uber: Lyft always chose to enter the markets already occupied by Uber, rather than exploring a new but smaller market, in the beginning. More evidence can be found from the launch choices of other ride-sharing platforms, such as Juno and Via; they both started in New York; similarly, SitBaq and Summon both started in San Francisco and the Bay area. So why do second mover ride-sharing platforms always choose to enter these large cities to face intensive competition, rather than exploring a new but smaller market?

Second, no matter the entry order and initial funds, Uber always tends to dominate the market. Before Uber started its first launch in San Francisco in 2010, it raised$1.3M in the angel round . Almost two years later, June 1, 2012, Lyft, a ride-sharing platform providing almost identical service, announced its first launch in the same city, San Francisco; and this time, with Uber already proving the potential of ride-sharing business, Lyft raised $7.3M for its debut launch. However, a more successful initial funding round didn't help Lyft become the market leader. Figure 1 is a report of Uber& Lyft market share in major cities in 2016 (Peltier 2016). We can see from the figure that market share of both firms are around the same ratio 20:80, so Lyft doesn't gain much market share over Uber. Although most of the cites where Uber makes the first entry, there are also several special cases (Table 4). For these four cities, Uber and Lyft entered at almost the same time; for Miami and Austin, Lyft even entered several days ahead. But looking at Figure 1, Lyft only owned around 16% to 17% of the market, for Houston it was especially low, only 3%. Why is the market share so constant across the country, and why does Uber always dominate the market even when Lyft was the a first mover?

This research could also join the recent emerging studies on ride-sharing platforms (e.g. Li, Hong, and Zhang 2016; Hall, Horton, and Knoepfle 2017; Hall and Krueger 2018; Cramer and Krueger 2016; Greenwood and Wattal 2015; Chen, Mislove, and Wilson 2015).

The purpose of this study is to apply mathematical programming method in economic models addressing the following questions: 1) What determines the expansion path of city-based platforms? 2)What kind of markets are capable of holding more entrants? and 3) How can a later entrant take over market leadership?

The rest of the paper is arranged as follows: In section 2, I will propose the fundamental model setups applied in later chapters; then, in section 3, I solve for equilibriums in static games under different market conditions. In section 4, I extend the static game to a dynamic game, and provide numerical experiments in section 5. Finally, in section 6, I will state my conclusions and discuss future research potential.

2 The General Rules

Before we go in to the static game, I will go through three fundamental rules, which will be applied in the later static game and dynamic game, taking account of market size, pre-entry uncertainty and the market-splitting rule. These rules will build a general model framework for the city-based platform entry problem.

Transaction Volume

Suppose each city is a market, and platforms face a line of potential cities ranked by population size. As in Table 5, the largest city in the U.S is New York City; its population is approximately twice the population of the second largest city, Los Angeles; and the third largest city Chicago, is nearly one-third the population of NYC, and so on. This phenomenon (rule) is called "Zipf's Law for Cites" (Zipf et al. 1949; Xavier 1999).

Also, suppose platforms compete over a fixed size of population in each city, the market rank K has $\frac{N}{K}$ service consumer and $\frac{M}{K}$ service provider on each side. Consider a fully efficient trading platform, so that every service provider is capable of interacting with every service counsumer. The potential transaction volume of such a platform will be similar to what it is in traditional two-sided market literature (Rochet and Tirole 2003): simply multiply the supply side and demand side together which equals $\frac{NM}{K^2}$. From this setup, the market potential transaction volume is substantially enlarged in large cities. For instance, the first largest market is 4 timesmthe second largest market and 9 times the third ($\frac{NM}{1}$ compares with $\frac{NM}{4}$ and $\frac{NM}{9}$).

Pre-entry Market Uncertainty

Before entry, the average benefit α_k of each potential transaction in the Kth market is unobservable to both platforms. However, platforms know the distribution of the benefit $\alpha_K \sim F(\alpha_K)$. Once market K was being explored, α_K would reveal to all. One can consider this α_K as platform specific city (exogenous) characteristics; because, for different types of businesses such as Groupon for deals, Airbnb for room-sharing, Uber for ride-sharing, etc., α_K should have different values. Here, α_K has two meanings: first, it is the platforms' average profit per potential transaction, which means it takes account the revenue and cost at the same time; second it also represents the utilization rate of a platform in the Kth market, because $\frac{NM}{K^2}$ is the ideal transaction volume, in reality, based on city culture and city characteristics, some cities may use the platform more frequently, other cities may not use the platform very much. So α_K also provides a scalar for each city's preference.

Market Share

Suppose users in one representative market i are facing a nested choice problem, there is no outside option available within this choice set. Before the second mover enters the market, the incumbent gets all the market share, because no other choice is available. And after the entry of the second mover, the utility of an average user choosing platform n in one transaction will be :

$$U_{nit} = \begin{cases} V_{nit} + \gamma x_{nt} & \text{, if platform n is a first mover} \\ V_{nit} + \gamma x_{nt} - h_n & \text{, if platform n is a second mover} \end{cases} \tag{1}$$

$$x_{nt} = \frac{m_{nt}}{m_{nt} + m_{-nt}} \tag{2}$$

Where $\gamma \in [0, \infty)$ represents the parameter of network effect larger γ means larger network effect, m_{nt}, m_{-nt} are platform sizes of the player and it's component, it is calculated by amount of users already captured by a platform; x_{nt} is the normalized relative firm size; V_{nit} is the average user utility gains from service provided by platform n; it is related to service price, service quality and other platform characteristics. And parameter h_n represents the dis-utility of switching from incumbent platform $-n$ to entrant platform n, if n is a second mover.

The market share of each platform based on logit choice model is:

$$S_{nit} = \begin{cases} \dfrac{e^{(V_{nit}+\gamma x_{nt})}}{e^{(V_{nit}+\gamma x_{nt})} + e^{(V_{nit}+\gamma x_{nt}-h_n)}} & \text{, for platform n is a first mover} \\ \dfrac{e^{(V_{nit}+\gamma x_{nt}-h_n)}}{e^{(V_{nit}+\gamma x_{nt})} + e^{(V_{nit}+\gamma x_{nt}-h_n)}} & \text{, for platform n is a second mover} \end{cases} \tag{3}$$

If we consider a homogeneous case, that switching cost and average service utility are the same for both platforms, so that $h_n = h_{-n} = h$ and $V_{nit} = V_{-nit} = V$, the share functions will become:

$$S_{int} = \begin{cases} \dfrac{(e^{x_{nt}})^{\gamma}}{(e^{x_{nt}})^{\gamma} + p(e^{x_{-nt}})^{\gamma}} & \text{, for platform n is a first mover} \\ \dfrac{p(e^{x_{nt}})^{\gamma}}{(e^{x_{-nt}})^{\gamma} + p(e^{x_{nt}})^{\gamma}} & \text{, for platform n is a second mover} \end{cases} \tag{4}$$

Where $p = e^{-h}, \in (0, 1]$[1], when $h = 0$, $p = e^{-p} = 1$, there is no switching cost; when

[1] Note that their might be the case of second mover advantage $p > 1$, but it is beyond the discussion of this paper. In this paper I regard the second mover advantage as ahead of disclosure of hidden market information, or the chance of avoiding a bad return market, which has already been included in the

$h \to \infty$, $p = \lim_{h\to\infty} e^{-h} = 0$, the switching cost of a new platform is extremely large, users are fully unacceptable for a second mover.

So the final rule of market share is concluded as follows: When a platform n decides to enter a new market without any incumbent, it will capture the entire market; when a platform n decides to enter a market already occupied by another incumbent $-n$, they will split the market:

$$S_{nit} = \begin{cases} 1, & \text{if market i has 0 incumbent} \\ \dfrac{p(e^{x_{nt}})^\gamma}{(e^{x_{nt}})^\gamma + p(e^{x_{-nt}})^\gamma}, & \text{if market i has 1 incumbent} \end{cases} \tag{5}$$

$$S_{-nit} = 1 - S_{nit} \tag{6}$$

Therefore, if there are two firms in the market, they will split the market based on their current firm sizes, parameter $p \in (0, 1]$, and parameter $\gamma \in [0, \infty]$. Note here, I basically assume that platforms are open to multi-homing, but with some degree of difficulty. In the utility function, the multi-homing difficulty is represented by dis-utility of using the entrant platform. Moreover, parameter p basically is a measurement of second mover disadvantage; but in the utility function, it is the switching cost per transaction from one platform to another. One might consider this switching cost has many sources, for example, the monetary cost induced by the platform that users have to pay some amount of money or lose the opportunity of earning some amount of benefit

model.

by switching to a new platform. Another example is adaption cost; users who are used to one platform will incur some dis-utility when switching to another one. Usually, researchers might consider firms offering heterogeneous products to have larger second mover disadvantages towards each other. Somehow, platforms providing similar goods can also establish a barrier to preventing customers from switching to another platform. For example Lyft and Uber offered power-drive bonus for drivers in some cities. If a driver completes a certain amount of rides within certain a period of time, the platform will pay she an extra bonus as reward. This method on the one hand keeps the driver active during rush hours; on the other hand, it creates an opportunity cost, which helps to make the drivers stick to the platform to earn the bonus. Another example is cellular service companies, usually customer has to pay a cancellation fee if she wants to switch to another service provider, and the service of the two companies could be very similar.

This market splitting rule makes sure that new entrants with zero market size will still gain some market share as a second mover, and provides a testable structure for different types of business models. Both γ and p should be treated as exogenous parameters related only to the type of platform, which means in this research, I will mainly discuss the case of homogeneous service platforms.

Consider a special case similar to the situation that the first mover has already been in the market for some time and gained some users, so it has some degree of network effect scaled by γ. Then the second mover just completes its seed round and starts to explore the markets. For the markets already occupied by the first mover, they will split the market following the rule in Equations 5 and 6. For example, refer to Figure 2a when $\gamma = 2$ and $p = 0.25$; the overall network effect of this market is very large, as the switching cost; the first mover will win 96.7% of the market and the second mover will only get 3.3%. Another scenario is described as when a small firm occupies a market first, then a big company enters the market as a second mover, refer to Figure 2b, where, as long as the network effect γ is not very large, an incumbent can still hold most of her current market even if she is facing a market giant.

3 Static Game

In this section, I will illustrate the above settings more clearly in a two-player static game.

First, I will make the following basic assumptions to build the static game 1) Firms are risk-neutral[2]; 2) A firm's average benefit from each potential transaction is drawn from one same distribution, i.e., the expectation of profit per potential transaction is the same for all markets: $E(\alpha_i) = \alpha \ \forall i \in M$, M is the set that includes all feasible markets 3) Once a market is being explored by a player, it will generate profits in every subsequent period; And 4) $p = 1$ no switching cost[3]

[2]This assumption is made to simplify the problem; the firms are not necessarily risk-neutral, but a firm's preference on risks is beyond the discussion of this paper.

[3]The purpose of inducing the measurement of switching cost, is to make both players have some degree of disadvantages when acting as a second mover, since in static game, only Player 2 has a chance to become second mover; network effect from period 0 will already make she disadvantaged when competing in the same market with Player 1. So, here $p = 1$ for simplicity

Game Setups 1 (Static Game)

1. **Time 0,** First mover, Player 1 with size $m_{10} = 0$ enters market K reveals market K's profit parameter α_k, and gain a profit $\frac{NM}{K^2}\alpha_k$, update $m_{11} = \frac{NM}{K^2}\alpha_k$;

2. **Time 1,** Second mover. Player 2 with size $m_{21} = 0$ comes into existence and both firms simultaneously decide which market to enter.

   (a) Player 1 gains profits from market K;

   (b) Since all markets have the same $E(\alpha)$, the dominant strategies for player 2 will be among the first several markets: K, K+1 and K+2, so it is the same with player 1's dominant strategies: market K+1 and K+2

   (c) If player 2 decides to enter market K, they will split the market based on their current relative size m_{11}, m_{21}, and γ, refer to Equation 7;

The payoff matrix of this static game can be seen in Table 6, where S_1 means the market share of Player 1 as first mover; $1 - S_1$ means the market share of Player 2 as second mover.

$$S_1 = \frac{(e^1)^\gamma}{(e^1)^\gamma + (e^0)^\gamma} \tag{7}$$

3.1 Static Equilibriums

Proposition 1 When Player 2 chooses action K, Player 1 will always choose K+1; when Player 2 chooses action K+2, Player 1 will always choose K+1; when Player 2 chooses

K+1, if and only if $K < \frac{2\sqrt{S_1}-1}{1-\sqrt{S_1}}$ Player 1 will choose K+1, otherwise Player1 will choose K+2.

Proposition 2 When Player 1 choose action K+1, Player 2 will choose K, K+1 or K+2; when Player 1 choose action K+2, Player 2 will choose K, K+1 or K+2.

Proposition 3 Four PSNE (pure strategy Nash equilibrium) and two MSNE (mixed strategy Nash equilibrium) exist in the myopia static game. They are PSNE (K+1,K), (K+1,K+1), (K+1,K+2), (K+2,K+1); and MSNE in anti-coordination game, (K+1,K+2) and (K+2,K+1), (K+1,K) and (K+2,K+1)

Proposition 4 For each pair of $\{\alpha_K, \gamma\}$, there exist a threshold $\bar{K}$ for $K \in \mathbf{Z}^+$, below which $\forall K < \bar{K}$, Player 2 will choose to compete in larger cities K or K+1, above which $\forall K > \bar{K}$, Player 2 will choose to enter a smaller city K+2 to avoid direct competition.

