

1、求极限. (32 分)

$$(1) \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2+2}} + \frac{1}{\sqrt{n^2+3}} + \cdots + \frac{1}{\sqrt{n^2+n+1}} \right)$$

$$(2) \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^{n+1}$$

$$(3) \lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right)$$

$$(4) \lim_{x \rightarrow 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right)$$

2、求导数. (20 分)

$$(1) \text{ 设 } y = \ln \tan \frac{x}{3} + e^{\sqrt{x}} \sin x^2, \text{ 求 } y'.$$

$$(2) \text{ 设函数 } y = y(x) \text{ 由方程 } e^y - xy = e \text{ 所确定, 求 } y'(0).$$

3、已知 $f(x) = \begin{cases} x^2 \cos \frac{1}{x^2} & x \neq 0 \\ a & x = 0 \end{cases}$ 在 $x=0$ 处连续, 求 a 的值, 并讨论此时 $f(x)$ 在

$x=0$ 处是否可导, 若可导, 则求出 $f'(0)$; 若不可导, 说明理由. (16 分)

$$4、\text{ 设 } \lim_{x \rightarrow +\infty} f'(x) = 3, \text{ 求 } \lim_{x \rightarrow +\infty} [f(x+5) - f(x)]. \quad (16 \text{ 分})$$

5、设某同学在操场跑步时速度函数为 $S(t) = 2t^3 - 9t^2 + 12t$, 时间 $t \in [0, 3]$. 试判断该同学在这段时间内有几次加速过程和几次减速过程? 并给出具体时间段以及加速度为零的时刻. (16 分)

明华林

081825030

L1

-8

设 $x=n+1$
 $\lim_{n \rightarrow \infty} x \rightarrow \infty$

$$\begin{aligned} \text{极限} &= \lim_{n \rightarrow \infty} \left(\frac{x-1}{x} \right)^x \\ &= \lim_{n \rightarrow \infty} \left(1 - \frac{1}{x} \right)^x \\ &= \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{x} \right)^{-x} \right]^{-1} \\ &= e^{-1} \end{aligned}$$

$$\begin{aligned} (3) \text{极限} &= \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{x^3} \\ &= \lim_{x \rightarrow 0} \frac{\sin x (1 - \cos x)}{x^3 \cos x} \\ &= \lim_{x \rightarrow 0} \frac{\tan (1 - \cos x)}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{x - \frac{1}{2}x^2}{x^2} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} (4) \text{极限} &= \lim_{x \rightarrow 0} \frac{x - \ln(1+x)}{x \ln(1/x)} \\ &= \lim_{x \rightarrow 0} \frac{1 - \frac{1}{1+x}}{\ln(1/x) + \frac{1}{1+x}} \end{aligned}$$

$$\begin{aligned} \text{设 } 1+x=t \\ \text{当 } x \rightarrow 0, t \rightarrow 1 \\ \text{极限} &= \lim_{t \rightarrow 1} \frac{t-1}{\ln t + \frac{1}{t-1}} \\ &= \lim_{t \rightarrow 1} \frac{1}{\ln t + \frac{1}{t-1}} \end{aligned}$$

$$\begin{aligned} 2. (1) y' &= \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x} \cdot \frac{1}{3} + \frac{e^x}{2\sqrt{x}} \sin x^2 + e^x \cdot 2x \cos x^2 \\ &= \frac{1}{3 \sin^2 x \cos^2 x} + \frac{e^x}{2\sqrt{x}} \sin x^2 + e^x \cdot 2x \cos x^2 \end{aligned}$$

$$\begin{aligned} \because S'(t) \text{ 为速度} \\ \therefore S''(t) \text{ 为加速度} \\ S''(t) = 12t + 8 \\ \text{当 } S''(t) = 0 \text{ 时} \end{aligned}$$

$$t = \frac{2}{3}$$

1. (2) 解 $y = y(x)$

$y'(0)$ 即当 $x=0$ 时 y 的导数

$$e^y = e$$

$$y = 1$$

$$y'(0) = 0$$

-9

1. 3. 当 $x \rightarrow 0$ 时

$$f(x) \rightarrow \cos \frac{1}{x} = 0$$

若 $x=0$ 处连续

$$\therefore a=0$$

~~$x=0$ 处不相等~~

$$\begin{aligned} f(x) &= 2x \cos \frac{1}{x} + x \cdot \sin \frac{1}{x} - \left(-\frac{2}{x} \right) \\ f(x) &= \frac{\cos \frac{1}{x}}{x^2} \end{aligned}$$

设 $\frac{1}{x} = t$

$$f(x) = \frac{\cos t}{t^2}$$

当 $t \rightarrow 0$ 时

$$f(x) \rightarrow \frac{\cos t}{t^2}$$

~~$t \rightarrow 0$~~

$$f(x) \rightarrow \frac{1}{t^2}$$

~~$t \rightarrow 0$~~

$$f(x) \rightarrow \infty$$

$$\begin{aligned} \text{设 } f(x) &= 3x \\ (\lim_{x \rightarrow 0} [f(x+t) - f(x)]) &= -16 \\ &= 15 \end{aligned}$$

5. 解 $S(t) = 6t^2 - 18t + 2$

$$S'(t) = 0$$

$$t = 1 \text{ 或 } 2.$$

$$\begin{array}{c} \nearrow \\ \searrow \end{array}$$

~~$t \in [0, 3]$ 有 2 次加速 1 次减速~~

$t \in [0, 1) \cup (2, 3]$ 为加速

$t \in [1, 2]$ 为减速