

1、求极限. (32 分)

$$(1) \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2+2}} + \frac{1}{\sqrt{n^2+3}} + \cdots + \frac{1}{\sqrt{n^2+n+1}} \right)$$

$$(2) \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^{n+1}$$

$$(3) \lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right)$$

$$(4) \lim_{x \rightarrow 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right)$$

2、求导数. (20 分)

$$(1) \text{ 设 } y = \ln \tan \frac{x}{3} + e^{\sqrt{x}} \sin x^2, \text{ 求 } y'.$$

$$(2) \text{ 设函数 } y = y(x) \text{ 由方程 } e^y - xy = e \text{ 所确定, 求 } y'(0).$$

3、已知 $f(x) = \begin{cases} x^2 \cos \frac{1}{x^2} & x \neq 0 \\ a & x = 0 \end{cases}$ 在 $x=0$ 处连续, 求 a 的值, 并讨论此时 $f(x)$ 在

$x=0$ 处是否可导, 若可导, 则求出 $f'(0)$; 若不可导, 说明理由. (16 分)

$$4、\text{ 设 } \lim_{x \rightarrow +\infty} f'(x) = 3, \text{ 求 } \lim_{x \rightarrow +\infty} [f(x+5) - f(x)]. \quad (16 \text{ 分})$$

5、设某同学在操场跑步时速度函数为 $S(t) = 2t^3 - 9t^2 + 12t$, 时间 $t \in [0, 3]$. 试判断该同学在这段时间内有几次加速过程和几次减速过程? 并给出具体时间段以及加速度为零的时刻. (16 分)

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1. 求极限

$$(1) \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2+2}} + \frac{1}{\sqrt{n^2+3}} + \dots + \frac{1}{\sqrt{n^2+n+1}} \right)$$

解：原式=

$$(3) \lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right)$$

$$\text{解：} = \lim_{x \rightarrow 0} \frac{\sin x - \sin x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x(1 - \cos x)}{x^3}, \quad \because x \rightarrow 0, 1 - \cos x \sim \frac{1}{2}x^2$$

$$= \frac{1}{2}$$

2. 求导数

$$(1) \text{设 } y = \ln \tan \frac{x}{3} + e^{\sqrt{x}} \sin x^2, \text{求 } y'$$

$$y = \frac{1}{\tan \frac{x}{3}} \times \sec^2 \frac{x}{3} \times \frac{1}{3} + \frac{1}{2}x^{-\frac{1}{2}} e^{\sqrt{x}} \sin x^2 + e^{\sqrt{x}} \cdot \cos x^2 \cdot 2x$$

$$= \frac{\sec^2 \frac{x}{3}}{3 \tan \frac{x}{3}} + e^{\sqrt{x}} \left(\frac{\sin x^2}{2\sqrt{x}} + \cos x^2 \cdot 2x \right)$$

$$(2) \text{设函数 } y = y(x) \text{ 由方程 } e^y - xy = e \text{ 确定, 求 } y'(0).$$

$$\text{解：} y' e^y - (y + xy') = 1$$

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$$3. \text{解：} f(x) = \begin{cases} x^2 \cos \frac{1}{x^2} & x \neq 0 \\ a & x=0 \end{cases}$$

$$\therefore \lim_{x \rightarrow 0} x^2 \cos \frac{1}{x^2} = 0, \therefore a=0.$$

即得

$$f'(x) = -2x \cos \frac{1}{x^2} + x^2 (-\sin \frac{1}{x^2}) \cdot (-2x^{-3})$$

$$= 2x \cdot \cos \frac{1}{x^2} + x \cdot \sin \frac{1}{x^2}$$

$$f'(0) = 0$$

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$$(2) \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^{n+1}$$

$$\text{解：} \because n \rightarrow \infty, \frac{n}{n+1} \rightarrow 1, \therefore \left(\frac{n}{n+1} \right)^{n+1} \rightarrow 1.$$

∴ 原式=1

$$(4) \lim_{x \rightarrow 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right)$$

$$\text{解：} \because x \rightarrow 0, \ln(1+x) \sim x,$$

$$\therefore \lim_{x \rightarrow 0} \left(\frac{1}{\ln(1+x)} - \frac{1}{x} \right) > 0$$

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$$4. \text{设 } \lim_{x \rightarrow \infty} f'(x) = 3, \text{且 } \lim_{x \rightarrow \infty} [f(x+5) - f(x)]$$

$$\text{解：} \begin{array}{c} \uparrow f(x) \\ \downarrow f(x+5) \end{array} \quad \text{令 } x+5=a, x=b.$$

$$\text{即得 } \lim_{x \rightarrow \infty} [f(a) - f(b)].$$

$$\text{又：} b+5=a, \quad \therefore a-b=5.$$

$$f(a) - f(b) = f'(b)(a-b).$$

$$\text{又：} \lim_{x \rightarrow \infty} f'(x) = 3, \therefore f(a) - f(b) = 3 \times 5 = 15.$$

$$\therefore f(a) - f(b) = f(x+5) - f(x).$$

$$\therefore \lim_{x \rightarrow \infty} [f(x+5) - f(x)] = 15$$

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$$5. \quad s(t) = 2t^3 - 9t^2 + 12t.$$

$$s'(t) = 6t^2 - 18t + 12$$

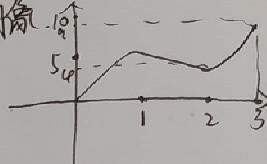
$$= 6(t-2)(t-1).$$

$[0, 1]$ 单调递增
 $(1, 2)$ 单调递减
 $(2, 3]$ 单调递增

$$s(0) = 0, s(1) = 5,$$

$$s(2) = 4,$$

$$s(3) = 9.$$



s(t) 大致图像
 有两次加速和一次减速过程.
 0~1s 加速
 1~2s 减速
 2~3s 加速

$$\text{令 } s'(t) = 0, \text{ 即 } 6(t-2)(t-1) = 0, \quad t_1 = 1, \quad t_2 = 2.$$

\therefore 1s, 2s 时, 加速度为0.