Intuitively, when players discover market K is good, Player 2 will be more likely to enter a good market in period 1. Similarly, when the market condition is welcoming for a second mover, that both the network effect and switching cost is lower, Player 2 will be more likely to face competition from Player 1. When K increases, the benefit from market size cannot cover the loss in competition; both players will tend to explore a new market, rather than compete in an old one. So, in this way one can expect the benefit of market size from high efficiency trading market to be held longer than a low efficiency trading setup, since the transaction volume is enlarged in the former one.

Figure 3 compares the evolution of PSNEs under fully a efficient transaction setup and less efficient transaction setups with different values of γ, K and α_K. We can see that PSNE (K+1, K+1) rarely happens- only when α_K is really below expectation

($\alpha_K = 0.1$), and the network effect of existing markets is really low ($\gamma < 0.3$), players tend to split the market in half, so that Player 2 will give up the first largest market and enter the second one. Only in this way, can we see two players simultaneously compete in the same market (K+1,K+1): mostly, a later entrant will either compete in the largest city (K) or explore a much smaller one (K+2).

From Figure 3 we can clearly see the evolution of threshold $\bar{K}$ with α_K and γ: When α_K increases, the margin for Player 1 to deviate to K+2 doesn't move, because from Proposition 1, we know that this line only related to the size of market share or at root related to γ; however the area of PSNE (K+1,K) becomes larger when α_K increases. If we take one horizontal slice of Figure 3 to take a look at the comparative statics, for example in the $\alpha_K = 1.0$ high transaction volume market scenario, if we hold $\gamma = 2$, and look at the change of different Nash equilibriums under different K, we might be able to get some insight in the firm's entry path. When markets are large, the second mover will always enter and compete in larger markets; then, with the decrease of market size, both firm will finally deviate to a non-aggressive strategy to avoid competition. And for low transaction markets, the expansion path would be similar, just with smaller market size, threshold $\bar{K}_1$ in all cases will always be smaller than $\bar{K}_2$ in a fully efficient market setup.

The same idea applies if we take a vertical slice from the figure, so that to hold market size constant and look at how network effect affects a firm's entry behavior, clearly, larger network effect will make second mover more disadvantaged when competing with the first mover.

So, in addition to the prediction in previous literature, that larger network effect will lead to more concentrated market structure, low market trading frequency, insufficient market size, or under-expected market return will also lead to market concentration.

Next, in Table 9 and Table 10, I list several specific numerical experiments under fully efficient platform-based markets versus inefficient non-platform-based markets respectively, with $E(\alpha_i) = 0.5$ and NM equals to some arbitrary positive [4] , to further illustrate the firm's behavior.

D_1, D_2 here represent Player 1's and Player 2's entry decisions, respectively. Market share of each firm is controlled by γ . From Tables 9 and 10 we can see how much the second mover favors the largest city in two-sided market - when $K = 1$, as long as the market yield is average level or above, the second mover in a platform-based market will always choose to enter the largest city (see case1-6). When conditions in the first market K are not good, $\alpha_k = 0.1$ is quite below the expectation value; player 2 has the advantage of information disclosure, he has the opportunity to avoid the bad ones, so he will enter market K+1, and K+2, and skip market K. In other scenarios, when K is larger, and city size is smaller, good profit and welcoming market environment can still be a strong incentive for Player 2 to enter a competitive market. However, in most of the cases when the benefit from a large market cannot cover players' loss from competition, player 2 will skip the second largest market and jump to the third to avoid direct competition with Player 1, because even without second mover disadvantages, the lack in existing network effect will still put him in an unfavorable competing position.

[4]In static game, the value of NM doesn't matter to the final Nash-equilibrium, because (refer to Table 6) NM will be cancelled from the nominator since it appears in all equations

Whereas the situation for Player 1 will be more favorable, holding the largest market K in hand, the network effect will give him many advantages in the further competition. So, when the market size is large enough, the dominant strategy for the first mover will always be continuing the previous exploration step in the next largest city, no matter the strategy of Player 2; when market size is not large enough (e.g. K=10) or the nature of the market doesn't have network effects (e.g. $\gamma = 0$, $p = 1$), there will be two MSNEs: two platforms will play anti-coordination game to avoid face to face competition.

An interesting scenario is described in (case 4 to 6), when $\alpha_k = E(\alpha) = 0.5$; one can regard these cases as: how would a risk-neutral second mover behave if the former incumbent doesn't reveal the market information?

3.2 Sequential Equilibriums

So far, I have discussed the static equilibriums under one-period static game, and the static game is based on the assumption that Player 1 does not foresee the advent of another competitor (or the discount rate is very large). In this subsection, I will briefly discuss the sequential game under the assumption that Player 1 does foresee the advent of Player 2's entry (or there is no discount rate). The game setup is similar:

Game Setups 2 (Sequential Game)

1. **Time 0,** Player 1 with size $m_{10} = 0$, chooses one among market K, K+1 and K+2 to enter, reveal the market information, gain a profit, and update m_{11}
2. **Time 1,** Second mover. Player 2 with size $m_{21} = 0$ comes into existence and both

firms simultaneously decide which market to enter.

The extensive game tree and payoff matrix of each node can be seen in Table 7, since before the first player enters the first market, no market information is known by either player, Player 1 will guess the state in the second period through expectation return $E(\alpha)$, so $E(\alpha)$ will be canceled from the payoff matrix. The subgame perfect Nash equilibrium is solved through backward induction in Figure 4, α_i here is the revealed market return after Player 1's first entry, i here could be K, K+1 or K+2.

In most of the cases, in the beginning period, Player 1 will choose the largest market K, under several scenarios, Player 1 will choose market K+1 or K+2; note here, if Player 1 doesn't enter market K in the beginning, market K will be a dominant strategy in the second period for Player 1 in the subgame, which means, in the real world, if the degree of network effect γ is uniformly distributed, most of the time, we will observe entries into the largest market. Or, in another perspective, if the degree of network effect is uncertain but follows a uniform distribution, for a start-up platform, entering the largest market is the safest strategy, because it has the highest probability to be the best choice. In Table 8 I summarize the average return of all tested numerical experiments of different value of K. Clearly, on average, entering the larger market will generate better return.

Static game gives us good insights to explain the phenomenon: Why in the beginning second movers like Lyft, Juno and Gett, etc., always prefer to be a follower, entering large cities and facing intense competition rather than exploring a new but smaller market. The nature of platform-based multi-side interactions greatly enlarges the amount of transaction in large cities, offsetting the disadvantage as a second mover and small network effect. On the contrary, the market structure in a non-platform-based or lower transaction efficiency market would be more concentrated, we would expect to see more natural monopolies in such markets.

Static game plots general equilibrium in a simple scenario; there are some caveats to serve the purpose of mimicking the real world, for example, firms can only enter one market at a time, the interactions take only one period, and it doesn't take entry cost into consideration. In this way, the static setup couldn't give answers when the firms' interactions are more dynamic or when there are more markets available.

So, next, I will present a model of dynamic sequential game to solve the above problems and answer the remaining questions.

4 Dynamic Game

The fundamental assumptions of the dynamic game inherit from the previous static game: 1) Firms are risk-neutral; 2) All markets' average transaction profits are drawn from one same distribution: $E(\alpha_i) = \alpha$, $\forall i \in M$, M is the set that includes all feasible markets. 3) Once a market is being explored by a player, it will generate profits in every period. Besides the assumptions from the static game, in the dynamic game I add two more assumptions: 4) The one-time fixed entry cost is diminishing in market size: $\frac{c}{K}$, for the Kth market, and for both players. Finally, without loss of generality 5) Players move in turn: player 1 moves the even turn, player 2 moves the odd turn;

The structure of entry cost plays a crucial role in designing the algorithm solves the firm's best behavior. Note that, K to the power 1 in the denominator :$\frac{C}{K}$, guarantees that the size of the feasible markets set M converges to an upper limit $\bar{K}$[5]:

$$\bar{K} = \tfrac{NM}{rc} E(\alpha_{\bar{k}}) \tag{8}$$

The expected present discount value $\frac{NM}{r\bar{K}^2} E(\alpha_K)$ of the last market $\bar{K}$ will just cover the one-time entry cost for the last market $\frac{c}{K}$.

Other cost structures, such as constant entry cost c for all K, may also have such properties, and have a different decision algorithm. Here I will just apply the simple and reasonable structure : $\frac{c}{K}$. Because for a platform to operate in a city, requires administrative costs, it would be reasonable to assume large cities have more administrative costs. Such as hiring more customer service staff, recruiting more programmers to maintain the platform, and purchasing more servers to store data, etc. And the costs don't necessarily happen locally.

Inducing the entry cost term in the dynamic game is very important, because, in the model, it adds a budget constraint for every decision, hence we have a constrained optimal decision to solve; And in reality, a firm's expansion is closely related to its money stock; it's very rare a city-based platform would enter all the markets at one time. As in the ride-sharing case, a platform's expansion decision is closely related to its funding rounds. And part of the expansion cost is reflected here as a one-time fixed cost, part of the expansion cost is reflected in the post-entry realization of α_K as average cost per potential transaction.

[5] Note here $\bar{K}$ is different from what it is in static games.

Game Setups 3 (Dynamic Game)

1. Before the game starts, p, γ, r[6], are predetermined parameters for both players;

2. Game starts at time 0, $t \in [0, T]$, $X_{n0} = \emptyset$ for $n = 1, 2$, $I_0 = \emptyset$, $J_0 = M$[7];

3. Player 1 starts at time 0 with initial fund u_{10}; Player 2 starts at time 1 with initial fund u_{21}; u_{nt} here represents how much money the player n owns at time t;

4. Within each turn, one player will first load the status of its current state.

   (a) Load the current states: u_{nt}, u_{-nt}, m_{nt}[8], m_{-nt}, I_t, J_t ,X_{nt}, X_{-nt};

   (b) Calculate S_{nit} and $1 - S_{nit}$ for $i \in I_t \cup J_t$ (Equation 5 and 6) for both players;

5. Second, player solves the following binary linear programming (BLP) problem to find the optimal market entry strategy set $(\boldsymbol{z_t^*}, \boldsymbol{y_t^*})$:

[6] Parameter r is the discount rate.

[7] I_t, J_t represent the sets of markets with 1 player, and markets with 0 players respectively. X_{nt} is the set of current markets entered by player n at time t

[8] In this game it equals to the amount of revenue one platform generated from previous $t-1$ period: R_{nt-1}.

$$
\begin{aligned}
\max_{y_{it}, z_{jt}} \quad & \sum_i y_{it} \tfrac{NM}{(K_i)^2} S_{nit} \alpha_{Ki} + \sum_j z_{jt} \tfrac{NM}{(K_j)^2} E(\alpha) \\
\text{s.t.} \quad & C_{nt} = \sum_i y_{it} \tfrac{c}{K_i} + \sum_j z_{jt} \tfrac{c}{K_j} \leq u_{nt}, \\
& y_{it} \tfrac{c}{K_i} \leq \tfrac{NM}{r(K_i)^2} S_{nit} \alpha_{Ki}, \forall\ i \in I_t, \\
& z_{it} \tfrac{c}{K_j} \leq \tfrac{NM}{r(K_j)^2} E(\alpha), \forall\ j \in J_t, \\
& y_{it} \in \{0, 1\}, \forall\ i \in I_t, \\
& z_{jt} \in \{0, 1\}, \forall\ j \in J_t.
\end{aligned}
$$

(a) I_t represents the set of markets already being explored by the other player at time t, so α_{Ki} is revealed to all; J_t represents the set of undeveloped markets at time t, so α_{Kj} is not revealed, and players can only make decision through expected profit $E(\alpha)$;

(b) Total cost of the entry has to be no larger than player's current money stock;

(c) The real present discounted value of a market must cover the entry cost for markets in I_t; the expected present discounted value of a market must cover the entry cost for markets in J_t;

(d) y_{it} and z_{jt} are decision variables of market entry; it is either 1 or 0 (enter or not);

(e) S_{nit} represents the player n's market share in market i at time t, when Player n is a first mover $S_{nit} = 1$

6. Third, update the status for next period and next player;

(a)

$$m_{nt+1} = R_{nt} = \sum_i \tfrac{NM}{(K_i)^2} S_{nit}\alpha_{Ki}, \forall i \in X_{nt+1}$$
$$m_{-nt+1} = R_{-nt} = \sum_j \tfrac{NM}{(K_j)^2} S_{-njt}\alpha_{Kj}, \forall j \in X_{-nt+1} \tag{9}$$

(b)

$$u_{nt+1} = u_{nt} + R_{nt} - C_{nt}$$
$$u_{-nt+1} = u_{-nt} + R_{-nt} - 0 \tag{10}$$

(c)

$$X_{nt+1} = X_t \cup \boldsymbol{y_t^*} \cup \boldsymbol{z_t^*}$$
$$X_{-nt+1} = X_{-nt}$$
$$I_{t+1} = I_t \setminus \boldsymbol{y_t^*} \cup \boldsymbol{z_t^*},$$
$$J_{t+1} = J_t \setminus \boldsymbol{z_t^*} \tag{11}$$

7. Repeat Step 4,5,6;

8. If $t = T$ end the game.

The basic idea behind this setup is that in a dynamic sequential game, each player's action will change the future states for both players, so in each period, players are making decisions upon a changing states.

First, note here m_{nt} and u_{nt} are totally two different things. Only by occupying (some of) the markets can one player make changes in her firm size; m_{nt} here, is in control of the player's market share when she has to split the market with the other one.

While u_{nt} is the platform's current cash flow, it can be regarded as the aggregation of platform's revenues and costs from time 0 to time t; u_{nt}, here, is in control of player's budget constraint of new entries.

Second, we can see from the above game setups that, a player's market share is not related to current market's characteristics, it is only related to the players' current state.

Third, once a game is initiated, no intervention is needed in the process of playing. The source of randomness comes from the realization of α_K, once the α is known to all, the "Optimal Solution" is somehow destined, for one bundle of parameters; as long as α_K are generated for each city, there will only exist one entry path for both players, although, from the firms' perspective, they might face a lot of uncertainties. While, in this research, I only discuss the situation when α_K is positive, that all markets will generate positive return in each period, there is no need for exit after entry. While if I allow for negative market return, the post-entry market information disclosure, may add more randomness to players' market positions, the later entrant has the chance to avoid the harmful markets, and so could have more opportunity to take the leadership. Moreover, if both platforms can gradually learn the properties of α_K during the entry process, they may have more chance to survive.

Fourth, it might seem that players are myopia, and only making decisions based on the current period. Actually, intuitively, in this game, the result of forward-looking should be the same with the myopic decision. Because the decision of market entry leads to two basic changes in the current state- the change of money stock, and the change of firm size, they are two separate channels- so for one player, either she enters one large market or enters several small markets because of her prediction on the other player's movement in the next period will lead to a same result, as long as the money spent on entry is the same, and market sizes are adding up to be the same, the change of status will always be the same. The player's market share when facing a later entrant in other player's period will be the same too. In the next period, when the other player moves, players' market share is already determined, so no matter which markets the second player enters, as long as the budget constraint allows, the adding up total market return will be the same. So, making optimal choice in current period and every period is making best action for the entire T periods.

Finally in Step 8, we can see that, the game doesn't stop at the exact time when $I_t = J_t = \emptyset$ such that no further empty markets are available. This is because, even after all the feasible markets have been explored. The firms will continue operating. One can see from the simulation results, the status of two players will tend to a steady state after they complete the entry process.

In the next section, I will present the results of several numerical experiments with different initial parameters to get insights on city-based platform behavior patterns in the market entry problem.

5 Numerical Results

In this section, I will describe some numerical experiments, which focus on two parts: First, I investigate the joint effect of switching cost and network effect on platforms' market entry decision. Second, I test the results under different initial funds.

5.1 Network Effect and Switching Cost

From previous setups we know that, γ and p jointly determine how players split the market. In homogeneous case these two are exogenous parameters related only to the type of platform's business model. From static game we know that γ and p eventually affect the platforms' willingness to compete. However, one has to note that, although γ and p control the market share together, p has a more sophisticated effect, because in a segregate markets setup, if the second player takes the advantage of entering first, it will be difficult for the first player to enter the market either. The initiation of dynamic games in this section will be set as follows:

1. Game starts with K=1, t=0;
2. $u_{10} = 20$, $u_{21} = 20$, $c = 20$, NM $= 20$, $r = 0.01$
3. α_{Ki} $\forall$ i are generated from uniform distribution $U(0,1)$, $E(\alpha_{Ki}) = 0.5$
4. The game will play 50 periods, T=50.

Several things should be noted from the above initiation: 1) In this case, the start fund for both players will just cover the entry cost of 1st market. 2) Refer to Equation 8, $\bar{K} = 50$.

The numerical results shown in this part will be arranged as 1) For each scenario, I will present an averaging result of 1000 simulations, and in each simulation, the programming will re-generate a new series of α_{Ki} $\forall$ i. (such as Figures 5 and 7) 2) Then, I will present a special case, in which series α_{Ki} $\forall$ i are fixed to illustrate the differences in players' choices (such as Figures 6 and 8). Moreover, in Table 11, I summarize the aggregate simulation results to provide a detailed view. Where "player order" represent the player's entry order, either as a first mover: "1", or as a second mover: "2", or no entry: "0". And "average market revenue", is the average total market revenue in the final period, it reflects the market size under each sub-category when the game ends (at time T=50). And the last column represents the percentage of total markets (50 in total) that are under each sub-category at the time when the game ends.

Switching Cost

When a second mover tries to enter a market with an incumbent, it is often in a disadvantaged position, and in this paper, this disadvantage is captured by switching cost, in the previous utility function Equation 1, Intuitively, the larger the switching cost, the more difficult it is for the market to accept a later entrant; in this way, the market tends to be more concentrated.

Such as in case 1 and case 2. Case 1 is a special setup wherein a later entrant is fully disadvantaged; users in case 1 totally reject to accept another platform at all. So the markets are extremely concentrated, in Table 11, case 1, we can see that none of the markets are entered by two firms, Player 1 captures most of the markets, and most of the revenues. Even if the market has some degree of network effect $\gamma = 1$, it won't affect firms' strategies at all. The reason Player 1 is able to gain more revenue, is because she took the exclusive occupation of the largest market first. In case 2, I slightly decrease the difficulty for multi-homing, let $p = 0.1$, we can see from Table 11 case 2 some fraction of the markets are entered by both firms, and they are all large markets, with average total revenue 9.25 and 3.99. Compare case 1 and case 2 in Figure 5; in case 2, on average player 2 is catching up with player 1. Also, in Figure 6, we can see that, the first 4 movements in period 0 to 3 in both cases are exactly the same, however in period 4 case 2, Player 1 turns to explore some of the territories of Player 2. And Player 2 with bad luck entered a bad market number 2 in her first period, so later on she can only explore some small markets because of insufficient cash flow, and finally after several periods of accumulation, Player 2 gathered enough money and picked up two best markets of Player 1 to enter: market 1 with large transaction volume and market 4 with great return.

Network Effect

Similarly, we can also expect that a larger network effect makes the market more concentrated. Users would be more likely to gather around larger platforms under the effect of networking. As in Case 3 and Case 4, here I test the effect of network effect. Case 3 is another special scenario where both network effect and switching cost equal zero. In this case, all markets will be split in half if entered by both players. Without all the disadvantages of being a second mover, from Figure 7 case 3 we can see that both players will eventually have the same money stock and the same firm size. However, in case 4 if the network effect increases by a little bit ($\gamma = 1$), we can see a big gap between two players: Player 2 is in an unfavorable situation, from Figure 8 Case 4, Player 1 is able to have massive expansion in almost every period, even when she is acting as a second mover, Player 1 will still gain most of the market share because of her big network effect. And Player 2 can only make scattered entries after Player 1.

From Case 1 to 4 we can see that, network effect and switching cost mutually determine the market structure. Either of these two parameters being high will finally lead to monopoly in most of the markets. And network effect and switching cost also provide a natural barrier for the later entrant; even if both players offer the same quality product and start with same amount of funding, lack of user base would make Player 2 much disadvantaged. This corresponds to the Uber and Lyft case, why did Uber always dominate the market leadership no matter the entry order in a city? This research provides a possible explanation: because Uber had already taken large amount of the market in other cities, the nation-wide network effect gives it a built-in advantage when conquering a market. Also we can expect Lyft to charge a lower price on the platform, in order to compensate the utility loss in network effect. When two platforms offer homogeneous services in the same market, later entrant might have to lower its price to attract more users; however, this will lead to shortages in money stock, and make it difficult for the small platform to expand in the future. So a later entrant either needs to raise more money or limit the amount of expansion. Next, I will talk about the influence of initial fund in the dynamic game.

5.2 Initial Fund

From case 1-4 we can see that, part of the success of Player 1 results from her prior occupation of the largest city. So we can presume that whether a player has sufficient fund to enter the largest city is very important in the dynamic of the two-player entry game. In this section, I will test the numerical experiments under different starting fund.

The initiation of dynamic games in this part will be set as follows:

1. Game starts with K=1, t=0;
2. $\gamma = 1$, $p = 0.8$, $c = 20$, NM $= 20$, $r = 0.01$
3. α_{Ki} $\forall$ i are generated from uniform distribution $U(0, 1)$, $E(\alpha_{Ki}) = 0.5$
4. The game will play 50 periods, T=50.

Again, the arrangement of numerical results present in this part will be: 1) For each scenario, I will present an averaging result of 1000 simulations, and in each simulation, the programming will re-generate a series of α_{Ki} $\forall$ i. 2). Then, I will present a special case, in which series α_{Ki} $\forall$ i are fixed to illustrate the differences in players' choices.

So the market environment in this part is fixed, with some level of network effect and some degree of switching cost. From Figure 2a we know that when $\gamma = 1$, $p = 0.8$, first mover at most will get around 70% of the market when facing a second mover.

First Mover without Sufficient Fund

In case 5, both firms don't have sufficient funds to entry the 1st market in the beginning, but Player 1 gets advantages from the first move. But without sufficient funds, the entry path for Player 1 moves in zigzags; then at around period 15, there is a leap in Player 1 's firm size or total revenue per period; we can treat this leap as on average when Player 1 finishes her accumulation of entry fund, and finally enters market 1. While the situation for Player 2 isn't so optimistic, she is at the edge of surviving; although it is still expanding, lack of network effect will put her in a disadvantageous position when competing with Player 1. However, in case 6, the situation is totally different; here Player 2's starting fund is just enough to cover the entry cost of the largest market, and Player 1 stays the same. With occupation of the largest market, large amounts of revenue are generated in each period, offering sufficient fund for Player 2 to explore the world. She will soon occupy all cities, such as in Figure 10 case 6 Player 2 has a very aggressive expansion, within 3 periods, she will expand to all the markets, and Player 1 this time will be at the edge of barely surviving, even if her initial fund is the same as in case 5. Facing a strong component, a first mover startup company such as Player 1 in case 6, could die soon, if she fails to seize the opportunity to attract the most majority target group (the largest city) at first. This is the real story for some of the start up companies, according to a report by venture capital database *CB Insights* (Insights 2014) 9% of the startups' failure results from failed geographical expansion.

First Mover with Sufficient Fund

However things will change again, in platform-based markets; large amount of money raising doesn't necessarily lead to market success. As in Figure 11, when Player 1 gets enough funds to enter the largest market in the beginning. We can see that Player 1 will dominate the market again, even if in Case 8 Player 2 raised double the amount of funds to start. Player 1 will still hold the market leadership. Such as in Figure 12, Player 1 enters the largest one in the beginning, then Player 2 with a greater amount of initial funds, tries to compete in market 1, she will only get about 30% of the market, for Player 2 to best allocate her 40 at the beginning, Player 2 will enter Market 1, 2, 3, and spend 36.66 out of 40, while the remaining 3.33 isn't enough to cover markets 4 and 5, so she will enter market 6 and spend the 3.33. In this special case, Player 2 has really bad luck; the second largest market is a bad one with only 0.061 average return, while for Player 1 after accumulating revenues from the largest market for 2 periods, starting from time 2, Player 1 will begin her massive expansion. And Player 2 with larger initial fund will only catch up a little bit in Case 8.

So, from the above cases we know that, in platform-based markets the entry timing is very important, if we treated cities as different groups of users, a majority of the transactions and revenue are generated by the largest group. Whoever capture this largest group of users, will be more likely to succeed in the following expansions, because benefit from these people secures the entry expense in other markets. Also, the network effect and switching cost of platform-based market will create a built-in barrier to prevent the entry of other competitors; in this way, first mover small startups may have a way to defend themselves from the impact of market giants.

Above all, the condition for a homogeneous service provider second mover to take market leadership in a platform-based market is very stringent. Not only does she need sufficient funds, she also has to enter the market at the right time plus have a little bit of luck.

Other Implications

Besides the factors mentioned above, the dynamic model also has explanatory power for other phenomena in the real world.

First, for example, in all 8 cases above, at some point of time during the game, the model predicts a massive expansion (refer to the 4th graph "Market entries per period" in the aggregation figures). In the beginning, both players will enter a small number of markets due to the limitation of budget constraints and high entry cost, then with the decrease of entry cost and accumulation of money stock, players at some point in time will make a massive expansion. This result can correspond to the expansion path of ride-sharing platforms in the real-world, as when, in 2010, Uber started with only one city, San Francisco. Not until almost a year later, it made its next expansion in another 3 cities: New York, Seattle, and Chicago. Then, in 2012, Lyft announced its first launch in San Francisco, and not until 6 months later did Lyft announce its second move, into Los Angeles, followed by several scatter entries in 2013. Based on their official launch record, both firms are expanding at an increasing rate, especially Lyft. Lyft had only 20 cities in the beginning of 2014, yet in April 2014 Lyft suddenly announced a massive 24-city expansion in 24 hrs, and in Jan 2017 it announced a 40-city expansion, followed by a 50-city massive launch only one month later. For Uber, in April 2014, it only occupied 47 cities, and Lyft had 60. Right now, they are both in more than 300 U.S. cities, occupying almost all the cities available in the U.S.

Second. from the second graph "Firm size" in the above figures, we can see that, after some point in time, the relative firm size between two firms will converge to a constant. That is because, after exploring the largest several cities, small expansion couldn't have much influential power on the firm size any more. Recall that firm size eventually determines the market share of two platforms when entering the same market, so that the market share of two firms will converge to a constant ratio. This may provide a explanation of Figure 1, why the market share in most of the cities across the U.S tend to be a constant 20:80.

Third, post-entry market information revelation gives the game a lot of uncertainties; it actually gives the second mover some source of advantages. As shown in the interesting case in Figure 10, case 5; when both firms start with the same amount of money, only sufficient to cover the second largest market, and it happens to be a really bad market, with $\alpha_2 = 0.061$ far lower than players' pre-entry expectation $E(\alpha) = 0.5$. Player 1, in this time, had really bad luck, such that her first entry is poor; not only will this market give her low return, but also it fails to establish a sizable network effect for Player 1 to defend the competition from later entrants. Because of this mistake, Player 1 lost the chance to explore the other large cities; the trivial return generated from the first period only made affordable some small markets in her next turn and, unfortunately, without proper market information, the next two entries for Player 1 are worse, with $\alpha_3 3 = 0.157$ and $\alpha_3 4 = 0.085$. But Player 2 is relatively lucky this time: as second mover, he has the chance to avoid market 2, and explore other markets; this time, he is much more lucky: although the market size is relatively smaller, the markets' returns are much better, and the good start gives Player 2 the chance to overcome the disadvantages as a second mover. We can see that, finally, Player 2 will enter market 1 after she accumulates sufficient funding. And in reality there are many examples of a first mover losing its market advantage because of expanding into the wrong market or at the wrong time. According to *CB Insights*, 13% of the startups' failures results from product mistiming.

Finally, from Figure 6, for some type of businesses without network effect, we might still observe a concentrated market structure in some small local market; that is because the market size is too small for both players to be profitable. I think this provides an explanation as to why usually we can only find one super mall in one suburban area.

6 Conclusion

In this paper, I study the relative importance of multiple factors on entry decision of city-based platforms with homogeneous products. I build a theoretical framework that incorporates the idea of city size and pre-entry uncertainty and find that, besides the strength of network effect, high switching cost, low market size, and low realized market return might also lead to market concentration. The static equilibrium in the two-player static game implies that, although the network effect and switching cost may pose a barrier for a late entrant, the multi-sided and efficient transaction properties related to a platform-based market could still make large markets attractive to second movers. Then I extend the one-period-two-player static game to a multi-period-two-player dynamic game. Consistent with the static-game prediction, the results of numerical experiments in the dynamic game also prove the importance of network effect, switching cost, market size, and realization of market return on market concentration. Moreover, the dynamic game also plots the expansion path of both players; under the structure of a platform-based market, the condition for a second mover to take over the market leadership becomes more stringent than a non-platform-based market. Capability of capturing the majority of the service target group or, say, the largest city ahead of its rival competitor is the crucial point in winning the market for both first mover and second mover. If a second mover lost the opportunity of capturing the largest market, she will have to raise a huge amount of money to overcome her disadvantage in the later competition. The content discussed in this paper can be applied to explain the expansion interaction of emerging city-based service platforms, such as Uber and Lyft, Groupon and LivingSocial, etc. This paper plots a scenario of homogeneous platform entry dynamic, where copy-and-paste is easy between digital platforms, and quality difference is difficult to achieve.

However, according to Equation 1 and 3 in Section 3, the utility function actually allows for heterogeneous services. Intuitively in such scenario, in additional to the conditions that are discussed in this paper, a second mover can also take the market leadership by lowering the service price or providing higher quality products. But the heterogeneous case is beyond the scope of this paper. This could be a good point for future study on city-based platform entry problems.

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Appendix

Proof of Proposition 1

$\frac{NM}{(K+1)^2}E(\alpha) + S_{11}\frac{NM}{K^2}\alpha_k > \frac{NM}{(K+2)^2}E(\alpha) + S_{11}\frac{NM}{K^2}\alpha_k$, so, Player 1 will choose K+1 when Player 2 choose K. And $\frac{NM}{(K+1)^2}E(\alpha) + \frac{NM}{K^2}\alpha_k > S_{10}\frac{NM}{(K+2)^2}E(\alpha) + \frac{NM}{K^2}\alpha_k$ so Player 1 will choose K+1 when Player 2 choose K+2.

When Player 2 choose K+1, Player 1 also choose K+1, iff $S_{10}\frac{NM}{(K+1)^2}E(\alpha) + \frac{NM}{K^2}\alpha_k > \frac{NM}{(k+2)^2}E(\alpha) + \frac{NM}{K^2}\alpha_k$, so $S_{10}\frac{1}{(K+1)^2} > \frac{1}{(K+2)^2}$, $(\frac{K+2}{K+1})^2 > \frac{1}{S_{10}}$, since $S_{10} \in [0.5, 1]$, $K < \frac{2\sqrt{S_{10}}-1}{1-\sqrt{S_{10}}}$, $\frac{2\sqrt{S_{10}}-1}{1-\sqrt{S_{10}}}$ is monotonically increasing in $S_{10} \in [0.5, 1]$, the threshold $\bar{K}$ for Player 1 choose K+1 is increasing with S_{10}

Proof of Proposition 2

When Player 1 choose K+1, the size of $(1-s_1)\frac{NM}{K^2}\alpha_k$, $(1-s_1)\frac{NM}{(K+1)^2}E(\alpha)$ and $\frac{NM}{(K+2)^2}E(\alpha)$ is changing with K, γ and α_K, $E(\alpha_K)$, So all three actions are possible best responses. When Player 1 choose K+2, same as above.

Proof of Proposition 3

From Proposition 1 and 2, when K+1 is dominant strategy for Player 1, there are three PSNEs. When Player1 has the incentive to deviate to K+2, there will be 2 anti-coordinates PSNEs and 2 MSNEs.

Table 1: Table of notations

Notation	Definition
i, j	Iteration of markets
n, -n	Iteration of players
t	Iteration of time/period
K	Cities' rank on population
α_K	Average transaction profit of the Kth market
γ	Degree of network effect
$-h$	Dis-utility of adapting new platform
V	Average utility per service
p	Degree of switching cost
r	Depreciation rate
c	Cost constant
M	sets for all feasible markets
X_t	sets of occupied markets for each player at time t
I_t	Set of markets with 1 player at time t
J_t	Set of markets with 0 player at time t
y_{it}	Decision variables for market i $\forall$ $i \in I_t$ at time t
z_{jt}	Decision variables for market j $\forall$ $j \in J_t$ at time t
y^*_{nt}, z^*_{nt}	Player n's optimal choice set at time t
u_{nt}	Player n's money stock at time t
m_{nt}	Player n's total market size at time t
x_{nt}	Player n's size ratio at time t
S_{nt}	Player n's market share at time t
R_{nt}	Player n's total revenue from all occupied markets at time t
C_{nt}	Player n's entry cost at time t

Table 2: Examples of local-based platforms

Platform(s)	Market	Side 1	Side 2
Uber, Lyft, Juno, Via, Didichuxing	Ride-sharing	Drivers	Riders
UberEats, Yelp Eat 24, Seamless, Meituan	Food Delivery	Restaurants	Diners
Groupon, Living Social, Yipit, CoolSavings	Online Deal & Promotion	Mechants	Deal-Shoppers
OfO, Mobike, CitiBike	Bike-sharing		Riders
Airbnb	Room-sharing	Room-owners	Tenants

Table 3: First 13 city launches of Uber & Lyft

Uber Cities	**Uber Launch Date**	**Lyft Cities**	**Lyft Launch Date**
San Francisco	7/10/2010	San Francisco	6/1/2012
New York	5/3/2011	Los Angeles	1/31/2013
Seattle	7/25/2011	Seattle	4/1/2013
Chicago	9/22/2011	Chicago	5/9/2013
San Diego	1/6/2012	Boston	5/31/2013
Los Angeles	3/8/2012	San Diego	7/2/2013
Philadelphia	6/6/2012	Washington	8/9/2013
Atlanta	8/24/2012	Atlanta	8/29/2013
Denver	9/5/2012	Minneapolis/St.Paul	8/29/2013
Dallas	9/14/2012	Indianapolis	8/29/2013
Boston	9/19/2012	Phoenix	9/5/2013
Minneapolis/St.Paul	10/25/2012	Charlotte	9/12/2013
Phoenix	11/15/2012	Denver	9/19/2013

Table 4: Cities entry time of Uber & Lyft

Cities	Uber Launch Date	Lyft Launch Date
Washington D.C	8/8/2013	8/8/2013
Houston	2/21/2014	2/19/2014
Miami	6/4/2014	5/23/2014
Austin	6/3/2014	5/29/2014

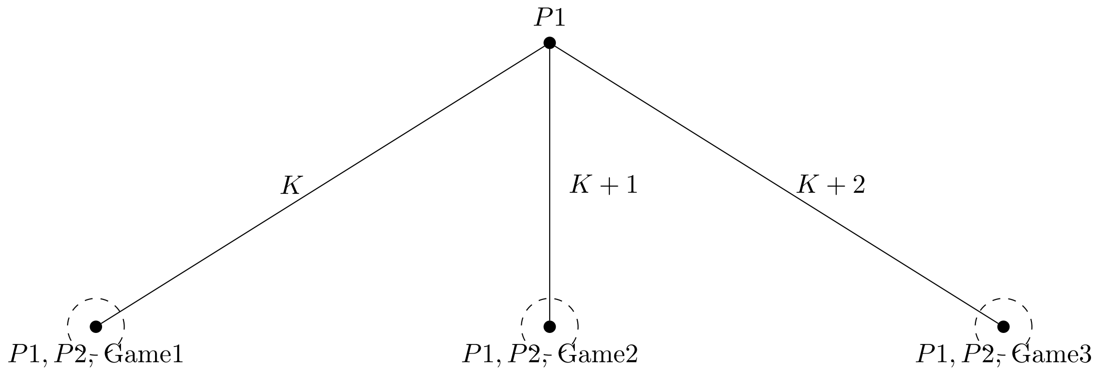
Table 5: U.S Top cities by population 2016 (in millions)

Rank	1	2	3	4	5	6
City	New York	Los Angeles	Chicago	Houston	Phoenix	Philadelphia
Population	8.538	3.976	2.705	2.33	1.615	1.568
Ratio to NYC	100.0%	46.6%	31.7%	27.3%	18.9%	18.4%
	1	$\approx \frac{1}{2}$	$\approx \frac{1}{3}$	$\approx \frac{1}{4}$	$\approx \frac{1}{5}$	$\approx \frac{1}{6}$

Data Source: US Census Bureau https://www.census.gov/en.html

Table 6: Static game payoff matrix

		P2		
		K	$K+1$	$K+2$
P1	$K+1$	$\frac{NM}{(K+1)^2}E(\alpha)+S_1\frac{NM}{K^2}\alpha_k$, $(1-S_1)\frac{NM}{K^2}\alpha_k$	$S_1\frac{NM}{(K+1)^2}E(\alpha)+\frac{NM}{K^2}\alpha_k$, $(1-S_1)\frac{NM}{(K+1)^2}E(\alpha)$	$\frac{NM}{(K+1)^2}E(\alpha)+\frac{NM}{K^2}\alpha_k$, $\frac{NM}{(K+2)^2}E(\alpha)$
	$K+2$	$\frac{NM}{(K+2)^2}E(\alpha)+S_1\frac{NM}{K^2}\alpha_k$, $(1-S_1)\frac{NM}{K^2}\alpha_k$	$\frac{NM}{(k+2)^2}E(\alpha)+\frac{NM}{K^2}\alpha_k$, $\frac{NM}{(K+1)^2}E(\alpha)$	$S_1\frac{NM}{(k+2)^2}E(\alpha)+\frac{NM}{K^2}\alpha_k$, $(1-S_1)\frac{NM}{(K+2)^2}E(\alpha)$



		P2		
		K	$K+1$	$K+2$
P1	$K+1$	$S_1\frac{1}{K^2}+\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{K^2}$	$\frac{1}{K^2}+S_1\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+1)^2}$	$\frac{1}{K^2}+\frac{1}{(K+1)^2}$, $\frac{1}{(K+2)^2}$
	$K+2$	$\frac{1}{(K+2)^2}+S_1\frac{1}{K^2}$, $(1-S_1)\frac{1}{K^2}$	$\frac{1}{(k+2)^2}+\frac{1}{K^2}$, $\frac{1}{(K+1)^2}$	$S_1\frac{1}{(k+2)^2}+\frac{1}{(K)^2}$, $(1-S_1)\frac{1}{(K+2)^2}$

(a) Game 1

		P2		
		K	$K+1$	$K+2$
P1	K	$S_1\frac{1}{K^2}+\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{K^2}$	$\frac{1}{K^2}+S_1\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+1)^2}$	$\frac{1}{K^2}+\frac{1}{(K+1)^2}$, $\frac{1}{(K+2)^2}$
	$K+2$	$\frac{1}{(K+2)^2}+\frac{1}{(K+2)^2}$, $\frac{1}{K^2}$	$\frac{1}{(k+2)^2}+S_1\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+1)^2}$	$S_1\frac{1}{(k+2)^2}+\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+2)^2}$

(b) Game 2

		P2		
		K	$K+1$	$K+2$
P1	K	$S_1\frac{1}{K^2}+\frac{1}{(K+2)^2}$, $(1-S_1)\frac{1}{K^2}$	$\frac{1}{K^2}+S_1\frac{1}{(K+2)^2}$, $\frac{1}{(K+1)^2}$	$\frac{1}{K^2}+S_1\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+2)^2}$
	$K+1$	$\frac{1}{(K+2)^2}+\frac{1}{(K+2)^2}$, $\frac{1}{K^2}$	$\frac{1}{(k+2)^2}+S_1\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+1)^2}$	$S_1\frac{1}{(k+2)^2}+\frac{1}{(K+1)^2}$, $(1-S_1)\frac{1}{(K+2)^2}$

(c) Game 3

Table 7: Sequential game with imperfect information

Table 8: Average return of sequential equilibriums under different value of K

	K	**1**		**2**		**3**		**5**		**10**	
		P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
t_0	K	0.917	0.142	0.263	0.042	0.122	0.025	0.049	0.011	0.012	0.004
	K+1	0.679	0.128	0.221	0.059	0.111	0.037	0.044	0.019	0.012	0.006
	K+2	0.600	0.125	0.181	0.056	0.086	0.032				

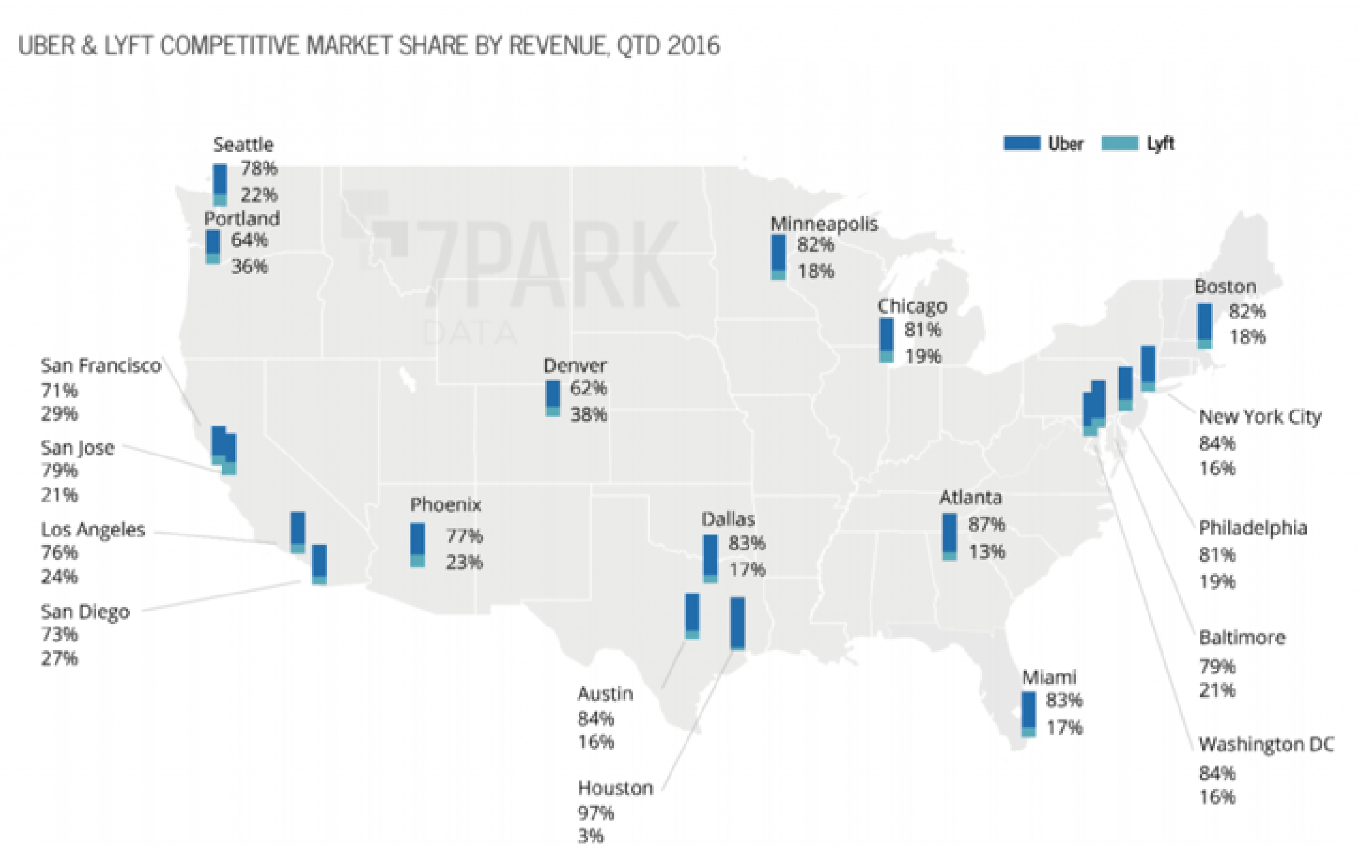
Table 9: PSNEs under different values of K, α, γ, p of platform-based markets

K		**1**	**2**	**3**	**5**	**10**
				(Case 1: $\gamma = 0$, $p = 1$, $S_1 = 0.5$, $1 - S_1 = 0.5$)		
	D1	K+1	K+1	K+1	K+1	K+1
	D2	K	K	K	K	K
				(Case 2: $\gamma = 1$, $p = 1$, $S_1 = 0.73$, $1 - S_1 = 0.27$)		
$\alpha_k = 1.0$	D1	K+1	K+1	K+1	K+1, K+2	K+1, K+2
	D2	K	K	K	K, K+1	K+2, K+1
				(Case 3: $\gamma = 2$, $p = 1$, $S_1 = 0.88$, $1 - S_1 = 0.12$)		
	D1	K+1	K+1	K+1	K+1	K+1
	D2	K	K+2	K+2	K+2	K+2
				(Case 4: $\gamma = 0$, $p = 1$, $S_1 = 0.5$, $1 - S_1 = 0.5$)		
	D1	K+1	K+1	K+1, K+2	K+1, K+2	K+1, K+2
	D2	K	K	K, K+1	K+2, K+1	K+2, K+1
				(Case 5: $\gamma = 1$, $p = 1$, $S_1 = 0.73$, $1 - S_1 = 0.27$)		
$\alpha_k = 0.5$	D1	K+1	K+1	K+1	K+1, K+2	K+1 , K+2
	D2	K	K	K+2	K+2, K+1	K+2 , K+1
				(Case 6: $\gamma = 2$, $p = 1$, $S_1 = 0.88$, $1 - S_1 = 0.12$)		
	D1	K+1	K+1	K+1	K+1	K+1
	D2	K	K+2	K+2	K+2	K+2
				(Case 7: $\gamma = 0$, $p = 1$, $S_1 = 0.5$, $1 - S_1 = 0.5$)		
	D1	K+1	K+1, K+2	K+1, K+2	K+1, K+2	K+1, K+2
	D2	K+1	K+2, K+1	K+2, K+1	K+2, K+1	K+2, K+1
				(Case 8: $\gamma = 1$, $p = 1$, $S_1 = 0.73$, $1 - S_1 = 0.27$)		
$\alpha_k = 0.1$	D1	K+1	K+1	K+1	K+1, K+2	K+1, K+2
	D2	K+2	K+2	K+2	K+2, K+1	K+2, K+1
				(Case 9: $\gamma = 2$, $p = 1$, $S_1 = 0.88$, $1 - S_1 = 0.12$)		
	D1	K+1	K+1	K+1	K+1	K+1
	D2	K+2	K+2	K+2	K+2	K+2

Table 10: PSNEs under different values of K, α, γ, p, of non-platform-based markets

K		**1**	**2**	**3**	**5**	**10**
		(Case 1: $\gamma = 0$, $p = 1$, $S_1 = 0.5$, $1 - S_1 = 0.5$)				
	D1	K+1	K+1	K+1	K+1	K+1
	D2	K+2	K+2	K+2	K+2	K+2
		(Case 2: $\gamma = 1$, $p = 1$, $S_1 = 0.73$, $1 - S_1 = 0.27$)				
$\alpha_k = 1.0$	D1	K+1	K+1,K+2	K+1,K+2	K+1, K+2	K+1, K+2
	D2	K	K,K+1	K+2,K+1	K+2, K+1	K+2, K+1
		(Case 3: $\gamma = 2$, $p = 1$, $S_1 = 0.88$, $1 - S_1 = 0.12$)				
	D1	K+1	K+1	K+1	K+1	K+1,K+2
	D2	K+2	K+2	K+2	K+2	K+2,K+1
		(Case 4: $\gamma = 0$, $p = 1$, $S_1 = 0.5$, $1 - S_1 = 0.5$)				
	D1	K+1,K+2	K+1,K+2	K+1, K+2	K+1, K+2	K+1, K+2
	D2	K,K+1	K,K+1	K+2, K+1	K+2, K+1	K+2, K+1
		(Case 5: $\gamma = 1$, $p = 1$, $S_1 = 0.73$, $1 - S_1 = 0.27$)				
$\alpha_k = 0.5$	D1	K+1	K+1,K+2	K+1,K+2	K+1, K+2	K+1 , K+2
	D2	K+2	K+2,K+1	K+2,K+1	K+2, K+1	K+2 , K+1
		(Case 6: $\gamma = 2$, $p = 1$, $S_1 = 0.88$, $1 - S_1 = 0.12$)				
	D1	K+1	K+1	K+1	K+1	K+1,K+2
	D2	K+2	K+2	K+2	K+2	K+2,K+1
		(Case 7: $\gamma = 0$, $p = 1$, $S_1 = 0.5$, $1 - S_1 = 0.5$)				
	D1	K+1,K+2	K+1, K+2	K+1, K+2	K+1, K+2	K+1, K+2
	D2	K+2,K+1	K+2, K+1	K+2, K+1	K+2, K+1	K+2, K+1
		(Case 8: $\gamma = 1$, $p = 1$, $S_1 = 0.73$, $1 - S_1 = 0.27$)				
$\alpha_k = 0.1$	D1	K+1	K+1	K+1,K+2	K+1, K+2	K+1, K+2
	D2	K+2	K+2	K+2,K+2	K+2, K+1	K+2, K+1
		(Case 9: $\gamma = 2$, $p = 1$, $S_1 = 0.88$, $1 - S_1 = 0.12$)				
	D1	K+1	K+1	K+1	K+1	K+1,K+2
	D2	K+2	K+2	K+2	K+2	K+2,K+1

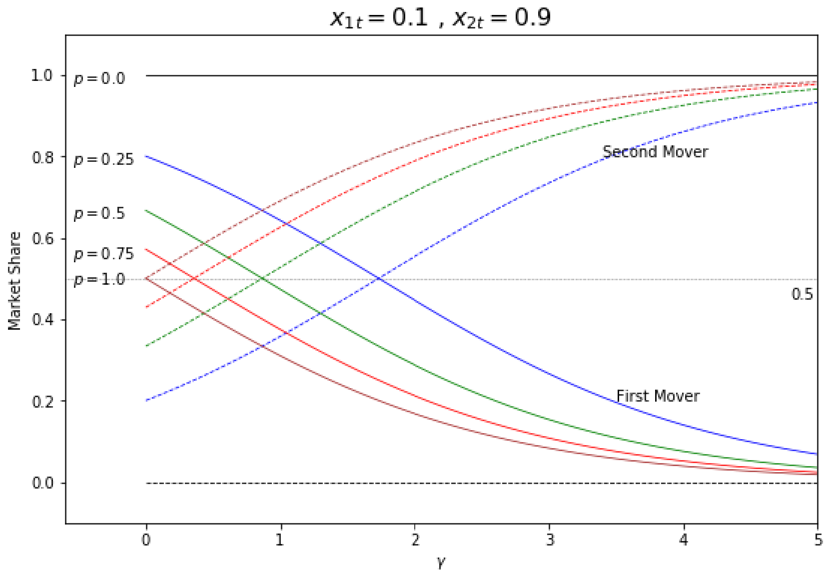
Figure 1: Uber & Lyft competitive market share by revenue QTD 2016



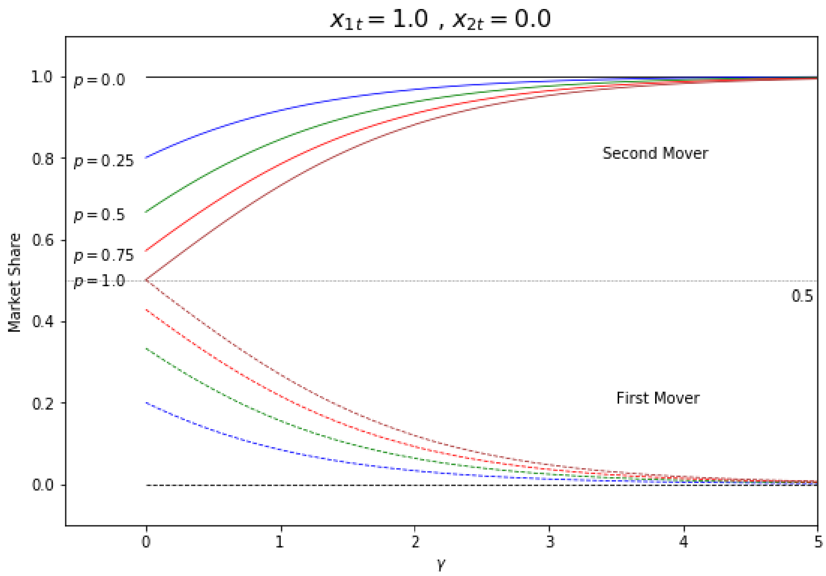
Source: https://skift.com/2016/10/12/uber-and-lyfts-growth-is-slowing-in-most-major-u-s-cities/

Table 11: Simulation results of dynamic games

Case	p	γ	u_{10}	u_{21}	Player 1 Order	Player 2 Order	Average Mkt. Revenue	% of Total Mkts
1	0	1	20	20	0	1	4.51	36.12%
					1	0	11.86	63.88%
					1	2	0	0.00%
					2	1	0	0.00%
2	0.1	1	20	20	0	1	0.92	40.12%
					1	0	1.35	52.00%
					1	2	9.25	2.84%
					2	1	3.99	5.04%
3	1	0	20	20	0	1	0.18	11.06%
					1	0	0.15	41.33%
					1	2	15.05	35.10%
					2	1	2.05	12.51%
4	1	1	20	20	0	1	0.12	11.76%
					1	0	0.42	46.24%
					1	2	13.13	27.16%
					2	1	3.27	14.84%
5	0.8	1	10	10	0	0	0.56	0.70%
					0	1	1.59	18.70%
					1	0	5.53	46.16%
					1	2	6.28	16.33%
					2	1	2.61	15.11%
6	0.8	1	10	20	0	1	3.29	56.08%
					1	0	0.26	11.21%
					1	2	3.59	12.45%
					2	1	9.14	20.23%
7	0.8	1	20	20	0	1	0.23	12.75%
					1	0	0.68	50.52%
					1	2	11.71	22.02%
					2	1	3.78	14.71%
8	0.8	1	20	40	0	1	0.15	20.20%
					1	0	0.19	40.00%
					1	2	11.73	15.39%
					2	1	4.70	24.41%



(a) Second mover size $m_2 = 0$



(b) First mover size m_1 very small

Figure 2: Market share under different value of γ and p

(a) Platform-based market

(b) Non-platform-based market

Figure 3: PSNEs under different values of α_K, γ and K

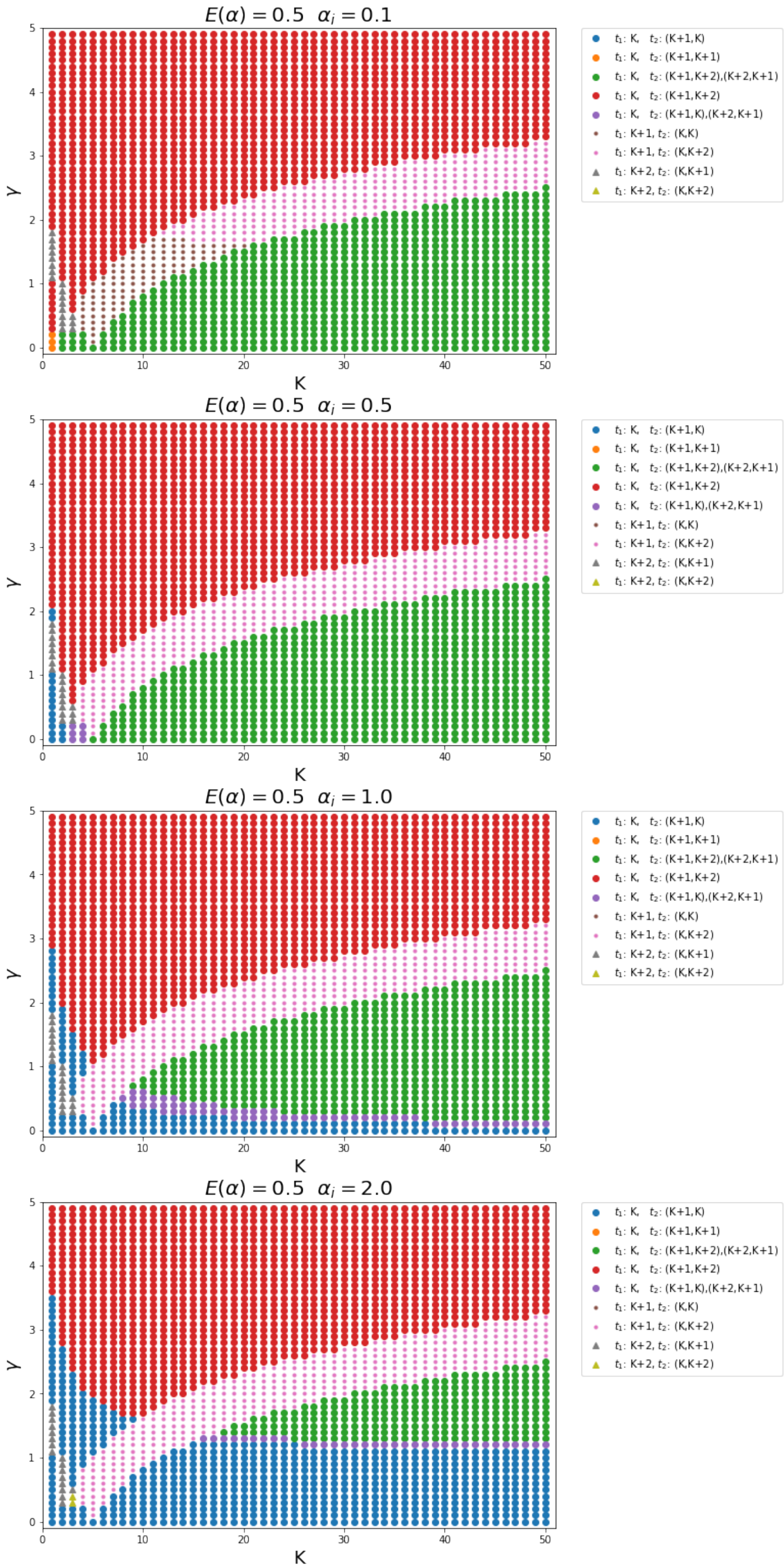
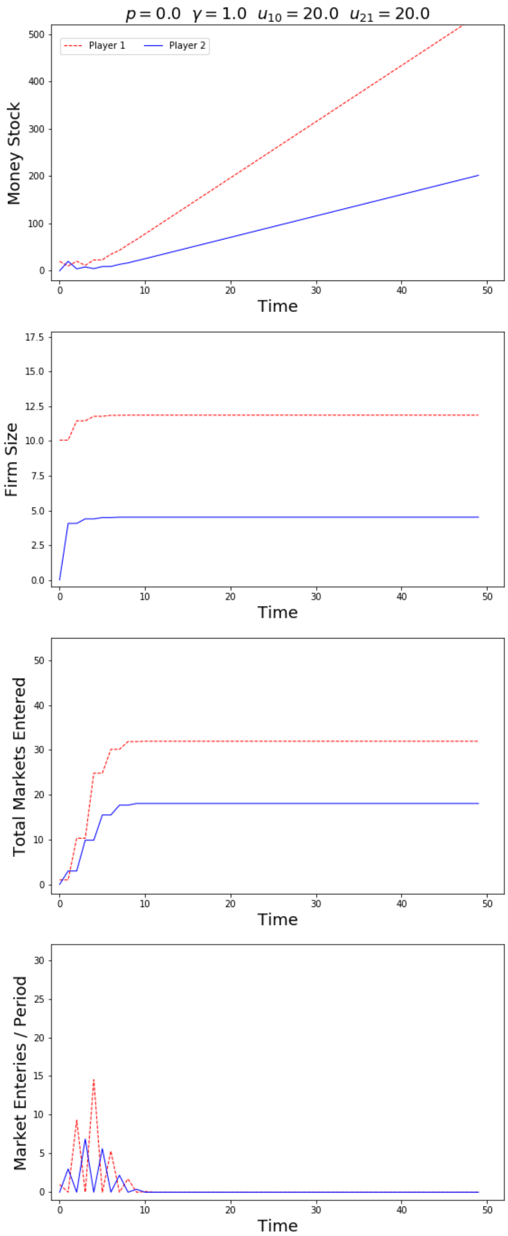
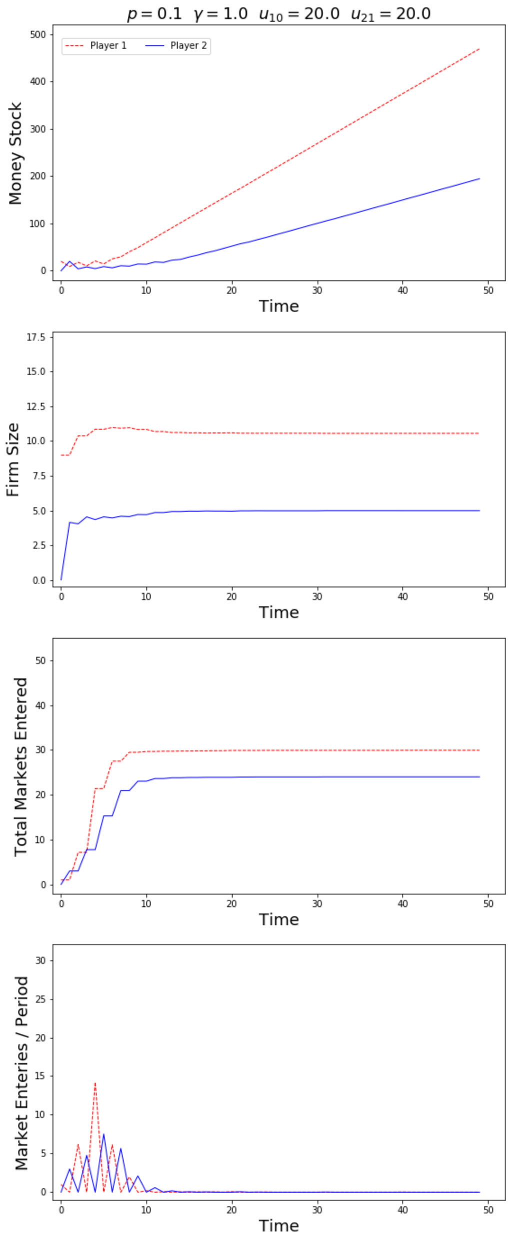


Figure 4: SPNEs under different values of α_K, γ and K

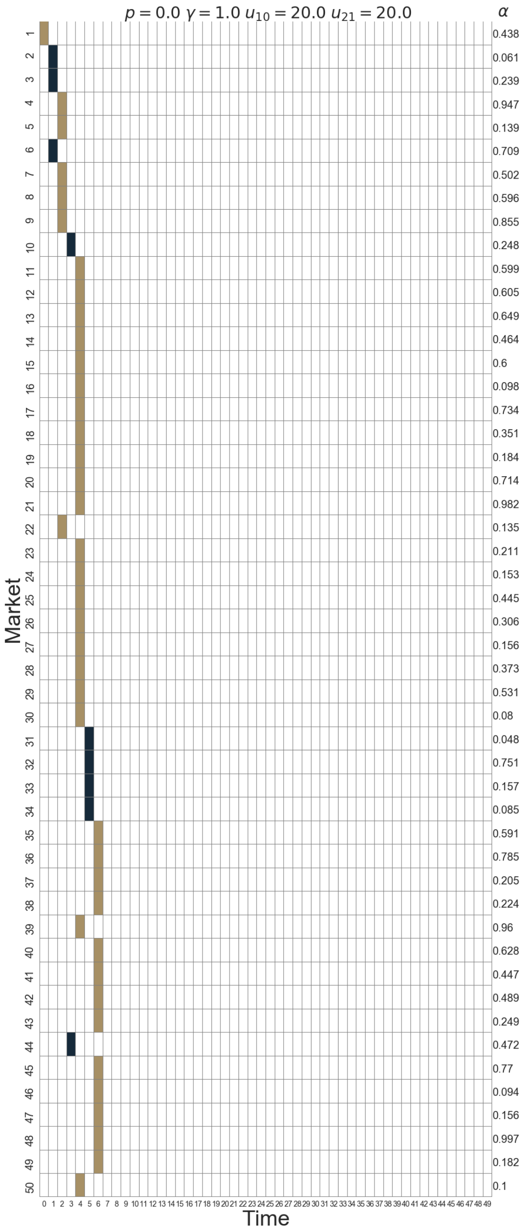


Case 1: $p = 0$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$

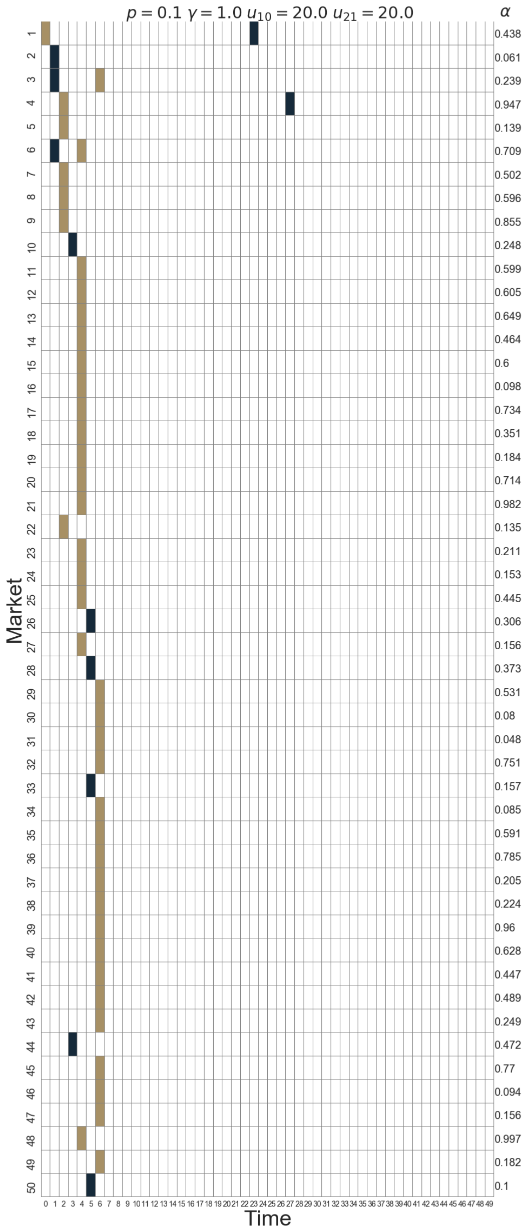


Case 2: $p = 0.1$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$

Figure 5: Simulation results of different p, (aggregation result of 1000 simulations)

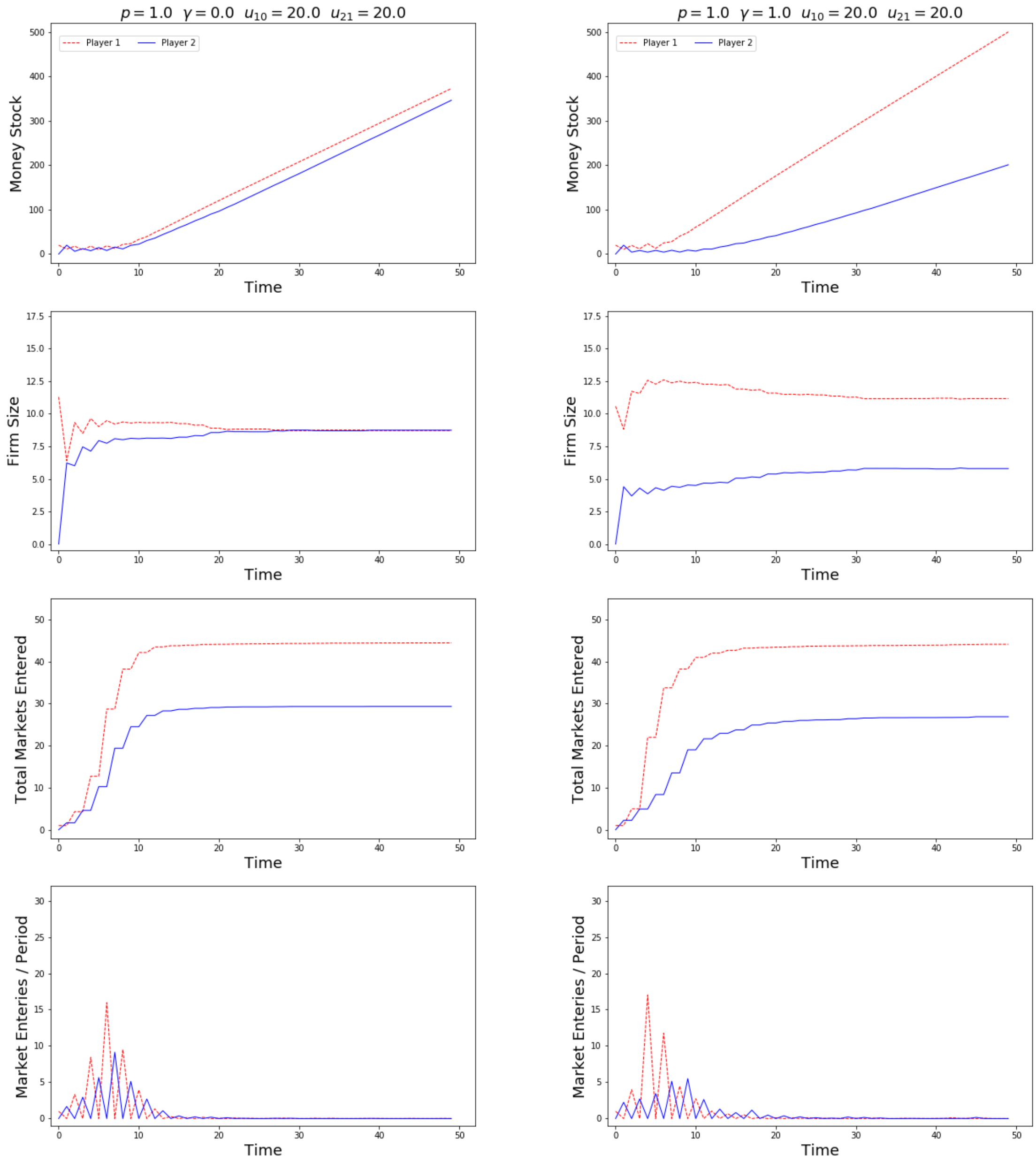


Case 1: $p = 0$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$



Case 2: $p = 0.1$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$

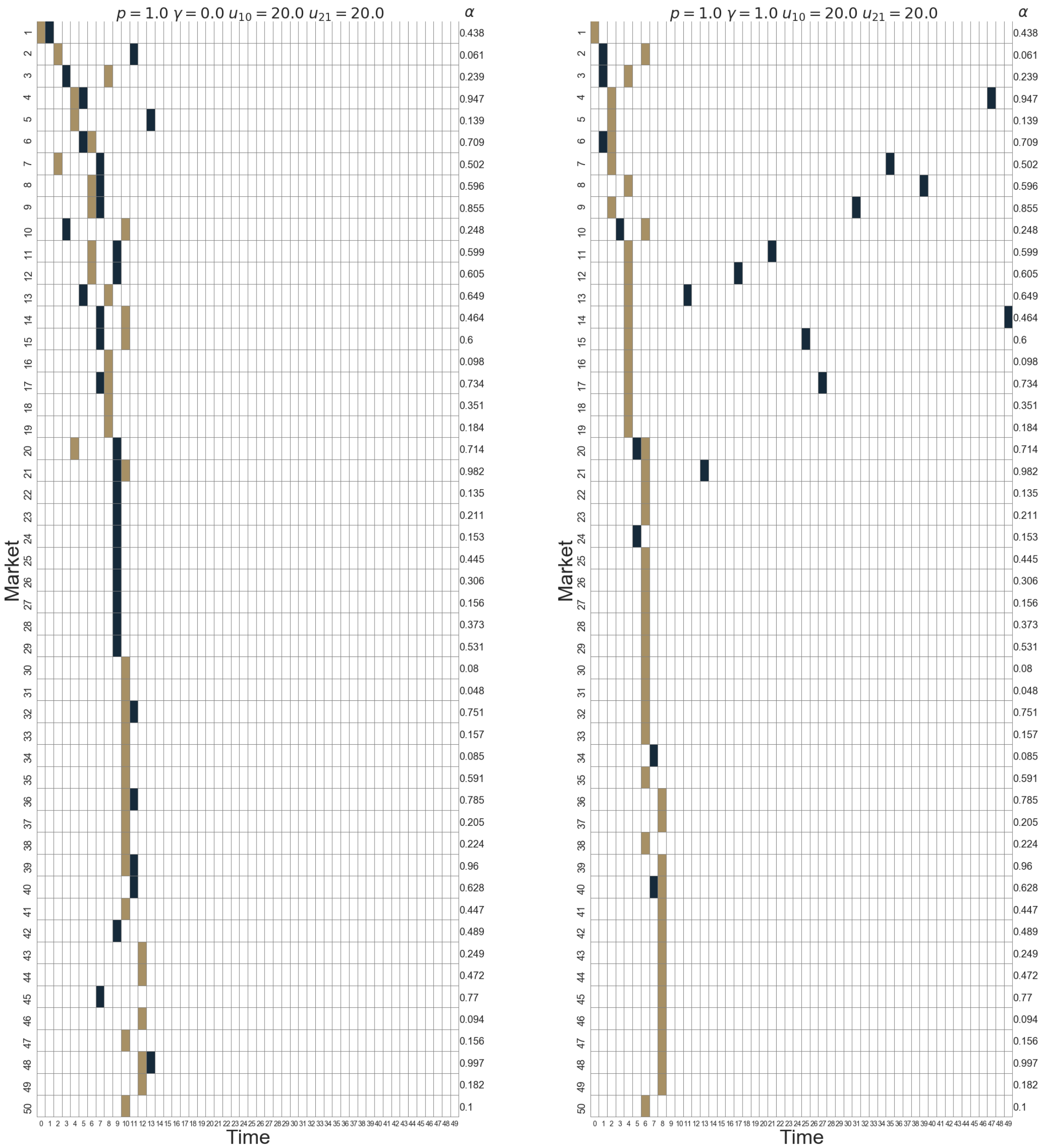
Figure 6: Simulation results of different p



Case 3: $p = 1.0$, $\gamma = 0$, $u_{10} = 20$, $u_{21} = 20$

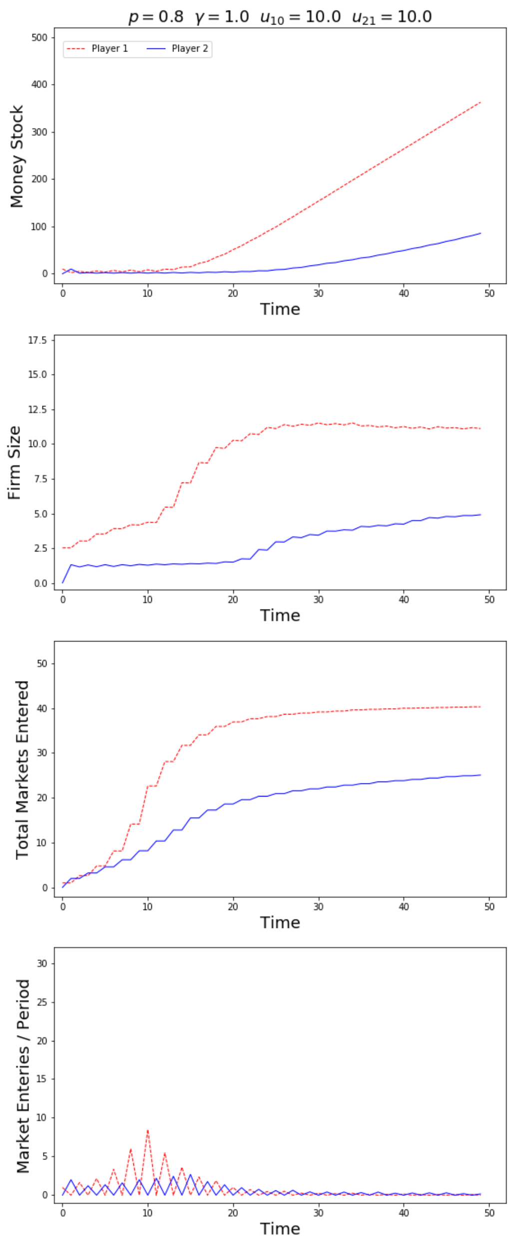
Case 4: $p = 1.0$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$

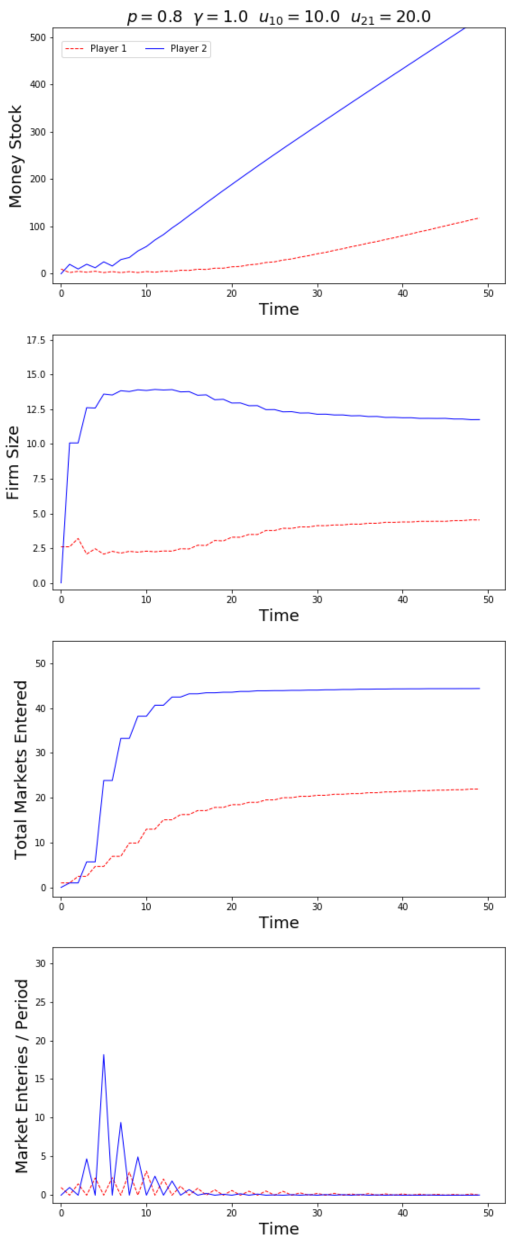
Figure 7: Simulation results of different γ, (aggregation result of 1000 simulations)



Case 3: $p = 1.0$, $\gamma = 0$, $u_{10} = 20$, $u_{21} = 20$ Case 4: $p = 1.0$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$

Figure 8: Simulation results of different γ, (one special case)

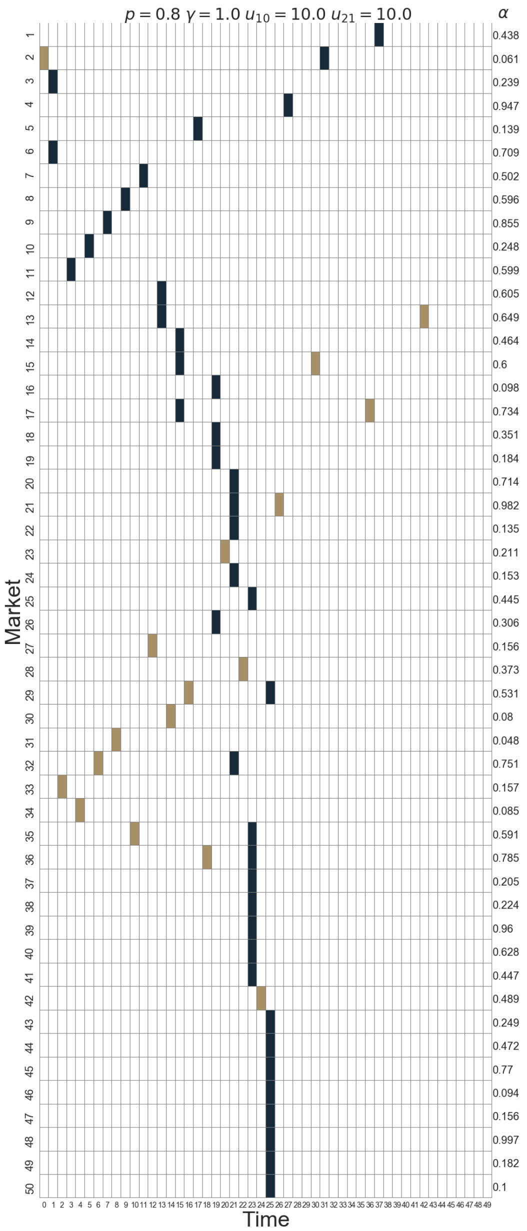




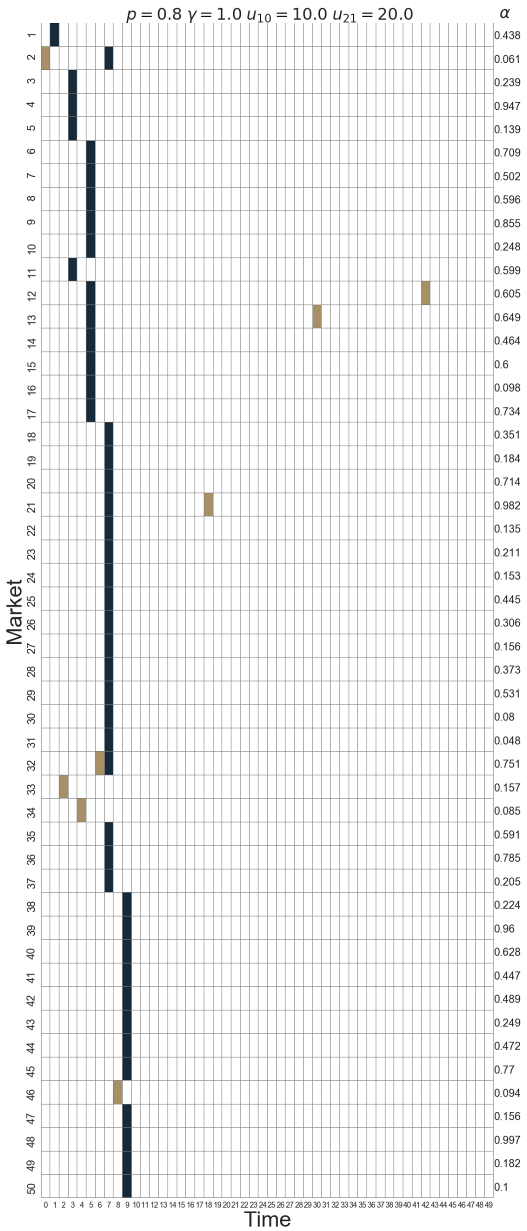
Case 5: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 10$, $u_{21} = 10$

Case 6: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 10$, $u_{21} = 20$

Figure 9: Simulation results of different initial fund, (aggregation result of 1000 simulations)

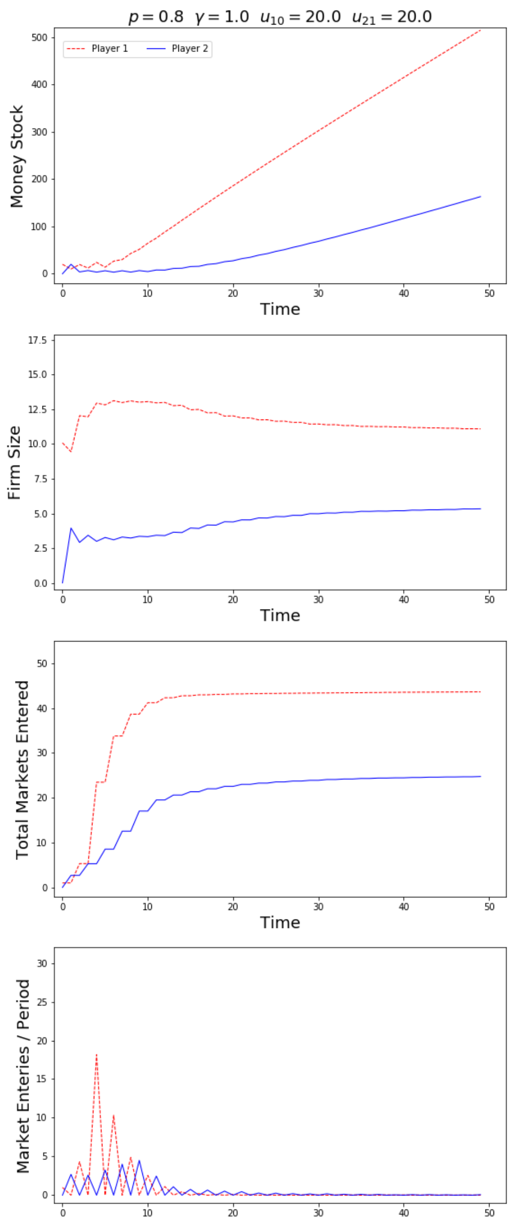


Case 5: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 10$, $u_{21} = 10$

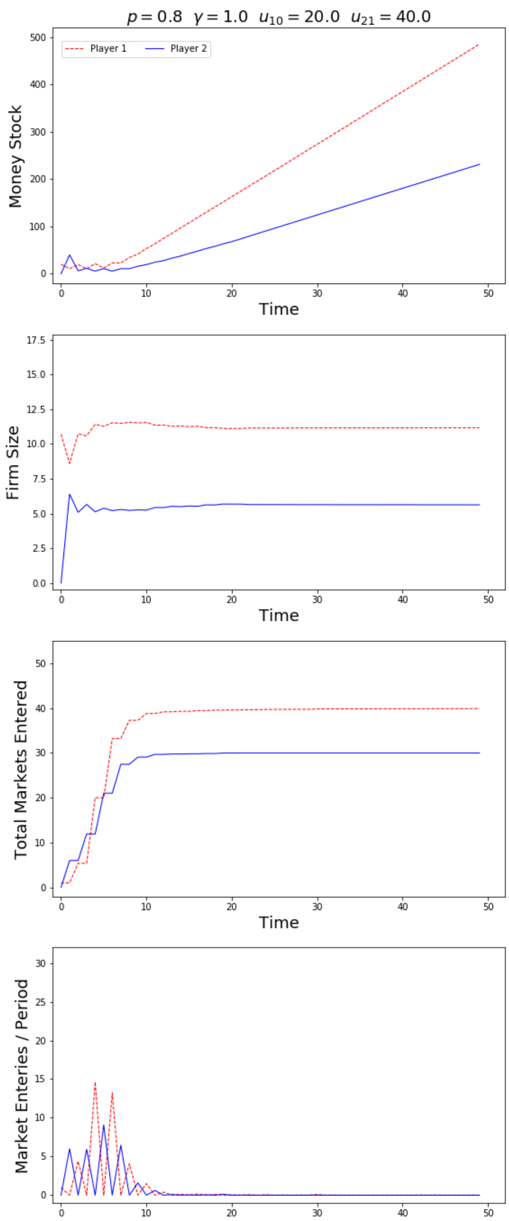


Case 6: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 10$, $u_{21} = 20$

Figure 10: Simulation results of different initial fund, (one special case)

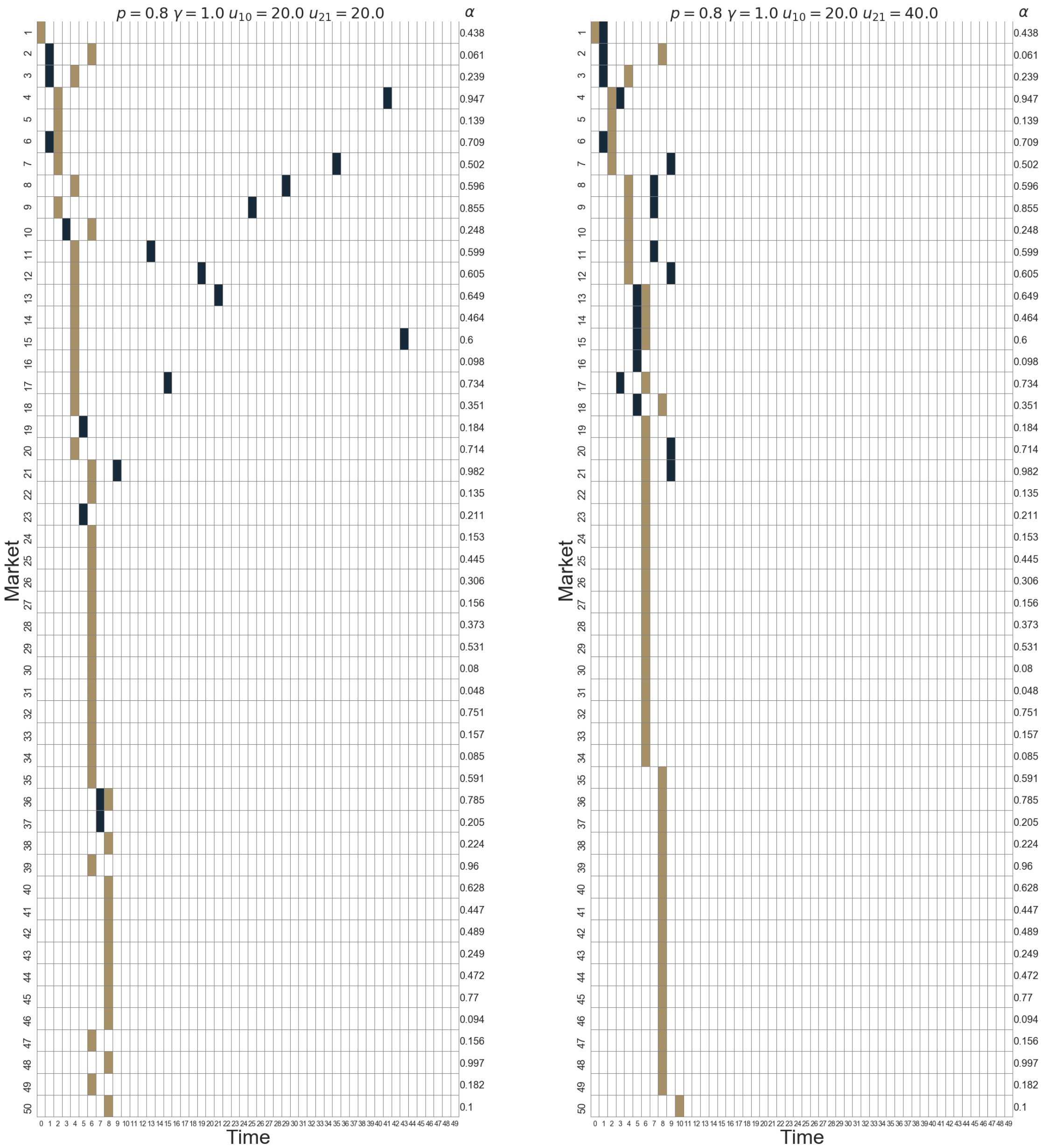


Case 7: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$



Case 8: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 40$

Figure 11: Simulation results of different initial fund, (aggregation result of 1000 simulations)



Case 7: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 20$ Case 8: $p = 0.8$, $\gamma = 1.0$, $u_{10} = 20$, $u_{21} = 40$

Figure 12: Simulation results of different initial fund, (one special case)